THE MAPINATOR CLASSIFICATION

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Abstract. The paper uses data from the mapinator project to estimate how finely the market is able to partition the skill distribution of new minted economics phds. Using a modified directed search model, and a collection of off the shelf clustering algorithms, we argue that the market separates the supply side into a collection of communities of universities and treats graduates from two universities in the same community as if they had the same market value. The communities identified by these algorithms form the mapinator classification. The theoretical model provides conditions under which a tier structured market can be detected from an aggregated adjacency matrix.

A common approach used to model decentralized trading markets is to imagine that buyers and sellers have continuously distributed types. A production function defines the surplus that a particular pair of types can earn by matching (or trading) with each other. If utility is transferable, and the surplus function is super modular in types, simple posted price models will support equilibrium with decentralized trade in which matching is assortative and trades are efficient.[1] Trading frictions may prevent fully efficient matching. However under reasonable conditions trade is often constrained efficient.[2]

This paper is based on the premise that it is too costly for market participants to finely distinguish beween applicants. The question we address is whether it is possible to understand market outcomes as the result of a very coarse partition of graduates into types that is based solely on public information. More precisely, we assume that universities form communities in the sense that graduates from universities in the same community will experience the same distribution of outcomes.

A related question is whether graduates from universities in the same community can be viewed as if they were commodities being traded in a distinct 'market'. What it means to constitute a distinct market is that employers view graduates from the same community as having a common value. Communities and their respective values are then determined by employers.

Ultimately, this paper and the associated mapinator project itself are an attempt to measure these market perceptions.

The economics job market is a good environment in which to study these issues for a couple of reasons. First, the market itself is very 'static'. As most academic jobs begin in mid summer, hiring tends to concentrated in time, roughly slightly before, and for a few months after the new year. Offers are not accepted or rejected

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[1]Becker (1973); Hopkins (2005); Siow (2003). See also the survey by Chade, Eeckhout, and Smith (2017).

[2]Peters (2010); Eeckhout and Kircher (2010)

immediately, so making one involves a costly commitment for employers. Evaluating quality of graduates is difficult. Unlike the standard assortative matching models in the literature, coarse information creates coordination frictions.

These issues suggest that a relatively standard frictional matching model is a reasonable way to model market outcomes. We develop this model in the first part of the paper and use it to predict what the cross market adjacency matrix should look like. In the current version of the paper we use this to focus on estimation of the community structure - i.e., identifying which graduates belong in which markets. Ultimately we want to estimate the values the market assigns to these graduates as well.

However, the community structure itself provides useful insights. For example, we find that two academic communities dominate production of graduates. The identities of these communities will not be a surprise. However two things stand out. The number of graduates from these two communities is so large that they are hired widely outside the academic research community. Only half the graduates of top tier universities are hired by institutions that graduate ph'd students.

In fact graduates from these top 'tiers' have a relatively small chance of being rehired by academic departments.

The Adjacency matrix. The easiest way to understand the data is to look at the visualization at https://sage.microeconomics.ca. It shows a directed graph that illustrates the transaction flows of economics ph'd graduates between universities from 2008 to the present - essentially a visualization of the worldwide market. As with all network data, it is hard to interpret visually.

To see how this paper will attempt to understand the data, it is useful to try to visualize it using an *adjacency matrix.* The data from the directed graph can be converted into a *raw* adjacency matrix by creating a matrix which has one row for every market participant on the demand side of the market - every university, every private sector firm and every government adjacency who hires phd graduates in economics.

The matrix also has one column for every university that graduates students - the supply side of the market. Any cell (i, j) of this matrix contains the number of graduates from the university represented by cell j who were hired by the institution represented by row i.

In the data used here, there are 752 universities that both produce graduates themselves, and also hire graduates from other universities. There are 627 universities that hire graduates at the assistant professor level, but don't themselves produce any graduates. We'll refer to them here as *teaching universities.* There are 195 private sector institutions and 156 government institutions as well. The other three rows in the matrix track placements at universities which are not at the assistant professor level.

The number of columns in the raw adjacency matrix would be just 752, the number of universities that place graduates.[3]

Let A represent this raw adjacency matrix, with representative entry a_{ij} interpreted as above, the number of hires by i from institution j. The basic objective in the paper is to sort the various institutions into communities based on the idea that members of the same community will have similar placements.

[3]There are many teaching universities that are also good research departments, for example Vassar College. Nothing about this paper refers to or measures research quality.

A *model* is a finite collection of communities $\mathcal{C}$ (including a specification of the members of each community). The paper attempts to identify the best model using maximum likelihood.

Once we have chosen a model with a small number of communities, we can create something we'll call the compressed adjacency matrix. Each cell a_{ij} in the compressed matrix records the number of graduates from community j that were hired by community i. Of course, the compressed matrix has the advantage that it makes the raw data easier to read. However, it also makes it possible to look for patterns in the data that can reveal some of the economics of the underlying data. If the communities have a tier structure, we'll show how that structure will manifest itself in the compressed matrix. We can then use the tier structure to estimate some of the economic parameters that underly that structure.

A priori we don't know the right community model. If we could find that model, we don't know whether it will have a tier structure. However, to clarify, we'll just illustrate the compressed adjacency matrix associated with the 'best' model we estimated. It appears in the table below.

We'll defer for the moment the question of what 'best' model means so we can explain how to interpret the model. Our best model has 5 communities of research universities. One difference between what we are doing here, and the analysis in some of the statistical literature (for example, Karrer and Newman (2011); Peixoto (2014); Wang and Bickel (2017)) is that our communities interact in different ways. Research universities graduate economists and also hire them. Private sector businesses hire them but don't graduate them. Research universities who are in the same community in our model should have similar placement rates in the private sector.

Looking down the column for Tier 3, for example, you can see that there were 43 graduates of Tier 3 universities from this sample of the data who were hired by Tier 1 universities. Tier 1 universities hired mostly from other Tier 1 universities.

	Tier 1	**Tier 2**	**Tier 3**	**Tier 4**	**Tier 5**	**Row Totals**
Tier 1	703	104	43	15	2	867
Tier 2	660	317	102	37	11	1127
Tier 3	620	564	256	95	27	1562
Tier 4	442	630	326	219	35	1652
Tier 5	20	59	61	61	67	268
Other Academic	416	404	355	317	220	1712
Government	545	441	247	117	48	1398
Private Sector	672	388	202	103	34	1399
Teaching Universities	380	555	452	274	135	1796
Postdoc	559	574	560	369	229	2291
Sessionals	141	210	169	164	119	803
Column Totals	5158	4246	2773	1771	927	14875

The Model. For the purposes of this paper, what a tier structure means is that the value of graduates from the same tier is the same. As a consequence, graduates of the highest tier, tier 1 here, have the highest value. The community clustering algorithms we are using know nothing about tiers. When the algorithm spits out what it thinks is the best *model* the communities are just given arbitrary names. The corresponding compressed adjacency matrix shows no obvious patterns at all.

So our first objective is to turn to matching theory to ask what a tier based adjacency matrix should look like.

The model is a frictional matching model that combines the ideas in Julien, Kennes, and King (2000) with the mathematical approach in Peters (2010). Universities make offers to applicants without knowing exactly what other offers the applicant might have. Offers are rejected with positive probability as a result so universities face a risk return trade off when they make their offers. The adjacency matrix here gives a hint - one community graduates more students than any of the others. The matrix above is a re-ordering of the compressed adjacency matrix delivered by our algorithms according to the rule that the community with the most graduates is also the highest tier.

Matching theory provides more strucute. We start with a collection of organizations, $D \cup S$. The set D consists of universities that produce graduate students. These graduates can be hired by any organization, including any university. The set S consists of organizations that hire graduates but don't graduate their own applicants, for example, government, the private sector and teaching universities. We'll refer to them as sinks. So in the trading network, universities in D act as both buyers and sellers, organizations in S are buyers only.

The organizations in $D \cup S$ try to hire graduates by making offers to them. A university who hires a graduate from a university that is in community t receives a payoff v_t. If an organization's offer is rejected, it receives a payoff normalized to 0. Offers have values that are independently and randomly drawn from a distribution that depends on which community the hiring organization comes from.

Once organizations have made their offers to graduates, each graduate accepts the most valuable offer they receive. Any randomly drawn offer, x, has the same value to all graduates. It is assumed that the payoff a graduate receives from accepting an offer x is just x. An applicant who does not receive any offer receives a payoff normalized to zero.

Hiring institutions are organized into communities. We'll assume there are K communities in all, with $k < K$ academic communities. Since each institution in any academic tier produces graduates that all have the same value we just refer to each academic community as a tier and number them from 1, the tier with the highest value, to k, the low value tier. The nonacademic communities are arbitrarily assigned consecutive indices that exceed k. The number of graduates from academic tier t us m_t, with $m = \sum_{t=1}^{k} m_t$. When needed we'll use the notation m to refer both to the set of graduates and the number of graduates.

Offers for institutions in each community i have values that are independently drawn from the same distribution F_i which we'll assume is monotonically increasing and differentiable with support on $[0, 1]$.

Institutions who are hiring know which community they belong to, but they don't know the communities of their competitors. Each potential competitor is assumed to be in community i probability ρ_i. The probability that each competitor has an offer whose value is less than or equal to x is then

$$F(x) = \sum_{t=1}^{K} \rho_t F_t(x).$$

Offers and matching. A market is a matching game of incomplete information. Organizations commonly know the set of graduates on the market, and make an offer to one and only one of them. After receiving offers, applicants accept the best offer they receive. Organizations privately know the value of their own offer, but not the offer values of the other organizations. They see the number of organizations participating and believe that each of them is a member of tier t with probability ρ_t.

A strategy rule for an organization j is a mapping from $[0,1] \to \triangle(m)$. Let $\tilde{\pi}_j^i(x)$ be the probability with which organization j with an offer of value x makes an offer to applicant i. We'll restrict attention to equilibrium that satisfy two symmetry restrictions: first, since every graduate from tier t is viewed to have the same ex ante value v_t we'll assume that $\tilde{\pi}_j^i(x) = \tilde{\pi}_j^{i'}(x)$ for every x and for every pair i and i' in m_t; and second, $\tilde{\pi}_j^i(x)$ does not depend on j.

Exploiting this symmetry we can again, with a slight abuse, simplify the strategy rule and write $\tilde{\pi}_t(x)$ to be the probability with which an organization with an offer of value x makes it to *some* applicant from tier t. The probability the organization makes this offer to any particular applicant in tier t is $\frac{\tilde{\pi}_t(x)}{m_t}$:

A symmetric equilibrium for this game is a vector valued rule $\{\tilde{\pi}_t(x)\}_{t\in T}$ such that $\sum_t \tilde{\pi}_t(x) = 1$ for all x and $\tilde{\pi}_t(x) > 0$ only if

$$v_t\left(1-\int_x^1 \frac{\tilde{\pi}_t(\tilde{x})}{m_t} dF(\tilde{x})\right)^{n-1} \geq v_{t'}\left(1-\int_x^1 \frac{\tilde{\pi}_{t'}}{m_{t'}}(\tilde{x})\, dF(\tilde{x})\right)^{n-1}$$

for every $t' \neq t$ for almost every $x \in [0,1]$.

Equilibrium. The problem is now to characterize the equilibrium. This problem is effectively the same problem as the one discussed in Peters (2010), except that the distribution of types across graduates has only finite support.

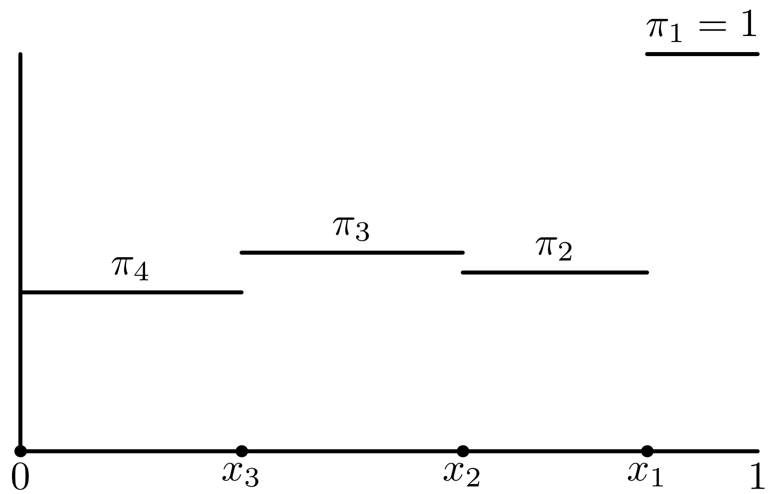
Recall that tiers are ordered such that $v_1 > v_2 > \dots v_k$.

Theorem. *There is a finite collection of cutoffs $\{x_0, x_1, \dots, x_k\}$ and a set of positive constants $\{\pi_j\}_{j=1,k}$ such that $x_0 = 1$, $\pi_1 = 1$, and for each $t > 0$ if $x \in [x_t, x_{t-1})$*

$$\tilde{\pi}_j(x) = \begin{cases} \pi_j \prod_{j<i\leq t}(1-\pi_i) & j \leq t \\ 0 & \textit{otherwise.} \end{cases} \tag{0.1}$$

Any department or institution who has an offer x will randomize when choosing where to make its offer, except when $x \in [x_1, 1)$. In the latter case the department will make its offer for sure to some applicant in tier 1. For offers whose value exceeds x_t offers are made with positive probability to each tier whose applicants have a value that is at least v_t.

The following figure illustrates what the theorem says:



The horizontal axis represents the various values for x, the value of an organization's offer. The vertical axis measures a probability between 0 and 1.

When $x \in (x_1, 1]$ the value of the organization's offer is very high, the offer is made with probability 1 to a graduate from tier 1. Within the tier, the offer is made with equal probability to each graduate from that tier.

If the organization has an offer x in the interval $(x_2, x_1]$ then the offer is made to a tier 2 graduate with probability π_2 and to a tier 1 graduate with probability $(1 - \pi_2)$. It is worth noting that by construction, conditional of choosing to make an offer to a tier one applicant, the organization behaves exactly as if it had an offer whose value were in the interval $(x_1, 1]$.

This same logic applies to the all other offers. For example if, as in the diagram, there are four tiers and an offer is in the interval $[0, x_3]$,the offer goes to a tier 4 applicant with probability π_4. However, conditional on choosing to make an offer to an applicant in one of the higher tiers, say tier 2, the organization behaves exactly the same way as it would have if its offer value were in the interval $(x_2, x_1]$, that is, it would make an offer to a tier 2 graduate with probability π_2.

So the unconditional probability with which such an offer goes to a tier 2 graduate is $\pi_2 (1 - \pi_3) (1 - \pi_4)$.

Theorem is proved in full in the appendix. The logic is based on Peters (2010).

Using the main theorem, we get the following corollary which is proved in Lemma 0.1 as part of the proof of Theorem :

Corollary 1. *The constants* $\{\pi_t\}_{t=1,K}$ *and* $\{x_t\}_{t=0,K-1}$ *can be found by solving the following system of equations with* $\pi_1 = 1$, $x_k = 0$ *and* $x_0 = 1$.

$$\pi_t = \frac{\pi_{t-1}}{\pi_{t-1} + \left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} \frac{m_{t-1}}{m_t}}$$

and

$$v_{t+1} = v_t \left(1 - \frac{\pi_t}{m_t} \left(F(x_{t-1}) - F(x_t)\right)\right)^{n-1}$$

for each $t = 1, K - 1$.

The proof of Lemma 0.1 solves this system recursively which verifies existence and uniqueness.[4]

Large Markets. The equilibrium can be further simplified by taking advantage of the fact that there is a lot of placement data. Specifically, we'll let the number of hiring organizations, n, go to infinity, while the ratio of graduates from each tier, m_t to the number of available positions remains constant.[5] In particular, we'll assume that $m_t = \alpha_t (n-1)$ where the coefficient α_t is constant while n becomes large.

For example, the x_t satisfy

$$v_{t+1} = v_t \left(1 - \frac{\pi_t}{m_t} \left(F(x_{t-1}) - F(x_t)\right)\right)^{n-1}.$$

Substituting this ratio gives

$$v_{t+1} = v_t \left(1 - \frac{\pi_t}{\alpha_t (n-1)} \left(F(x_{t-1}) - F(x_t)\right)\right)^{n-1}$$

From this, the sequence of equilibrium outcomes π_t and x_t depend on n. Since, these values can be found recursively and the various expressions are all continuous, we can take limits recursively to get the following Lemma:

Theorem 2. *The limit values of π_t and x_t exist for each $t = 1, k-1$ and are given by*

$$\lim_{n\to\infty} \pi_t^{(n)} = \frac{\alpha_t}{\sum_{s=1}^{t} \alpha_s} \tag{0.2}$$

for $t = 1, k-1$, while $\lim_{n\to\infty} x_t^{(n)} = x_t^{\infty}$ is given by the solution to

$$F(x_t^{\infty}) = 1 - \sum_{j=1}^{t-1} \left(-\log\left(\frac{v_{j+1}}{v_j}\right)\right) \sum_{s=1}^{j} \alpha_s \tag{0.3}$$

for $t = 1, k-1$, while $x_K^{\infty} = 0$.

The next result is a shortcut that is used in the trading probability functions below. Recall that the strategy rule of a university when making an offer is a piecewise constant function. The probability that an offer is made to an applicant from some tier t is some constant π_t multiplied by the probability that the applicant doesn't make an offer to a lower tier applicant. In the limit this probability has the same kind of structure as described above.

Corollary 3. *If $x \in \left[x_{i-1}^{(n)}, x_i^{(n)}\right]$ and $t \leq i$, $\tilde{\pi}_t^{(n)}(x) = \pi_t^{(n)} \prod_{s=t+1}^{i} \left(1 - \pi_s^{(n)}\right)$. Then for any $x \in \left[x_{i-1}^{\infty}, x_i^{\infty}\right]$*

$$\lim_{n\to\infty} \pi_t^{(n)} \prod_{s=t+1}^{i} \left(1 - \pi_s^{(n)}\right) = \frac{\alpha_t}{\sum_{s=1}^{i} \alpha_s}.$$

[4] Here the term 'active tier' means tiers which graduate students who are hired with some probability.

[5] It is impossible to know exactly what the number of positions is in each trading year in the economics job market because there are many different websites that universities use to advertise their openings. However if it is assumed that the market is roughly in balance, a reasonable guess is somewhere between 3 and 4 thousand openings each year.

One of the limitations of the outcome data is that it records successful offers, but says nothing about the offers that were rejected. One of the most important considerations for a department when making an offer is the probability it will be accepted. The only way to know this is to infer it from the outcome data. To this end we have the following result:

Theorem 4. *For* $x \in [x_{i-1}, x_i]$

$$Q_t(x) = e^{-\left(G_i^t(x) + \kappa_i^t\right)}$$

where

$$G_i^t(x) = \left(F(x_{i-1}) - F(x)\right)\frac{1}{\sum_{s=1}^{i}\alpha_s}$$

and

$$\kappa_i^t = \sum_{j=t}^{i-1}\left(F(x_{j-1}) - F(x_j)\right)\frac{1}{\sum_{s=1}^{j}\alpha_s} =$$

$$\sum_{j=t}^{i-1}\left(-\log\left(\frac{v_j}{v_{j-1}}\right)\right)\sum_{s=1}^{j-1}\alpha_s$$

The Sampling Distribution of the Adjacency Matrix. From Corollary 1, the cross tier placement rates can be calculated from knowledge of the various distributions F_i.

The probability an organization who makes an offer of value x to an applicant in tier t successfully hires the applicant is given by

$$Q_t(x) = \left(1 - \int_x^1 \frac{\tilde{\pi}_t(\tilde{x})}{m_t} dF(\tilde{x})\right)^{n-1}. \tag{0.4}$$

Using Theorem and Corollary 1 we can write this as

$$Q_t(x) =$$

(0.5)

$$\left(1 - \left(F(x_{i-1}) - F(x)\right)\frac{\pi_t}{m_t}\prod_{l=t+1}^{i}(1-\pi_l) - \sum_{j=t}^{i-1}\left(F(x_{j-1}) - F(x_j)\right)\frac{\pi_t}{m_t}\prod_{l=t+1}^{j}(1-\pi_l)\right)^{n-1}$$

when $x \in [x_i, x_{i-1}]$

The probability that an organization from tier i hires an applicant from tier t is given by the following formula:

$$q_i^t = \int_0^1 \tilde{\pi}_t(x')\, Q_t(x')\, dF_i(x') \tag{0.6}$$

which can be written as

$$q_i^t = \sum_{s=t}^{k}\int_{x_s}^{x_{s-1}} \pi_t \prod_{l=s+1}^{k}(1-\pi_l)\, Q_t(\tilde{x})\, dF_i(\tilde{x}). \tag{0.7}$$

A randomly drawn offer made by a university from tier i will result in the offer being accepted with probability q_i^t which, of course depends on n. When this occurs, the hire is recorded in the adjacency matrix by adding 1 to the matrix in cell (i, t). Ultimately the sum recorded in each cell of the adjacency matrix is a random variable. It is tempting to assume that this number has a binomial distribution. However q_i^t is just the probability with which the university expects to hire from tier t. In the finite game, the probability a 1 is added to a cell depends on the number currently in the cell, since a previous hire will remove one of the available applicants from tier t.

Similarly the probability that a 1 is added to cell (i, t') depends on how many hires have been added to cell (i, t), since a previous hire removes a competitor for the next university. As a consequence, the best we can do to understand the empirical distribution of outcomes in the adjacency matrix is to use a large market approximation since the impact of outcomes of others has a vanishing impact on the probability of any other offer being accepted.

Then using our previous limit results we get:

Corollary 5. *The limit*

$$\lim_{n\to\infty} q_i^t =$$

$$\sum_{i=t}^{k} \int_{x_i}^{x_{i-1}} \frac{\alpha_t}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^t(x)+\kappa_i^t\right)} dF_\tau\left(\tilde{x}\right)$$

At this point the results are such that we can begin to provide a description of the adjacency matrix. Assuming we could sample a finite number of M_i outcomes associated with universities from tier i, as they appear in the adjacency matrix above, then each outcome will be independently placed in one of the k cells representing the k different tiers that produce graduates. The outcome will be added to cell t with probability q_i^t. Failures aren't observable, so they aren't recorded in the adjacency matrix. The random vector for row i that would result would have a multinomial distribution with parameters M_i,k, and

$$\left\{ \frac{q_i^t}{\sum_{s=1}^{k} q_i^s} \right\}_{t=1,k}$$

While sampling from the distribution we can't control the tier from which the observation is drawn, with each draw being an institution from tier i with probability ρ_i. Then after M draws from this distribution the adjacency matrix becomes a random vector of dimension $K \times k$ distributed multinomial with parameters M, $K \times k$ and

$$\left\{ \frac{\rho_i q_i^t}{\sum_{j=1}^{K} \sum_{s=1}^{k} \rho_j q_j^s} \right\}_{i=1,K;t=1.k} . \tag{0.8}$$

What the Adjacency Matrix should look like. The raw parameter values in (0.8) impose only one basic restriction - values are distinct and ordered. However one part of this assumption is that when an applicant receives and offer of value x, his or her assessment of that offer doesn't depend on what tier the offer comes from. In other words, tier membership itself doesn't matter to an applicant.

As illustrated in Table , the rows of the matrix look quite different for different tiers. For example, tier one schools seem to be a lot more successful than other schools at hiring their own graduates. The way this apparent tier effect is measured is by assuming that the tier draw their offer values from different distributions. Since tier one schools seem more successful than others, this is attributed to the fact that the distribution of offers stochastically dominates the distributions of the other tiers.

To this end we have the following

Proposition 6. *Suppose that $F_1 \succ F_2 \cdots \succ F_k$ (where $\succ$ is in the sense of stochastic dominance). Then $q_s^1 > q_{s'}^1$ whenever $s' > s$*

This theorem just says that top tier universities are more likely to hire top tier graduates than second tier universities are when the offer distribution for the top tier stochastically dominates the offer distribution of second tier. Similarly for the lower tiers. If one of them has a stronger offer distribution, they will be more successful hiring top tier students (though not as successful as tier 1 itself would be). This logic can't be used to address the question whether tier 1 would be more likely to hire from tier 2 than tier 2 is because the hiring probability for tier 1 at tier 2 is not increasing in its offer value.

In any case, a stronger restriction on the appearance of the adjacency matrix can be had by considering the supply side. The main theorem is:

Proposition. *Suppose that for $t > 1$*

$$\frac{\alpha_{t-1}}{\alpha_t}\frac{v_t}{v_{t-1}} > 1.$$

Then $q_\tau^{t-1} - q_\tau^t > 0$.

The value $\frac{v_t}{v_{t-1}}$ is a fraction, since values are decreasing in i. Making offers to higher value applicants is risky though because there is more competition so that offers may not be accepted. The condition used in the theorem says roughly that the number of students that graduate from a tier rises more rapidly than the value does - a type of elasticity condition. When it holds, all the tiers will hire more from higher tiers than lower ones.

Referring back to Table, it is easy to see from the column totals that the higher the tier, the more graduates they have. This is consistent with Proposition . However, since we don't know the values that the market assigns it is impossible to verify the assumption in Proposition directly.

Proposition also makes a prediction that is constant across tiers. That is, if the condition in the assumption holds for any pair of tiers t and $t-1$, then all tiers will hire more from tier $t-1$ than they do from tier t. In the adjacency matrix in Table , this prediction holds for tiers 1 to 3, government and the private sector. It fails to hold for tiers 4 and 5, and for teaching universities. This isn't inconsistent with the model, which is silent on what happens when an initial offer is rejected.

Lower tier universities, which have relatively few placements, and teaching universities likely need to fill teaching needs whatever the tier they hire from. So they are more likely to make additional offers after their first offers are rejected. Since many tier 1 graduates are hired early in the market, it is likely that the condition in Proposition no longer holds once the teaching market clears.

Finding the number of communities. The next step in the process is to use a method of find the appropriate community model. We use the *stochastic block model* to do this, following Karrer and Newman (2011); Peixoto (2014); Wang and Bickel (2017).

In most of the literature the approach is to take the entire network graph and create from it a square adjacency matrix which has as many rows and columns as their are nodes in the graph. In other words there is a distinct row and column for every institution that either hires or graduates phd students in economics. This isn't what we want here because we already know the indentities of the institutions that graduate phd students (and the ones who don't). As some institutions only hire, large blocks of this adjacency matrix would be full of zeros.

Perhaps more important, we already know pretty well how to allocate the non-academic institutions in the placement data. We expect all the government institutions and teaching universities, for example, to hire in pretty much the same way as other institutions in their communities. For this reason, we'll work with an adjacency matrix which has one row for every institution, but only has a column for every academic university that graduates students.

We'll refer to this adjacency matrix as the *raw* adjacency matrix and continue to refer to it as A, with enteries a_{ij} giving the number of graduates from *academic* institution j that were hired by institution i.

Since we will assign the institutions who don't produce graduates to communities manually, finding the best community model involves choosing the number of different academic communities, then assigning each iniversity to one of the communities.

Once we have an assignment of institutions to communities, then we can create a new called the *compressed* adjacency matrix. Each element i, j of this matrix will contain the number of graduates from academic community that were hired by academic community i. the dimension of this matrix is $K \times k$ where k is the number of academic communities and K is just the sum of k and the number of communities we have predefined to contain the institutions who don't graduate students. Table in the introduction is a compressed adjacency matrix with 5 academic communities. Since the number of non-academic communities is fixed, we can refer to a community $\mathcal{C}^k$ consisting of k academic tiers. We don't know which institutions belong to each tier, we have to estimate that.

So we can define as estimate $\hat{\mathcal{C}}^k$ as a particular assignment of universities into the k communities. The elements c, c' of the compressed matrix $A[\hat{\mathcal{C}}^k]$ associated with this assignment is given by

$$\sum_{i \in c} \sum_{j \in c'} a_{ij}.$$

To evaluate and estimate we assume that the institutions in the same community will have the same success at hiring from the other communities. So if we take the random variable

$$\hat{a}_{ic'} = \sum_{j \in c'} a_{ij}$$

we can assume it has a poisson distribution with mean $\lambda_{cc'}$ which is the same for each member i of community c. Then the probability that community member i hires $\hat{a}_{ic'}$ graduates from community c' is

$$\frac{e^{\lambda_{cc'}} \lambda^{\hat{a}_{ic'}}}{\hat{a}_{ic'}!}$$

Each member of community i generates a single observation from this distribution. The maximum likelihood estimator for $\lambda_{cc'}$ is then

$$\hat{\lambda}_{cc'} = \frac{\sum_{i \in c} \hat{a}_{ic'}}{n_c}$$

where n_c is the number of institutions in community c.

Using this, we can compute the likelihood of the set of observations $\hat{a}_{ic'}$ given the estimate $\hat{\lambda}_{cc'}$. This is just

$$\prod_{i \in c} \frac{e^{\hat{\lambda}_{cc'}} \hat{\lambda}^{\hat{a}_{ic'}}}{\hat{a}_{ic'}!}.$$

Now we need to do this calculation for all institutions c and all academic institutions c' to give the likelihood of the compressed adjacency matrix given the community $\hat{\mathcal{C}}^k$, which is

$$G(\hat{\mathcal{C}}^k) = \prod_{c \in \mathcal{C}} \prod_{c' \in \mathcal{C}} \frac{e^{\hat{\lambda}_{cc'}} \hat{\lambda}^{\hat{a}_{cc'}}}{\hat{a}_{cc'}!}. \tag{0.9}$$

Every assignment of universities to tiers will generate a different value in (0.9). Our estimator is going to be the assignment of universities to communities that maximizes (0.9), i.e

$$\hat{\mathcal{C}}^k_* = \arg\max_{\hat{\mathcal{C}}^k} G\left(\hat{\mathcal{C}}^k\right) \tag{0.10}$$

This is a well defined problem, but not a solvable one without approximation. Furthermore, this calculation, even if it could be solved, is only part of the problem since we don't know what k is.

Note on Poisson vs Multinomial. The poisson process described above may seem far removed from the theoretical model we started the paper with. The model predicts probabilities with which universities in the same community will hire graduates from different communities. In fact, the poisson model is doing just the right thing. Its estimates will provide maximum likelihood estimates of the probabilities. The relationship between the poisson and the multinomial is well known in statistics. However, since it gives insight into how to estimate the theoretical model, we'll explain it here.

Start with poisson. If we fix a community $\hat{\mathcal{C}}^k$ and want to estimate the poisson means associated with each community, we collect a random sample of placements to fill the adjacency matrix $A\left[\hat{\mathcal{C}}^k\right]$. We are sampling from a large market, so (hopefully) it is reasonable to assume that all placements are independent. To the extent that the sampling is unbiased, it should be equally likely to select each placement to add to the data. The number of placements it comes up with is also random, since that depends on the energy of the research assistants collecting the data. The idea should be that the number of samples drawn is independent of which particular outcomes are chosen. Then once we collect all the outcomes, we can imagine placing them in a vector $\hat{a}$ which has $K * k$ elements in it.

The probability of the observed data is

$$\Pr\{\hat{a}\} = \prod_{i=1}^{Kk} \frac{e^{-\lambda_i} \lambda_i^{\hat{a}_i}}{\hat{a}_i!} = \frac{e^{-\sum \lambda_i} \lambda_1^{a_1} \cdots \lambda_{Kk}^{a_{Kk}}}{a_1! \cdots a_{Kk}!} \tag{0.11}$$

There are n observations in the sample, so

$$\Pr\{n\} = e^{-\sum \lambda_i} \sum_{a: \sum a_i = n; a_i \geq 0} \frac{\lambda_1^{a_1} \cdots \lambda_{Kk}^{a_{Kk}}}{a_1! \cdots a_{Kk}!}.$$

From the multinomial theorem

$$\left(\sum_{i=1}^{Kk} \lambda_i\right)^n = n! \sum_{a:\sum_i a_i = n; a_i \geq 0} \frac{\lambda_1^{a_1} \cdots \lambda_{Kk}^{a_{Kk}}}{a_1! \cdots a_{Kk}!}.$$

Multiplying by both sides by $e^{-\sum \lambda_i}$ and dividing both sides by $n!$ gives

$$\Pr\{n\} = e^{-\sum \lambda_i} \frac{\left(\sum_{i=1}^{Kk} \lambda_i\right)^n}{n!}.$$

So, the probability of the adjacency matrix $A^{\mathcal{C}}$ conditional on n is

$$\frac{\frac{e^{-\sum \lambda_i} \lambda_1^{a_1} \cdots \lambda_{Kk}^{a_{Kk}}}{a_1! \cdots a_{Kk}!}}{e^{-\sum \lambda_i} \frac{\left(\sum_{i=1}^{Kk} \lambda_i\right)^n}{n!}} =$$

$$\frac{n! \left(\frac{\lambda_1}{\sum \lambda_i}\right)^{a_1} \cdots \left(\frac{\lambda_{Kk}}{\sum \lambda_i}\right)^{a_{Kk}}}{a_1! \cdots a_{Kk}!}. \tag{0.12}$$

This is just the multinomial distribution. The important point is that choosing the λ_i to maximize (0.12) will also maximize (0.11). So both approaches will yield the same estimate of the best community model.

Assigning universities to communities given a fixed number of tiers. We worked with the poisson model since it was simpler. The assignment algorithm then started by assigning all universities to the same tier. At this stage, each university places different numbers of applicants to other universities, government, the private sector, and teaching universities (as well as the other sinks). Then the likelihood of the data was calculated. This initial assignment of all universites to the same tier, and the resulting likelihood was defined to be the status quo.

Then, one by one, each university was randomly selected to be reassigned to a randomly chosen tier so that the likelihood could be recalculated. If the likelihood increased after the reassignment, the new assignment along with its corresponding likelihood was adopted as the new status quo and the process was repeated. If the likelihood didn't increase, the status quo was unchanged, and a new reassignment occurred.

This process continued until the likelihood stopped rising.

To illustrate how this process worked, the following table illustrates how consistently the algorithm categorized the various universities in the case where there were 5 tiers. The algorithm was run 165 times using bootstrapped (with replacement) samples of 10000 placements from the mapinator data. Each university was placed into the tier to which the algorithm most often put them. Cell (i, j) in the table gives the proportion of times that a university who was most often placed in tier i was allocated to tier j.

For the top tier, the algorithm was remarkably consistent. Universities in tier 1 were placed in tier 1 by the algorithm over 98% of the time. This is less true for lower tier universities. For example, Tier 4 universities where classified as such just 63% of the time.

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5
Tier 1	0.982054	0.00548355	0.00398804	0.00847458	0.0
Tier 2	0.0433359	0.893875	0.057396	0.00539291	0.0
Tier 3	0.00131279	0.0534789	0.755407	0.163822	0.0259794
Tier 4	0.0	0.00381175	0.134479	0.633615	0.228095
Tier 5	0.0	3.31686e-5	0.00202328	0.202295	0.795648

The corresponding table when there were assumed to be just 4 types is given below. Generally with 4 possible tiers the algorithm was more consistent

	Tier 1	Tier 2	Tier 3	Tier 4
Tier 1	0.9509	0.0490998	0.0	0.0
Tier 2	0.0385484	0.899714	0.0617377	0.0
Tier 3	0.000514249	0.0466252	0.873109	0.0797514
Tier 4	0.0	0.0	0.0711762	0.928824

In the appendix, the actual categorization of each university is listed along with a confidence measure. The confidence measure is similar to the one above. It measures the proportion of times that a particular university was placed into the tier to which it was finally assigned.

Number of Tiers. To estimate the number of tiers we used a penalized likelihood approach (Wang and Bickel (2017)). The estimation procedure is

$$\min_k \left\{ -\ln\left(G(\hat{\mathcal{C}}_*^k)\right) + \lambda \frac{k(k+1)}{2} n \ln(n) \right\}$$

where n is the total number of placements in the sample (14875 from table) and G and $\hat{\mathcal{C}}_*^k$ are defined in table 0.10.

The variable λ is a tuning parameter. Obviously this is chosen to get an interior solution. However to find it, we used the heuristic procedure defined in Wang and Bickel (2017). Define

$$\beta_k(\lambda) = -\ln\left(G(\hat{\mathcal{C}}_*^k)\right) + \lambda \frac{k(k+1)}{2} n \ln(n).$$

We calculated this ratio for each k between 2 and 10. Then we calculated

$$w_k(\lambda) = \frac{\beta_k(\lambda)}{\sum_{k'=2}^{10} \beta(k')}.$$

We then picked λ to maximize the entropy measure

$$-\sum_{k=2,10} w_k(\lambda) \ln(w_k(\lambda)).$$

The resulting value for the tuning parameter was .001. The calculation yields an 'optimal' of 5. The categorization by departments given by this calcuation is given in the appendix.

Literature. The only paper we have so far found that uses placement data to evaluate departments is Amir and Knauff (2008). They chose 54 departments and created faculty lists for them. They then looked up graduating institutions for the faculty members in each department and looked for links between them by linking the graduating department to the current institution. They then calculated the value of each department as a weighted average of the value of all its graduates.

Weights were calculated using a google page rank like method - all institutions start arbitrary weights. Given those weights, the value of the graduates of each institution can be calculated. The calculated values form the basis of a new set of weights which can used to repeat the calculation, until a fixed point is established.

Their objective was to create a ranking of a relatively small set of institutions. Our categorization creates a type of ranking as well since we investigate a tiered market structure. However, our objective is different since we are trying to determine whether placements can be understood as a reflection of a very coarse information partition.

Mechanically, our approach differs in that it contains a lot more data. Secondly, rather than pre-selecting departments to study, our data generates them automatically since we are just tracing placements of graduates who registered with econjobmarket. We also deal with initial placements rather than employee lists.

The largest difference comes from the fact that by selecting a small group of departments and looking for links between them, their results perhaps understate the impact of top tier departments. As can be seen from table , the top tier universities have a market share of 35% (divide the number of placements by the top tier by the total number of placements). The first two tiers together supply 64% of all the economics phds who are hired anywhere.

The other result here, which is hard to see in the analysis in Amir and Knauff (2008), is that only 14% of graduates of tier 1 universities actually end up being hired by tier 1 universities. The rest are widely spread. This provides a much better indication of how influential the top tier departments are.

Conclusion. A frictional matching model is designed that can be used to analyse network trading data. It predicts an adjacency matrix with a lower diagonal structure. Our informal attempt to look for communities finds a matrix that does have this appearance. This is hardly a test, but it is at least consistent with the theory.

If we accepted the matrix above as an accurate description of the tiers that exist in the market, then the parameters of the model, in particular the values v_i, could be estimated using maximum likelihood. However both the number of tiers and the elements of each tier are themselves estimates, so the corresponding distribution theory is unclear.

Tier based matching has potentially detrimental effects. The values v_i of the tiers, and the members of each tier are market based assessments. They have little to do with candidate specific observables. When a department who is not in the top tier invests to try to improve the quality of its graduate program, the market won't respond by immediately rewarding them. This can lead to a kind of statistical discrimination in which lower tier department have no incentive to improve.

Similarly phd candidates who want to work on niche problems will not be admitted to top tier schools possibly restricting innovation in research.

One the other hand, to the extent that coarse types and market tiers are the consequence of the fact that evaluating many applicants is just too costly to be practical, then this market organization may be constrained efficient. In that case, quantifying the tier structure may be a useful way to reduce evaluation costs since the beleaguered recruiting committee can skip the applications of graduates they will never hire in an automated way and focus on the more difficult job of evaluating signals.

We suggested in the discussion that the stochastic block model could be used to measure the degree of clustering. To the extent that they work, coarse communities of universities (like the communities in our example adjacency matrix) are consistent with the idea that evaluation costs are high. Without a specific model of what these costs are it is impossible to try to use the communities to estimate these costs. We leave this for future research.

Finally, estimation of the parameters of the model here are in one sense straightforward. The first step would be to adapt the stochastic block model so that community membership is based on maximum likelihood estimation using the best model parameters instead of communitying only on Poisson success rates estimated independently for every cell in the adjacency matrix (at least in the cases where the tier based model actually imposes restrictions on these rates). Whether the asymptotic properties of the SBM estimator still hold after this modification is an open question. Secondly, estimating the variance covariance of the model estimators using the Jacobian of the likelihood function no longer works since the members of each community are being estimated along with the model parameters. Again, for future research.

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0.1. **Appendix: The communities.**

Tier 1.

University	**Confidence Values**
`Any`	`Any`
Yale University	(0.9830508474576271, 1)
Columbia University	(0.9830508474576271, 1)
Harvard University	(0.9830508474576271, 1)
Duke University	(0.9830508474576271, 1)
Massachusetts Institute of Technology	(0.9661016949152542, 1)
University of California, Berkeley	(0.9830508474576271, 1)
University of Wisconsin, Madison	(0.9830508474576271, 1)
London School of Economics and Political Science	(0.9830508474576271, 1)
University of Pennsylvania	(0.9830508474576271, 1)
University of Michigan	(0.9830508474576271, 1)
University of California Los Angeles (UCLA)	(0.9830508474576271, 1)
New York University	(0.9830508474576271, 1)
University of Maryland	(0.9830508474576271, 1)
Princeton University	(0.9830508474576271, 1)
Stanford University	(0.9830508474576271, 1)
University of Chicago	(0.9830508474576271, 1)
Northwestern University	(0.9830508474576271, 1)

Tier 2

University `Any`	**Confidence Values** `Any`
University of Mannheim	(0.9661016949152542, 2)
University of Southern California	(0.9830508474576271, 2)
University of Toronto	(0.8644067796610169, 2)
University of Bonn	(0.9491525423728814, 2)
Stockholm School of Economics	(0.923728813559322, 2)
University of Oxford	(0.7627118644067797, 2)
Ohio State University	(0.9322033898305085, 2)
University of Illinois at Urbana-Champaign	(0.8813559322033898, 2)
Vanderbilt University	(0.9830508474576271, 2)
European University Institute	(0.9152542372881356, 2)
Washington State University	(0.788135593220339, 2)
Bocconi University	(0.9576271186440678, 2)
University of Washington	(0.9830508474576271, 2)
Washington University in St. Louis	(0.9830508474576271, 2)
Ãcole d'Ãconomie de Paris	(0.8983050847457627, 2)
Michigan State University	(0.9830508474576271, 2)
University of Minnesota, Twin Cities	(0.8559322033898305, 2)
Boston University	(0.576271186440678, 2)
University of California, San Diego	(0.7288135593220339, 2)
University of Virginia	(0.9830508474576271, 2)
University of Texas at Austin	(0.9830508474576271, 2)
Johns Hopkins University	(0.9830508474576271, 2)
Arizona State University	(0.7796610169491525, 2)
University of Rochester	(0.9830508474576271, 2)
National University of Singapore	(0.7711864406779661, 2)
Cornell University	(0.8898305084745763, 2)
University of British Columbia	(0.9830508474576271, 2)
University of Nottingham	(0.9576271186440678, 2)
Tilburg University	(0.9576271186440678, 2)
Texas A&M University, College Station	(0.9830508474576271, 2)
Brown University	(0.9406779661016949, 2)
University of California, Davis	(0.9830508474576271, 2)
Boston College	(0.9830508474576271, 2)
Universitat Pompeu Fabra	(0.9491525423728814, 2)
Carnegie Mellon University	(0.9745762711864407, 2)
Pennsylvania State University	(0.8474576271186441, 2)
University of Warwick	(0.9576271186440678, 2)
Toulouse School of Economics	(0.9661016949152542, 2)
Maastricht University	(0.8220338983050847, 2)
University College London	(0.9661016949152542, 2)
Purdue University	(0.9830508474576271, 2)

Tier 3.

University	**Confidence Values**
`Any`	`Any`
University of North Carolina, Chapel Hill	(0.7711864406779661, 3)
University of Melbourne	(0.9406779661016949, 3)
The New School	(0.8050847457627119, 3)
University of Oslo	(0.7203389830508475, 3)
University of Massachusetts, Amherst	(0.7457627118644068, 3)
University of California, Santa Cruz	(0.8983050847457627, 3)
Rice University	(0.923728813559322, 3)
Peking University	(0.8796296296296296, 3)
INSEAD	(0.9152542372881356, 3)
Rutgers, The State University of New Jersey	(0.9406779661016949, 3)
University of Florida	(0.8898305084745763, 3)
Georgia Institute of Technology	(0.7711864406779661, 3)
ETH Zurich	(0.8389830508474576, 3)
Georgetown University	(0.8813559322033898, 3)
Indiana University Bloomington	(0.8898305084745763, 3)
University of Vienna	(0.9322033898305085, 3)
Seoul National University	(0.5423728813559322, 3)
University of Manchester	(0.788135593220339, 3)
Hong Kong University of Science and Technology	(0.8983050847457627, 3)
West Virginia University	(0.8728813559322034, 3)
Vrije Universiteit Amsterdam	(0.5338983050847458, 3)
Simon Fraser University	(0.7796610169491525, 3)
UniversitÃ di Bologna	(0.6610169491525424, 3)
Stockholm University	(0.8898305084745763, 3)
University of Illinois at Chicago	(0.923728813559322, 3)
University of Pittsburgh	(0.8559322033898305, 3)
City University of London	(0.576271186440678, 3)
University of Notre Dame	(0.5508474576271186, 3)
London Business School	(0.8728813559322034, 3)
CEMFI	(0.9152542372881356, 3)
Utrecht University	(0.6271186440677966, 3)
George Mason University	(0.9322033898305085, 3)
University of Georgia	(0.9152542372881356, 3)
University of Essex	(0.5084745762711864, 3)
Sciences Po	(0.9152542372881356, 3)
University of Glasgow	(0.6271186440677966, 3)
University of Zurich	(0.788135593220339, 3)
University of California, Irvine	(0.8220338983050847, 3)
Erasmus University Rotterdam	(0.5847457627118644, 2)
Tsinghua University	(0.788135593220339, 3)
University of Oregon	(0.9406779661016949, 3)
Katholieke Universiteit Leuven	(0.8644067796610169, 3)
University of Amsterdam	(0.9152542372881356, 3)
Singapore Management University	(0.9067796610169492, 3)
University of Connecticut	(0.8983050847457627, 3)
McMaster University	(0.7627118644067797, 3)
Clemson University	(0.8220338983050847, 3)
Ludwig-Maximilians-Universitat MÃ¼nchen (LMU) (University of Munich)	(0.9152542372881356, 3)
Syracuse University	(0.8559322033898305, 3)
University of California, Riverside	(0.8559322033898305, 3)
North Carolina State University	(0.8983050847457627, 3)
University of Missouri, Columbia	(0.8983050847457627, 3)
University of Arizona	(0.5847457627118644, 2)
Queen Mary University of London	(0.5423728813559322, 3)
Georgia State Universit	(0.9491525423728814, 3)

Universities	**Confidence Values**
Indian Statistical Institute, Delhi	(0.6355932203389831, 4)
Lancaster University	(0.6779661016949153, 4)
Eastern Kentucky University	(0.4406779661016949, 4)
University of Gothenburg	(0.6864406779661017, 3)
Bentley University	(0.59, 4)
Insper (Institute of Education and Research)	(0.7263157894736842, 4)
UniversitÃ Degli Studi di Milano	(0.6694915254237288, 4)
Universidad de los Andes	(0.5338983050847458, 3)
Korea University	(0.7, 4)
Rutgers University, Newark	(0.6206896551724138, 4)
University of Reading	(0.7288135593220339, 4)
Hong Kong Baptist University	(0.4322033898305085, 4)
UniversitÃ degli Studi di Trento	(0.6525423728813559, 4)
University of Cologne	(0.5847457627118644, 4)
Fordham University	(0.7586206896551724, 4)
Wilfrid Laurier University	(0.7333333333333333, 4)
Colorado State University	(0.5726495726495726, 4)
UniversitÃ© catholique de Louvain	(0.5254237288135593, 4)
Osaka University	(0.5866666666666667, 4)
Leiden University	(0.7173913043478261, 4)
University of Adelaide	(0.6694915254237288, 4)
University of Leicester	(0.5169491525423729, 3)
University of St. Gallen	(0.6610169491525424, 4)
Hanyang University	(0.7346938775510204, 4)
UniversitÃ degli Studi di Pavia	(0.5508474576271186, 4)
Wuhan University	(0.6625, 3)
University of Texas at Arlington	(0.576271186440678, 4)
UniversitÃ© Paris 1 PanthÃ©on-Sorbonne	(0.5593220338983051, 4)
Drexel University	(0.7008547008547009, 4)
University of Miami	(0.4871794871794872, 4)
Concordia University	(0.4915254237288136, 3)
Sogang University	(0.6923076923076923, 4)
University of Kentucky	(0.4827586206896552, 4)
University of Rhode Island	(0.6810344827586207, 4)
Waseda University	(0.6, 4)
Ca' Foscari University Venice	(0.7288135593220339, 4)
Auburn University	(0.5677966101694915, 3)
UniversitÃ degli Studi di Firenze	(0.5517241379310345, 4)
University of Kent	(0.6694915254237288, 4)
Trinity College Dublin, University of Dublin	(0.6440677966101695, 4)
University of Edinburgh	(0.5847457627118644, 4)
Graduate Institute of International and Development Studies	(0.7033898305084746, 4)
National Research University Higher School of Economics, Moscow	(0.6727272727272727, 4)
Norwegian School of Economics (NHH)	(0.6694915254237288, 3)
University of St Andrews	(0.7126436781609195, 4)

University of Liverpool	(0.6966292134831461, 4)
University of Southampton	(0.6610169491525424, 4)
Universidad Complutense de Madrid	(0.6120689655172414, 4)
Mississippi State University	(0.7415730337078652, 4)
City University of Hong Kong	(0.4395604395604396, 3)
Durham University	(0.7033898305084746, 4)
Shiv Nadar University	(0.75, 4)
Freie UniversitÃ¤t Berlin	(0.6864406779661017, 4)
Radboud University	(0.7222222222222222, 4)
UniversitÃ degli Studi di Verona	(0.7372881355932203, 4)
Carleton University	(0.4661016949152542, 4)
Tufts University	(0.6436781609195402, 4)
University of Nevada, Las Vegas (UNLV)	(0.8181818181818182, 4)
Hebrew University of Jerusalem	(0.6101694915254237, 4)
UniversitÃ© Laval	(0.7372881355932203, 4)
Toronto Metropolitan University	(0.4915254237288136, 3)
Wayne State University	(0.7288135593220339, 4)
New Mexico State University	(0.7619047619047619, 4)
New York University Shanghai	(0.6666666666666667, 4)
UniversitÃ della Svizzera Italiana	(0.6949152542372881, 4)
California Institute of Technology (Caltech)	(0.5084745762711864, 3)
Lehigh University	(0.7435897435897436, 4)
Aalto University	(0.5169491525423729, 4)
University of Nebraska, Lincoln	(0.6016949152542373, 3)
Bilkent University	(0.7372881355932203, 4)
University of Innsbruck	(0.5593220338983051, 4)
Universitat de ValÃ¨ncia	(0.4871794871794872, 4)
University of Jena	(0.6016949152542373, 4)
University of Ottawa	(0.6101694915254237, 4)
Sun Yat-Sen University	(0.7676767676767677, 4)
University of Bath	(0.7692307692307692, 4)
Copenhagen Business School	(0.5689655172413793, 3)
University of Waterloo	(0.6752136752136752, 4)
Miami University	(0.5853658536585366, 4)
King's College London	(0.5689655172413793, 4)
University of Sydney	(0.5181818181818182, 4)
University of Iowa	(0.5254237288135593, 3)
Brandeis University	(0.7349397590361446, 4)
Tel Aviv University	(0.7830188679245283, 4)
Deakin University	(0.6436781609195402, 4)
University of New Mexico	(0.5847457627118644, 4)
Vienna Graduate School of Finance (VGSF)	(0.5254237288135593, 4)
Ghent University	(0.576271186440678, 4)
University of Southern Denmark	(0.7171717171717172, 4)
Universidade Nova de Lisboa	(0.7008547008547009, 4)
University of Utah	(0.4491525423728814, 3)
University of Memphis	(0.6741573033707865, 4)
Pontifical Catholic University of Rio de Janeiro	(0.6442307692307692, 4)

University of Hong Kong	(0.4912280701754386, 3)
University of Cincinnati	(0.6470588235294118, 4)
Royal Melbourne Institute of Technology (RMIT)	(0.4915254237288136, 4)
University of Sheffield	(0.6864406779661017, 4)
University of North Carolina, Charlotte	(0.5585585585585586, 4)
Royal Holloway, University of London	(0.6440677966101695, 4)
University of South Carolina	(0.4915254237288136, 4)
UniversitÃ degli Studi di Torino	(0.7033898305084746, 4)
University of South Florida	(0.6551724137931034, 4)
University of Bern	(0.6581196581196581, 4)
Instituto Tecnologico AutÃ³nomo de Mexico (ITAM)	(0.608695652173913, 3)
KoÃ§ University	(0.6525423728813559, 4)
Clark University	(0.6949152542372881, 4)
Hunter College, CUNY	(0.5119047619047619, 4)
IMT School for Advanced Studies Lucca	(0.6779661016949153, 4)
Indira Gandhi Institute of Development Research (IGIDR)	(0.4915254237288136, 5)
York University	(0.6355932203389831, 4)
University of International Business and Economics	(0.5595238095238095, 3)
University of Queensland	(0.5254237288135593, 3)
Ball State University	(0.5714285714285714, 4)
Fudan University	(0.4545454545454545, 4)
University of East Anglia	(0.7372881355932203, 4)
University of Surrey	(0.711864406779661, 4)
University of Exeter	(0.8061224489795918, 4)
University of Haifa	(0.527027027027027, 4)
Binghamton University (SUNY)	(0.7288135593220339, 3)
Universitat de Barcelona	(0.6779661016949153, 4)
Center for Research in Economics and Statistics (CREST)	(0.6810344827586207, 4)
UniversitÃ© du QuÃ©bec Ã MontrÃ©al (UQAM)	(0.6403508771929825, 4)
University of Manitoba	(0.7475728155339806, 4)
University of Groningen	(0.4830508474576271, 3)
Howard University	(0.641025641025641, 4)
University of Oklahoma	(0.5847457627118644, 3)
University of Victoria	(0.7068965517241379, 4)
University of New South Wales (UNSW), Sydney	(0.5677966101694915, 3)
University of North Carolina, Greensboro	(0.7333333333333333, 4)
Humboldt University Berlin	(0.5847457627118644, 4)
ESMT Berlin	(0.5961538461538462, 4)
Imperial College London	(0.5423728813559322, 4)
Florida International University	(0.5847457627118644, 4)
Central European University	(0.6694915254237288, 4)
National Central University	(0.6206896551724138, 4)
University of Helsinki	(0.6016949152542373, 4)
CERGE-EI (Center for Economic Research and Graduate Education) Economics Institute)	(0.5338983050847458, 4)
East China Normal University	(0.6785714285714286, 4)
Collegio Carlo Alberto	(0.7631578947368421, 4)
New York University Abu Dhabi	(0.7368421052631579, 4)

Dongbei University of Finance and Economics	(0.660377358490566, 4)
University of Tennessee, Knoxville	(0.5847457627118644, 4)
University of Alberta	(0.5084745762711864, 3)
City University of New York Graduate Center	(0.4915254237288136, 4)
Vienna University of Economics and Business	(0.6779661016949153, 4)
Southern Methodist University	(0.6724137931034483, 3)
Shandong University	(0.8392857142857143, 4)
University at Albany (SUNY)	(0.6666666666666667, 4)
University of Maine, Orono	(0.6153846153846154, 4)
University College Dublin	(0.7203389830508475, 4)
University of Guelph	(0.6271186440677966, 4)
Nanyang Technological University	(0.4745762711864407, 4)
University of Bristol	(0.6068376068376068, 4)
Zhongnan University of Economics and Law	(0.7796610169491525, 4)
University of Wisconsin, Milwaukee	(0.6052631578947368, 4)
UniversitÃ Cattolica del Sacro Cuore	(0.6779661016949153, 4)
University of Sussex	(0.5593220338983051, 3)
University of Birmingham	(0.6440677966101695, 4)
University of Nevada, Reno	(0.7372881355932203, 4)
Universidad de Alicante	(0.7033898305084746, 4)
Queens' University Belfast	(0.7542372881355932, 4)
University of Mississippi	(0.7058823529411765, 4)
University of Technology Sydney	(0.6363636363636364, 4)
Texas Tech University	(0.5677966101694915, 3)
State University of New York at Buffalo	(0.6486486486486486, 4)
Heidelberg University	(0.7033898305084746, 4)
Frankfurt School of Finance & Management	(0.52, 4)
University of New Hampshire	(0.5643564356435644, 4)
Indian Institute of Technology, Kanpur	(0.77, 4)
Louisiana State University	(0.6271186440677966, 3)
Tulane University	(0.6610169491525424, 3)
Johannes Kepler University Linz	(0.5254237288135593, 4)
Baruch College, City University of New York	(0.5444444444444444, 4)
Northeastern University	(0.4615384615384615, 3)
University of York	(0.6101694915254237, 4)
University of Arkansas	(0.576271186440678, 4)
Azyein University	(0.6464646464646465, 4)
University of Tokyo	(0.6837606837606838, 4)
Colorado School of Mines	(0.4871794871794872, 5)
Middle Tennessee State University	(0.5254237288135593, 3)
Claremont Graduate University	(0.711864406779661, 4)
University of SÃ£o Paulo	(0.7291666666666667, 4)
Oklahoma State University	(0.5084745762711864, 3)
Birkbeck College, University of London	(0.5084745762711864, 4)
University of Delaware	(0.5641025641025641, 3)
Queensland University of Technology	(0.6440677966101695, 4)
Technische UniversitÃ¤t Berlin	(0.5508474576271186, 4)
Universidad de Navarra	(0.4786324786324786, 3)

UniversitÃ© Libre de Bruxelles	(0.5084745762711864, 4)
University of Strathclyde	(0.6864406779661017, 4)
University of Colorado, Denver	(0.6071428571428571, 4)
Sapienza UniversitÃ di Roma	(0.7288135593220339, 4)
Emory University	(0.6694915254237288, 3)
University of Louisville	(0.7272727272727273, 4)
Universidad del Rosario	(0.8035714285714286, 4)
University of Texas at Dallas	(0.5, 4)
Cardiff University	(0.6610169491525424, 4)
FundaÃ§Ã£o Getulio Vargas (FGV)	(0.6324786324786325, 4)
UniversitÃ© Paris-Dauphine	(0.6355932203389831, 4)
Temple University	(0.6779661016949153, 3)
University of Wyoming	(0.4745762711864407, 4)
Dalhousie University	(0.7247706422018349, 4)
New Economic School	(0.4871794871794872, 4)
HEC Paris	(0.547008547008547, 4)
University of Western Australia	(0.7288135593220339, 4)
SUNY Buffalo State	(0.7087378640776699, 4)
Hong Kong Polytechnic University	(0.6938775510204082, 4)
UniversitÃ© de GenÃ¨ve	(0.6837606837606838, 4)
Pontificia Universidad CatÃ³lica de Chile	(0.6428571428571429, 4)
ESSEC Business School	(0.5689655172413793, 4)
University of Konstanz	(0.7033898305084746, 4)
Utah State University	(0.7, 4)

Tier 4. Tier 5

University	**Confidence**
`Any`	`Any`
Helwan University	(0.8333333333333333, 5)
Newcastle University	(0.6372549019607843, 5)
Kwara State University	(0.84, 5)
Indian Institute of Technology, Indore	(0.9166666666666667, 5)
University of Canterbury	(0.8333333333333333, 5)
University of Jammu	(0.82, 5)
Norwegian University of Life Sciences	(0.8275862068965517, 5)
Eastern Mediterranean University	(0.6440677966101695, 5)
Linn universitetets	(0.8529411764705882, 5)
University of Vaasa	(0.6326530612244898, 4)
Stevens Institute of Technology	(0.75, 5)
Politecnico di Torino	(0.7840909090909091, 5)
Indiana Institute of Technology	(0.891566265060241, 5)
LUISS Guido Carli	(0.6630434782608696, 4)
RAND Corporation	(0.8666666666666667, 5)
Universitie de Toulouse 1 Capitole	(0.8804347826086957, 5)
Universidad de Cantabria	(0.8488372093023256, 5)
University of North Texas	(0.543859649122807, 4)
Yildiz Technical University	(0.890625, 5)
Technische UniversitÃ¤t Dresden	(0.8390804597701149, 5)

Universitie de Toulon	(0.872093023255814, 5)
University of the Western Cape	(0.8813559322033898, 5)
Dublin Institute of Technology	(0.9, 5)
University of Portsmouth	(0.8653846153846154, 5)
Bangor University	(0.8545454545454545, 5)
Universiti Kebangsaan Malaysia	(0.9069767441860465, 5)
Ecole des Hautes Etudes en Sciences Sociales (EHESS)	(0.8482142857142857, 5)
Universitie Paris 2 Pantheon-Assas	(0.7627118644067797, 5)
International Centre for Education in Islamic Finance	(0.872093023255814, 5)
University of Bedfordshire	(0.8541666666666667, 5)
ESCP Business School	(0.6593406593406593, 5)
Stellenbosch University	(0.6610169491525424, 5)
Universitie Carlo Cattaneo	(0.896551724137931, 5)
University of Duisburg-Essen	(0.8192771084337349, 5)
Universidad de Valladolid	(0.9066666666666667, 5)
Universitie de Poitiers	(0.9264705882352941, 5)
University of Dundee	(0.8648648648648649, 5)
University of Botswana	(0.8620689655172414, 5)
University of Beira Interior	(0.7906976744186047, 5)
Technical University of Munich	(0.8053097345132743, 5)
Nnamdi Azikiwe University	(0.8727272727272727, 5)
University of Bergen	(0.6, 5)
Universitie degli Studi di Napoli Federico II	(0.6203703703703704, 5)
Universitie degli Studi di Siena	(0.7627118644067797, 5)
University of Cyprus	(0.7586206896551724, 5)
Marietta College	(0.8620689655172414, 5)
Central Michigan University	(0.6323529411764706, 5)
University of Science and Technology in China	Not found
University of Muenster	(0.8421052631578947, 5)
Indian Institute of Science	(0.7951807228915663, 5)
University of Waikato	(0.8795180722891566, 5)
ISM University of Management and Economics	(0.8548387096774194, 5)
Brunel University London	(0.7232142857142857, 5)
Istanbul University	(0.8571428571428571, 5)
Universidad Rey Juan Carlos	(0.8362068965517241, 5)
Sultan Qaboos University	(0.8703703703703704, 5)
Tarbiat Modares University	(0.8771929824561404, 5)
Cochin University of Science and Technology	(0.8955223880597015, 5)
University of Basel	(0.7333333333333333, 5)
Justus-Liebzig-Universitat	(0.9344262295081967, 5)
Kurukshetra University	(0.875, 5)
KTH Royal Institute of Technology	(0.75, 5)
EDHEC Business School	(0.7216494845360825, 5)
University of Limerick	(0.6633663366336634, 5)
Walden University	(0.8596491228070175, 5)
National Graduate Institute for Policy Studies (GRIPS), Japan	(0.5483870967741935, 5)
Institute of Technology Sligo	(0.875, 5)
University of Malaya	(0.7777777777777778, 5)

Instituto Politicnico Nacional (IPN)	(0.7966101694915254, 5)
University of East London	(0.7570093457943925, 5)
Lomonosov Moscow State University	(0.8333333333333333, 5)
Ferdowsi University of Mashhad	(0.8275862068965517, 5)
Gauhati University	(0.8813559322033898, 5)
University of Tirana	(0.8548387096774194, 5)
Universitie Paris-Est Marne-la-Vallee	(0.85, 5)
Universitie de Picardie Jules Verne	(0.8604651162790698, 5)
Leipzig University	(0.875, 5)
International Monetary Fund (IMF)	(0.81, 5)
Universitie d'Evry Val-d'Essonne	(0.8888888888888889, 5)
University of Galway	(0.8135593220338983, 5)
Libera Universitie di Bolzano	(0.8245614035087719, 5)
Auckland University of Technology	(0.7966101694915254, 5)
University of Lisbon	(0.7021276595744681, 5)
Technological University Dublin	(0.8777777777777778, 5)
Universiteit Antwerpen	(0.8645833333333333, 5)
Universitie degli Studi di Genova	(0.8681318681318681, 5)
Bielefeld University	(0.6964285714285714, 5)
University of Carthage	(0.8674698795180723, 5)
Universitie degli Studi dell'Insubria	(0.8588235294117647, 5)
Indiana University Purdue University Indianapolis (IUPUI)	(0.8333333333333333, 5)
ITMO University	(0.8571428571428571, 5)
University of Fort Hare	(0.8148148148148148, 5)
University of Namur	(0.574468085106383, 5)
Universidad Anahua	(0.9056603773584906, 5)
Universidade Tecnoligica Federal do Parana	(0.8846153846153846, 5)
IE University	(0.51, 5)
Universitie Politecnica delle Marche	(0.8202247191011236, 5)
Coventry University	(0.6440677966101695, 5)
Indian Institute of Technology, Delhi	(0.830188679245283, 5)
Azmir University of Economics	(0.5280898876404494, 4)
Swedish University of Agricultural Sciences	(0.7962962962962963, 5)
Universitie Claude Bernard Lyon 1	(0.8446601941747573, 5)
Old Dominion University	(0.8712871287128713, 5)
Middle East Technical University	(0.7610619469026549, 5)
Griffith University	(0.8316831683168317, 5)
Universitie de Strasbourg	(0.8425925925925926, 5)
Korea Advanced Institute of Science and Technology (KAIST)	(0.619047619047619, 5)
Oxford Brookes University	(0.8222222222222222, 5)
Universitie degli Studi di Ferrara	(0.7863247863247863, 5)
Acole Normale Superieure de Lyon	(0.8484848484848485, 5)
Thammasat University	(0.8476190476190476, 5)
Universitie degli Studi di Milano Biocca	(0.6810344827586207, 5)
Federal Reserve Bank of New York	(0.8360655737704918, 5)
Kangwon National University	(0.8936170212765957, 5)
Indian Institute of Management Kozhikode	(0.6730769230769231, 5)
University of Gottingen	(0.5384615384615385, 5)

Penn State Abington	(0.7166666666666667, 5)
Bharathidasan University	(0.8714285714285714, 5)
Universitie d'Orleans	(0.8488372093023256, 5)
Grenoble Ecole de Management	(0.5697674418604651, 4)
Marmara University	(0.8378378378378378, 5)
Academy of Postgraduate Education	(0.85, 5)
Technische UniversitÃ¤t Dortmund	(0.831858407079646, 5)
K. N. Toosi University of Technology	(0.8793103448275862, 5)
Saint Mary's University (SMU)	(0.8727272727272727, 5)
Julius-Maximilians-UniversitÃ¤t Warzburg	(0.6966292134831461, 5)
University of Roehampton	(0.8617021276595745, 5)
Universitie de Montpellier	(0.8521739130434783, 5)
Corvinus University of Budapest	(0.8936170212765957, 5)
Technion	(0.8615384615384615, 5)
Rabindra Bharati University	(0.8688524590163934, 5)
Heriot Watt University	(0.7567567567567568, 5)
Pakistan Institute of Development Economics	(0.8928571428571429, 5)
Massey University	(0.9113924050632911, 5)
University of the Aegean	(0.8627450980392157, 5)
Sorbonne Universitie	(0.92, 5)
Universiti Sains Malaysia	(0.8495575221238938, 5)
Hamburg University	(0.7425742574257426, 5)
University of Swansea	(0.8363636363636364, 5)
University of Huddersfield	(0.7173913043478261, 5)
Universitie Clermont Auvergne	(0.8407079646017699, 5)
Chinese Academy of Sciences	(0.8901098901098901, 5)
University of the West of Scotland	(0.9024390243902439, 5)
Norges Artiske Universitet	(0.8181818181818182, 5)
North-West University	(0.8333333333333333, 5)
Charles University	(0.4954128440366972, 4)
Anglia Ruskin University	(0.869047619047619, 5)
Universitie de Tunis	(0.8666666666666667, 5)
University of Iceland	(0.8666666666666667, 5)
Kyoto University	(0.6428571428571429, 5)
CY Cergy Paris Universitie	(0.7863247863247863, 5)
FUCAPE Business School	(0.8666666666666667, 5)
Near East University	(0.7818181818181818, 5)
International Islamic University, Islamabad	(0.8351648351648352, 5)
Universitat Ramon Llull	(0.6285714285714286, 5)
University of Freiburg	(0.8363636363636364, 5)
Universitie Cheikh Anta Diop de Dakar	(0.9047619047619048, 5)
Ecole Normale Superieure Paris-Saclay	(0.855421686746988, 5)
East Carolina University	(0.5263157894736842, 4)
University of Hull	(0.8245614035087719, 5)
Suffolk University	(0.58, 5)
Tallinn University of Technology	(0.85, 5)
University of Mazandaran	(0.8372093023255814, 5)
Zhejiang Gongshang University	(0.6428571428571429, 5)

Sant'Anna School of Advanced Studies	(0.6545454545454545, 5)
Wuhan University of Technology	(0.8375, 5)
Istanbul Bilgi University	(0.6607142857142857, 5)
Universitie degli Studi di Messina	(0.8732394366197183, 5)
University of Tasmania	(0.6239316239316239, 5)
Indian Institute of Management Bangalore	(0.9444444444444444, 5)
Tampere University	(0.851063829787234, 5)
University of Sfax	(0.8979591836734694, 5)
Southern Illinois University, Carbondale	(0.8547008547008547, 5)
RWTH Aachen University	(0.84375, 5)
Glasgow Caledonian University	(0.8928571428571429, 5)
Universitie Paris-Saclay	(0.6403508771929825, 5)
Indian Institute of Technology, Kharagpur	(0.7580645161290323, 5)
Indian Institute of Management Indore	(0.7196261682242991, 5)
University of Massachusetts, Boston	(0.7413793103448276, 5)
Technical University of Denmark	(0.8584905660377358, 5)
Leibniz Universitat Hannover	(0.8205128205128205, 5)
University of Dschang	(0.8426966292134831, 5)
Universitie de Bordeaux	(0.8034188034188034, 5)
Barcelona Graduate School of Economics	(0.5272727272727273, 5)
Illinois Institute of Technology	(0.8979591836734694, 5)
SKEMA Business School	(0.5882352941176471, 5)
ICFAI Foundation for Higher Education	(0.8095238095238095, 5)
Technical University of Crete	(0.8117647058823529, 5)
Pusan National University	(0.8977272727272727, 5)
University of Tsukuba	(0.5098039215686275, 5)
Edith Cowan University	(0.9444444444444444, 5)
Universitie de Technologie de Compiagne	(0.8588235294117647, 5)
Tecnologico de Monterrey	(0.8313253012048193, 5)
Paderborn University	(0.9056603773584906, 5)
Aston University	(0.7647058823529412, 5)
University of Passau	(0.74, 5)
University of New Orleans	(0.7627118644067797, 5)
Northern Illinois University	(0.6774193548387097, 5)
University of Johannesburg	(0.8125, 5)
Middle East Technical University	(0.851063829787234, 5)
Bond University	(0.859375, 5)
AgroParisTech	(0.6525423728813559, 5)
St. Thomas University	(0.9152542372881356, 5)
Jiwaji University	(0.8571428571428571, 5)
University of Sarajevo	(0.8070175438596491, 5)
University of Northern British Columbia	(0.6290322580645161, 5)
University of Pretoria	(0.7857142857142857, 5)
Ecole Polytechnique Federale de Lausanne	(0.6607142857142857, 5)
University College Cork	(0.8316831683168317, 5)
Aveiro University	(0.8518518518518519, 5)
University of Zagreb	(0.8072289156626506, 5)
Saint Petersburg University	(0.8666666666666667, 5)

Indian Institute of Technology, Madras	(0.8878504672897196, 5)
Manchester Metropolitan University	(0.6434782608695652, 5)
Quaid-i-Azam University	(0.8771929824561404, 5)
Galatasaray University	(0.8703703703703704, 5)
Tohoku University	(0.803921568627451, 5)
National University of Sciences and Technology (NUST)	(0.7454545454545455, 5)
Universitie Grenoble Alpes	(0.8469387755102041, 5)
Covenant University	(0.8521739130434783, 5)
University of Leeds	(0.5398230088495575, 4)
Cairo University	(0.5245901639344262, 5)
Otto-von-Guericke University Magdeburg	(0.8275862068965517, 5)
Harbin Institute of Technology	(0.8478260869565217, 5)
Universitie Paris 13	(0.8333333333333333, 5)
University of Cape Town	(0.6896551724137931, 5)
Towson University	(0.5625, 4)
University of KwaZulu-Natal	(0.8793103448275862, 5)
Drew University	(0.7592592592592593, 5)
UniversitÃ degli Studi di Bergamo	(0.7964601769911504, 5)
University of Jyvaskyla	(0.8736842105263158, 5)
University of Massachusetts, Lowell	(0.7843137254901961, 5)
Staffordshire University	(0.8090909090909091, 5)
Dublin City University	(0.8305084745762712, 5)
Vysoka Akola ekonomicka v Praze	(0.8833333333333333, 5)
UniversitÃ degli Studi di Palermo	(0.8235294117647059, 5)
Universidad de Granada	(0.711864406779661, 5)
University of Auckland	(0.5130434782608696, 5)
Linkping University	(0.6923076923076923, 5)
Universiti Malaysia Perlis	(0.890625, 5)
(INRAE)	(0.8818181818181818, 5)
Alexandria University	(0.8571428571428571, 5)
Universitie degli Studi di Macerata	(0.8717948717948718, 5)
Universidad Publica de Navarra	(0.7647058823529412, 5)
University of Missouri, Kansas City	(0.8421052631578947, 5)
Instituto Universitario de Lisboa, ISCTE-IUL	(0.7663551401869159, 5)
Budapest University of Technology and Economics	(0.8103448275862069, 5)
Federal University of ParanÃ¡	(0.8214285714285714, 5)
University of Southern Mississippi	(0.8658536585365854, 5)
UniversitÃ degli Studi del Molise	(0.845360824742268, 5)
Multimedia University	(0.7543859649122807, 5)
Technische Universitat Braunschweig	(0.8807339449541284, 5)
University of Turku	(0.8709677419354839, 5)
University of Graz / Karl Franzens University, Graz	(0.7764705882352941, 5)
Loughborough University	(0.5862068965517241, 4)
University of Fribourg	(0.8867924528301887, 5)
Meiji University	(0.9152542372881356, 5)
Pardee RAND Graduate School	(0.7094017094017094, 5)
Sari Agricultural Sciences and Natural Resources University	(0.8644067796610169, 5)
Instituto de Matematica Pura e Aplicada (IMPA)	(0.8555555555555556, 5)

Chung-Ang University	(0.6727272727272727, 5)
University of Alabama at Birmingham	(0.5789473684210526, 5)
Sciences Po Toulouse	(0.5172413793103448, 5)
Ruhr University Bochum	(0.8813559322033898, 5)
Universitie LumiÃ¨re Lyon 2	(0.8241758241758242, 5)
Limkokwing University	(0.8823529411764706, 5)
University of Crete	(0.6355932203389831, 5)
University of Newcastle	(0.8673469387755102, 5)
Lulea University of Technology	(0.8928571428571429, 5)
Universidad de Zaragoza	(0.8333333333333333, 5)
Universitat Jaume I	(0.8035714285714286, 5)
Hanken School of Economics	(0.8152173913043478, 5)
Universidade da Coruda	(0.8928571428571429, 5)
International Islamic University Malaysia	(0.8255813953488372, 5)
University of South Alabama	(0.6893203883495146, 5)
University of Otago	(0.6880733944954128, 5)
Institut Agro Montpellier	(0.8214285714285714, 5)
UniversitÃ degli Studi di Cagliari	(0.8048780487804878, 5)
Sam Houston State University	(0.6666666666666667, 4)
Bucharest Academy of Economic Studies	(0.8818181818181818, 5)
ENSAE Paris Tech	(0.6666666666666667, 5)
Norwegian University of Science and Technology	(0.8425925925925926, 5)
Universidad Nacional Autonoma de Mexico (UNAM)	(0.8253968253968254, 5)
University of the Witwatersrand	(0.8476190476190476, 5)
UniversitÃ di Pisa	(0.8, 5)
Universitie degli Studi G. DAnnunzio Chieti-Pescara	(0.7863247863247863, 5)
SOAS (School of Oriental and African Studies), University of London	(0.4913793103448276, 4)
Western Kentucky University	(0.7413793103448276, 5)
University of Porto	(0.7241379310344828, 5)
Obafemi Awolowo University	(0.8596491228070175, 5)
Banco de Portugal	(0.9, 5)
Ss. Cyril and Methodius University in Skopje	(0.875, 5)
University Putra Malaysia	(0.85, 5)
Universitie Toulouse Jean Jaura	(0.8, 5)
Univerzita Pardubice	(0.85, 5)
University of Lucerne	(0.819672131147541, 5)
Universidad de Salamanca	(0.8909090909090909, 5)
Kobe University	(0.8793103448275862, 5)
University of Gdansk	(0.83, 5)
Bournemouth University	(0.9245283018867925, 5)
University of Bamenda	(0.8888888888888889, 5)
Eindhoven University of Technology	(0.7058823529411765, 5)
Florida Atlantic University	(0.7264150943396226, 5)
California State Polytechnic University, Pomona	(0.6219512195121951, 5)
Universitie de Nantes	(0.8333333333333333, 5)
South Asian University	(0.8387096774193548, 5)
Southern Illinois University	(0.8615384615384615, 5)
Scuola Normale Superiore	(0.8571428571428571, 5)

Indian Statistical Institute, Kolkata	(0.8615384615384615, 5)
Universidad de Deusto	(0.7, 5)
WHU - Otto Beisheim School of Management	(0.8653846153846154, 5)
National Institute of Development Administration	(0.830188679245283, 5)
Universidad de Castilla-La Mancha	(0.8888888888888889, 5)
Allameh Tabataba'i University	(0.8214285714285714, 5)
Universitie degli studi di Modena e Reggio Emilia	(0.8314606741573034, 5)
Universitat Rovira i Virgili	(0.6239316239316239, 5)
University of Luxembourg	(0.5660377358490566, 5)
University of Lucknow	(0.9333333333333333, 5)
Ruhr Graduate School in Economics	(0.875, 5)
University of Minho	(0.5826086956521739, 5)
Philipps University Marburg	(0.9090909090909091, 5)
University of Saskatchewan	(0.5116279069767442, 5)
University of Warsaw	(0.8372093023255814, 5)
University of Central Florida	(0.59, 4)
Universiti Utara Malaysia	(0.8636363636363636, 5)
MINES ParisTech	(0.7964601769911504, 5)
Kiel Institute for the World Economy	(0.8173913043478261, 5)
Halle Institute for Economic Research (IWH)	(0.85, 5)
Stripe	(0.8305084745762712, 5)
Universidad Pontificia Comillas	(0.6875, 5)
Bayero University, Kano	(0.9433962264150943, 5)
Wageningen University	(0.5, 4)
University of the Punjab	(0.7173913043478261, 5)
Nelson Mandela University	(0.8725490196078431, 5)
Universitie de Rennes 1	(0.8478260869565217, 5)
International Trade Centre	(0.9069767441860465, 5)
Universitie de Lorraine	(0.8421052631578947, 5)
University of Hyderabad	(0.8771929824561404, 5)
Delft University of Technology	(0.5089285714285714, 4)
Manchester University	(0.8166666666666667, 5)
Devi Ahilya Vishwavidyalaya, Indore	(0.9655172413793103, 5)
University of Algiers 3	(0.8529411764705882, 5)
Texas A&M International University	(0.6170212765957447, 5)
Martin Luther University of Halle-Wittenberg	(0.8214285714285714, 5)
University of Wollongong	(0.8780487804878049, 5)
Kingston University London	(0.8765432098765432, 5)
Athens University of Economics and Business	(0.5258620689655172, 5)
Jawaharlal Nehru University	(0.7288135593220339, 5)
Chr. Michelsen Institute (CMI)	(0.5042735042735043, 5)
Universidad del Pais Vasco / Euskal Herriko Unibertsitatea	(0.8613861386138614, 5)
Universitie de Pau et des Pays de l'Adour	(0.8372093023255814, 5)
Universitie Paris-Est Crieteil	(0.67, 5)
Federal University of Rio de Janeiro	(0.85, 5)
University of Nairobi	(0.8644067796610169, 5)
University of Bradford	(0.9072164948453608, 5)
Macquarie University	(0.8596491228070175, 5)

University of Stavanger	(0.7, 5)
Pan-Atlantic University	(0.7966101694915254, 5)
SWPS University of Social Sciences and Humanities	(0.8636363636363636, 5)
Katholische Universitat Eichstatt-Ingolstadt	(0.8983050847457627, 5)
Bank of Finland	(0.8253968253968254, 5)
Swiss Tropical and Public Health Institute	(0.9361702127659574, 5)
Union College	(0.5245901639344262, 5)
Jaypee Institute of Information Technology	(0.9344262295081967, 5)
University of Aberdeen	(0.7431192660550459, 5)
University of Macedonia	(0.5818181818181818, 5)
Universitie degli Studi di Roma Tre	(0.7203389830508475, 5)
Aligarh Muslim University	(0.6666666666666667, 5)
Dalian University of Technology	(0.8644067796610169, 5)
Institute for Advanced Study in Toulouse	(0.8545454545454545, 5)
Sharif University of Technology	(0.7636363636363636, 5)
Universidad Politicnica de Madrid	(0.9038461538461538, 5)
University of Tabriz	(0.858695652173913, 5)
Universitat Politecnica de Catalunya	(0.819672131147541, 5)
Universitie degli Studi di Padova	(0.7672413793103448, 5)
University of Gloucestershire	(0.8518518518518519, 5)
Kiel University	(0.8666666666666667, 5)
Universitie Paris Nanterre	(0.7906976744186047, 5)
Universitie de Lille	(0.7203389830508475, 5)
National Research University Higher School of Economics, St. Petersburg	(0.6122448979591837, 4)
Telecom ParisTech	(0.8557692307692308, 5)
Amity University	(0.8947368421052632, 5)
Umea University	(0.7904761904761905, 5)
Jiankping University	(0.8160919540229885, 5)
Maulana Azad National Institute of Technology Bhopal (MANIT)	(0.8032786885245902, 5)
Babcock University	(0.8970588235294118, 5)
Indian Institute of Technology, Bombay	(0.7410714285714286, 5)
Aalborg University	(0.7373737373737374, 5)
Aristotle University of Thessaloniki	(0.8870967741935484, 5)

Appendix - Proof of Theorem.

Proof of Theorem :

Lemma 7. *Suppose F is monotonically increasing. There is a type x_1 such that if an organization has an offer of value $x > x_1$, then the only symmetric equilibrium strategy has them making the offer to one of the tier 1 applicants and choosing each applicant with equal probability.*

Proof. The payoff to making an offer to a tier 1 applicant is v_1 times the probability that it is accepted. It is accepted by the candidate if the candidate has no other offers, or if it is the highest value offer the candidate receives. So the payoff when an organization with an offer of value x makes an offer to a top tier applicant is

$$v_1 \left(1 - \int_x^1 \tilde{\pi}_1 (\tilde{x})\, dF(\tilde{x}) \right)^{n-1} \geq$$

$$v_1\left(1-\frac{(1-F(x))}{m_1}\right)^{n-1}.$$

This inequality follows from symmetry. If $\pi(\tilde{x})$ is the probability with which any organization with a value of $\tilde{x}$ makes an offer to *some* tier 1 applicant, and $\frac{1}{m_1}$ is the symmetric probability with which an offer is made to any particular tier 1 applicant, then the left hand side of the equation attains a maximum if $\pi(x)$ is uniformly 1.

Now choose x_1 such that

$$v_2=v_1\left(1-\frac{(1-F(x_1))}{m_1}\right)^{n-1}$$

then any offer whose value exceeds x_1 will earn a strictly higher expected payoff when it is offered to a tier 1 applicant than it would if it were offered to a tier 2 (or lower) applicant. □

An immediate corollary is that for x in $(x_1,1]$, $\tilde{\pi}(x)=\frac{1}{m_1}$ is the probability with which an organization with an offer of value x makes an offer to any one of the m_1 tier 1 applicants. Note that this mixing probability $\tilde{\pi}(x)$ is independent of x on this interval. Furthermore, x_1 is the solution to

$$1+m_1\left(\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}-1\right)=F(x_1) \tag{0.13}$$

if a positive solution exists. If no solution exists with x_1 in $[0,1]$ then $x_1=0$ and all offers go to candidates in the highest tier.

The next Lemma extends the argument to all other active tiers.

Lemma. *Suppose that there is a tier $t>1$ and cutoff $x_{t-1}>0$ such that every symmetric equilibrium has the property that there is a sequence of pairs $\{(x_k,\pi_k)\}_{k=1,t-1}$ with $\pi_1=1$, and $0<\pi_k<1$ such that for any tier $k\le t-1$*

- *the probability that any offer x goes to an applicant in tier k conditional on the offer going to an applicant in tier k or higher is π_k ;*
- *the expected payoff associated with making an offer x to any tier $k\ge 1$ is the same for every $x\le x_{k-1}$;*
- *the expected payoff associated with making an offer to tier k is strictly higher than the expected payoff associated with making an offer to tier $k+1$ for every $x>x_k$.*

Then there is a cutoff $x_t\ge 0$ such that (0.1) to (0.1) hold when π_t satisfies

$$\pi_t=\frac{\pi_{t-1}}{\pi_{t-1}+\left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}}\frac{m_{t-1}}{m_t}} \tag{0.14}$$

and x_t satisfies

$$v_{t+1}=v_t\left(1-\frac{\pi_t}{m_t}\left(F(x_{t-1})-F(x_t)\right)\right)^{n-1} \tag{0.15}$$

or

$$F(x_{t-1})-\frac{m_t}{\pi_t}\left(1-\left(\frac{v_{t+1}}{v_t}\right)^{\frac{1}{n-1}}\right)=F(x_t)$$

Proof. From Lemma 7, organizations whose offer x lies in $[x_1, 1)$ make an offer to an applicant in tier 1 with probability $\pi_1 = 1$ independent of their type in any symmetric equilibrium because it is a dominant strategy. So (0.1), (0.1) and (0.1) hold for tier 1 and cutoff x_1.

For $x < x_{t-1}$, let $\tilde{\pi}_t(x)$ be the probability with which the offer is made to a tier t applicant and $1 - \tilde{\pi}_t(x)$ be the probability the offer is made to an applicant from one of the higher tiers. By (0.1), organizations with offers of value $\tilde{x} > x_{t-1}$ do not make offers to any applicants from tier below tier $t-1$, for example, from tier t. Then the payoff associated with an offer to any tier t applicant is

$$v_t \left(1 - \frac{\int_x^{x_{t-1}} \tilde{\pi}_t(\tilde{x})\, dF(\tilde{x})}{m_t}\right)^{n-1}. \tag{0.16}$$

The expected payoff to an offer of value $x < x_{t-1}$ made to any higher tier $k \leq t-1$ is the same by (0.1). the payoff to making the offer to a tier $t-1$ applicant is

(0.17)

$$v_{t-1} \left(1 - \frac{\int_x^{x_{t-1}} (1 - \tilde{\pi}_t(\tilde{x}))\, \pi_{t-1} dF(\tilde{x})}{m_{t-1}} - \frac{\pi_{t-1}(F(x_{t-1}) - F(x_{t-2}))}{m_{t-1}}\right)^{n-1}.$$

If there is an open interval below x_{t-1} where other organizations make their offers to v_t for sure. Let $\underline{x}$ and $\overline{x}$ be the greatest lower and least upper bounds on this region. Then for $x \in (\underline{x}, \overline{x})$, the payoff from making the offer to v_{t-1} is given by

$$v_{t-1} \left(1 - \frac{\int_{\overline{x}}^{x_{t-1}} (1 - \tilde{\pi}_t(\tilde{x}))\, \pi_{t-1} dF(\tilde{x})}{m_{t-1}} - \frac{\pi_{t-1}(F(x_{t-1}) - F(x_{t-2}))}{m_{t-1}}\right)^{n-1} \geq$$

$$v_t \left(1 - \frac{\int_{\overline{x}}^{x_{t-1}} \tilde{\pi}_t(\tilde{x})\, dF(\tilde{x})}{m_t}\right)^{n-1} >$$

$$v_t \left(1 - \frac{\int_x^{x_{t-1}} \tilde{\pi}_t(\tilde{x})\, dF(\tilde{x})}{m_t}\right)^{n-1}.$$

The first inequality follows because $x \geq \overline{x}$ makes an offer to an applicant in tier $t-1$ with positive probability. The second follows since $x < \overline{x}$ and and a set $x' \in (\underline{x}, \overline{x})$ of positive measure make an offer to an applicant in tier t with probability 1.

This gives a profitable deviation.

A similar argument when $\tilde{\pi}_t(x)$ is zero an any open interval, implies that in any symmetric equilibrium

$$(v_t)^{\frac{1}{n-1}} \left(1 - \frac{\int_x^{x_{t=1}} \tilde{\pi}_t(\tilde{x})\, dF(\tilde{x})}{m_1}\right) = \tag{0.18}$$

$$(v_{t-1})^{\frac{1}{n-1}} \left(1 - \frac{\int_x^{x_{t-1}} (1 - \tilde{\pi}_t(\tilde{x}))\, \pi_{t-1} dF(\tilde{x})}{m_{t-1}} - \frac{\pi_{t-1}(F(x_{t-1}) - F(x_{t-2}))}{m_{t-1}}\right)$$

for almost all x.

Uniform equality requires the derivatives of these two functions to be equal, or

$$(v_t)^{\frac{1}{n-1}} \frac{\tilde{\pi}_t(x)}{m_1} = (v_{t-1})^{\frac{1}{n-1}} \frac{(1 - \tilde{\pi}_t(x))}{m_{t-1}} \pi_{t-1}$$

which gives

$$\left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} \frac{m_{t-1}}{m_t}\tilde{\pi}_t\left(x\right) = \left(1-\tilde{\pi}_t\left(x\right)\right)\pi_{t-1}$$

$$\left(\left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} \frac{m_{t-1}}{m_t} + \pi_{t-1}\right)\tilde{\pi}_t\left(x\right) = \pi_{t-1}$$

or

$$\tilde{\pi}_t\left(x\right) = \frac{\pi_{t-1}}{\pi_{t-1} + \left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} \frac{m_{t-1}}{m_t}}.$$

This verifies properties (0.1) for tier t.

Finally, define x_t to be the solution to

$$v_t\left(1-\frac{\pi_t\left(F\left(x_{t-1}\right)-F\left(x\right)\right)}{m_t}\right)^{n-1} = v_{t+1}$$

to get (0.15). □

Combining Lemmas 1 and 2 gives the proof of Theorem .

Proof of Theorem 2.

Proof. From Corollary 1, $\pi_1^{(n)} = 1$ for all n, so that

$$\pi_2^{(n)} = \frac{1}{1+\left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} \frac{\alpha_1}{\alpha_2}}.$$

Then since $v_t < v_{t-1}$

$$\pi_2^{\infty} = \frac{1}{1+\frac{\alpha_1}{\alpha_2}} = \frac{\alpha_2}{\alpha_1+\alpha_2}. \tag{0.19}$$

So suppose that for $t' < t$

$$\pi_{t'}^{\infty} = \frac{\alpha_{t'}}{\sum_{s=1}^{t'}\alpha_s}.$$

Then

$$\lim_{n\to\infty}\pi_t^{(n)} = \frac{\pi_{t-1}^{\infty}}{\pi_{t-1}^{\infty}+\frac{\alpha_{t-1}}{\alpha_t}} = \frac{\frac{\alpha_{t-1}}{\sum_{s=1}^{t-1}\alpha_s}}{\frac{\alpha_{t-1}}{\sum_{s=1}^{t-1}\alpha_s}+\frac{\alpha_{t-1}}{\alpha_t}} =$$

$$\frac{\frac{1}{\sum_{s=1}^{t-1}\alpha_s}}{\frac{1}{\sum_{s=1}^{t-1}\alpha_s}+\frac{1}{\alpha_t}} =$$

$$\frac{\frac{1}{\sum_{s=1}^{t-1}\alpha_s}}{\frac{\alpha_t}{\left(\sum_{s=1}^{t-1}\alpha_s\right)\alpha_t}+\frac{\left(\sum_{s=1}^{t-1}\alpha_s\right)}{\left(\sum_{s=1}^{t-1}\alpha_s\right)\alpha_t}} =$$

$$\frac{\alpha_t}{\sum_{s=1}^{\alpha_t}\alpha_s}.$$

This gives (0.2).

For (0.3), start with the basic formula that determines each of the cutoffs:

$$v_t = v_{t-1}\left(1-\frac{\pi_{t-1}^{(n)}}{\alpha_{t-1}\left(n-1\right)}\left(F\left(x_{t-1}^{(n)}\right)-F\left(x_t^{(n)}\right)\right)\right)^{n-1}$$

The right hand side of this equation is continuous in x_t and attains it maximum value v_{t-1} when $x_t^{(n)} = x_{t-1}^{(n)}$. I attains its minimum at $x_t^{(n)} = 0$. So $x_t^{(n)}$ either has a unique solution or is 0, in which case the right hand side exceeds the left.

Now for each case in which a solution exists, take roots of both sides of this equation to get

$$\left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}} = 1 - \frac{\pi_{t-1}^{(n)}}{\alpha_{t-1}(n-1)}\left(F\left(x_{t-1}^{(n)}\right) - F\left(x_t^{(n)}\right)\right),$$

or

$$\left(1 - \left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}}\right)\frac{\alpha_{t-1}(n-1)}{\pi_{t-1}^{(n)}} = \left(F\left(x_{t-1}^{(n)}\right) - F\left(x_t^{(n)}\right)\right).$$

This solution for $F\left(x_t^{(n)}\right)$ is given by

$$F\left(x_t^{(n)}\right) = F\left(x_{t-1}^{(n)}\right) - \left(1 - \left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}}\right)\frac{\alpha_{t-1}(n-1)}{\pi_{t-1}^{(n)}}.$$

The right hand side is jointly continuous in $x_{t-1}^{(n)}$, $\pi_t^{(n)}$ and $\left(\frac{1}{n-1}\right)$ for $n > 1$. From our previous result,

$$\lim_{n\to\infty} \pi_t^{(n)} = \frac{\alpha_{t-1}}{\sum_{s=1}^{t-1}\alpha_s}$$

Now take limits.

$$\lim_{(n-1)\to\infty}\left(1 - \left(\frac{v_t}{v_{t-1}}\right)^{\frac{1}{n-1}}\right)\frac{\alpha_{t-1}(n-1)}{\pi_{t-1}^{(n)}} =$$

$$\lim_{\tau\to 0}\frac{\left(1 - \left(\frac{v_t}{v_{t-1}}\right)^{\tau}\right)}{\tau}\lim\frac{\alpha_{t-1}}{\pi_{t-1}^{(n)}} =$$

$$-\log\left(\frac{v_t}{v_{t-1}}\right)\sum_{s=1}^{t-1}\alpha_s$$

by L'Hopitals rule.

This along with continuity implies

$$F\left(x_t^{\infty}\right) = F\left(x_{t-1}^{\infty}\right) - \left(-\log\left(\frac{v_t}{v_{t-1}}\right)\right)\sum_{s=1}^{t-1}\alpha_s.$$

The result (0.3) follows by substituting for $F\left(x_{t-1}^{\infty}\right)$, $F\left(x_{t-2}^{\infty}\right)$ and so on. □

Proof of 3.

Proof. From Lemma 2

$$\pi_t\prod_{s=t+1}^{i}(1-\pi_s) =$$

$$\frac{\alpha_t}{\sum_{s=1}^{t}\alpha_s}\prod_{l=t+1}^{i}\frac{\sum_{s=1}^{l-1}\alpha_s}{\sum_{s=1}^{l}\alpha_s} =$$

$$\frac{\alpha_t}{\sum_{s=1}^{t}\alpha_s}\frac{\sum_{s=1}^{t}\alpha_s}{\sum_{s=1}^{t+1}\alpha_s}\cdots\frac{\sum_{s=1}^{i-1}\alpha_s}{\sum_{s=1}^{i}\alpha_s} =$$

$$\frac{\alpha_t}{\sum_{s=1}^{i}\alpha_s}$$

□

Proof of Theorem 4:

Proof. From (0.5) whenever $x \in [x_i, x_{i-1}]$,

$$Q_t(x) =$$

$$\left(1-\left(F\left(x_{i-1}\right)-F\left(x\right)\right)\frac{\pi_t}{m_t}\prod_{l=t+1}^{i}\left(1-\pi_l\right)-\sum_{j=t}^{i-1}\left(F\left(x_{j-1}\right)-F\left(x_j\right)\right)\frac{\pi_t}{m_t}\prod_{l=t+1}^{j}\left(1-\pi_l\right)\right)^{n-1}.$$

We can use the fact that the x_i and π_i all have finite non 0 limits and treat them as if they were constants to simplify the notation and take limits in this expression. It can be rewritten as

$$\left(1-\left(F\left(x_{i-1}\right)-F\left(x\right)\right)\frac{\pi_t}{\alpha_t\left(n-1\right)}\prod_{l=t+1}^{i}\left(1-\pi_l\right)-\sum_{j=t}^{i-1}\left(F\left(x_{j-1}\right)-F\left(x_j\right)\right)\frac{\pi_t}{\alpha_t\left(n-1\right)}\prod_{l=t+1}^{j}\left(1-\pi_l\right)\right)^{n-1}.$$

Using the previous limits define

$$\kappa_i^t=\sum_{j=t}^{i-1}\left(F\left(x_{j-1}\right)-F\left(x_j\right)\right)\frac{\pi_t}{\alpha_t}\prod_{s=t+1}^{j}\left(1-\pi_s\right)$$

and

$$G_i^t\left(x\right)=\left(F\left(x_{i-1}\right)-F\left(x\right)\right)\frac{\pi_t}{\alpha_t}\prod_{s=t+1}^{i}\left(1-\pi_s\right)$$

so that we can write

$$Q_t(x) =$$

$$\left(1-\frac{\left(G_i^t\left(x\right)+\kappa_i^t\right)}{\left(n-1\right)}\right)^{n-1}$$

and we want to calculate

$$\lim_{(n-1)\to\infty}\left(1-\frac{\left(G_i^t\left(x\right)+\kappa_i^t\right)}{\left(n-1\right)}\right)^{n-1}.$$

This limit can be evaluated by taking the exponential of the limit of its log, so we want:

$$\lim_{(n-1)\to\infty}\left(n-1\right)\log\left(1-\frac{\left(G_i^t\left(x\right)+\kappa_i^t\right)}{\left(n-1\right)}\right)=$$

$$\lim_{y\to 0}\frac{\log\left(1-y\left(G_i^t\left(x\right)+\kappa_i^t\right)\right)}{y}=$$

$$\lim_{y\to 0}\frac{-\left(G_i^t\left(x\right)+\kappa_i^t\right)}{1-y\left(G_i^t\left(x\right)+K_i^t\right)}=-\left(G_i^t\left(x\right)+\kappa_i^t\right)$$

Then we have

$$Q_t\left(x\right)=e^{-\left(G_i^t(x)+\kappa_i^t\right)}$$

□

Appendix: Proof of Proposition 6.

Proof. From (0.4) and the fact that $\tilde{p}_1 > 0$ for all x, it is apparent that $Q_1(x)$ is strictly increasing in x.

Secondly, observe that

$$\tilde{\pi}_1(x) = \prod_{s=2}^{i} (1 - \pi_s)$$

when $x \in [x_i, x_{i-1}]$. Then as x increases, $\tilde{\pi}_1(x)$ is either constant, or increases since one of the terms in the product is dropped.

As a consequence the function $\tilde{\pi}_1(x) Q_1(x)$ is strictly increasing. So

$$\int_0^1 \tilde{\pi}_1(x') Q_1(x') dF_s(x') > \int_0^1 \tilde{\pi}_1(x') Q_1(x') dF_{s'}(x')$$

by stochastic dominance whenever $s' > s$. □

Appendix: Proof of Proposition.

Proof. For $t > 1$

$$\frac{\alpha_{t-1}}{\alpha_t} \frac{v_t}{v_{t-1}} > 1.$$

Then $q_\tau^{t-1} - q_\tau^t > 0$.

For $t > 1$

$$q_\tau^{t-1} - q_\tau^t =$$

$$\sum_{i=t-1}^{k} \int_{x_i}^{x_{i-1}} \frac{\alpha_{t-1}}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} dF_\tau(\tilde{x}) -$$

$$\sum_{i=t}^{k} \int_{x_i}^{x_{i-1}} \frac{\alpha_t}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^{t}(x)+\kappa_i^{t}\right)} dF_\tau(\tilde{x}) =$$

$$\int_{x_{t-1}}^{x_t} \frac{\alpha_{t-1}}{\sum_{s=1}^{t-1} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} dF_\tau(\tilde{x})$$

$$+ \sum_{i=t}^{k} \int_{x_i}^{x_{i-1}} \left(\frac{\alpha_{t-1}}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} - \frac{\alpha_t}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^{t}(x)+\kappa_i^{t}\right)} \right) dF_\tau(\tilde{x}) =$$

$$\int_{x_{t-1}}^{x_t} \frac{\alpha_{t-1}}{\sum_{s=1}^{t-1} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} dF_\tau(\tilde{x}) +$$

$$\sum_{i=t}^{k} \int_{x_i}^{x_{i-1}} \left(\frac{\alpha_{t-1}}{\sum_{s=1}^{i} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} - \frac{\alpha_t}{\sum_{s=1}^{i} \alpha_s} \frac{v_{t-1}}{v_t} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} \right) dF_\tau(\tilde{x}) =$$

$$\int_{x_{t-1}}^{x_t} \frac{\alpha_{t-1}}{\sum_{s=1}^{t-1} \alpha_s} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} dF_\tau(\tilde{x}) +$$

$$\sum_{i=t}^{k} \int_{x_i}^{x_{i-1}} \left(\frac{\alpha_{t-1} - \alpha_t \frac{v_{t-1}}{v_t}}{\sum_{s=1}^{i} \alpha_s} \right) e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} dF_\tau(\tilde{x}) > 0.$$

The substitution in the last expression comes from the basic property of the mixed strategy equilibrium

$$v_{t-1} e^{-\left(G_i^{t-1}(x)+\kappa_i^{t-1}\right)} = v_t e^{-\left(G_i^{t}(x)+\kappa_i^{t}\right)}$$

which just means that if an institution with an offer of value x is making the offer to both a tier t and tier $t-1$ applicant with positive probability, then it must receive the same expected payoff from both. □