UNOBSERVED MECHANISMS

LI, HAO AND MICHAEL PETERS
VANCOUVER SCHOOL OF ECONOMICS

Abstract. This paper considers the possibility that some buyers who participate in a selling mechanism do not know the trading rules that the seller is using to allocate goods. We consider the simplest model in which all bidders have identically and independently distributed private values, and model the mechanism design process as a game of imperfect information in which some players cannot observe the commitments of the seller. We show that independent of how likely it is that players are uninformed, standard auction formats cannot survive as equilibria unless they are augmented with messages that allow informed players to identify themselves. If there is a strictly positive probability that there is an uninformed buyer, there is an equilibrium in which the seller holds an auction among the informed bidders with a reserve price that varies with the actual number of uninformed bidders. The lowest of these reserve prices is strictly less than the optimal reserve price with full commitment. If no informed bidder meets the reserve price, the seller makes a take it or leave it offer to one of the uninformed bidders. This offer is at least as high as the optimal reserve price with full commitment.

1. Introduction

One of the important lessons of mechanism design is the 'power of commitment'. An optimal auction, for example, requires the seller to set a reserve price. If no buyer chooses to bid that price, the seller is supposed to commit himself never to trade. Ex post, the seller won't want to carry out that commitment, as the large literature on auctions with resale explains. Nonetheless, the seller will benefit ex ante if he can convince bidders that he will honor that commitment.

Commitment is something that is relatively easy to accomplish in digital markets. Price offers are generated by computer programs that cannot be easily re-written. Yet as pricing mechanisms begin to appear in digital markets, it is apparent that there is another important assumption required by mechanism design - one that is usually not discussed at all. Even if a seller can commit, this might be of limited benefit if buyers do not understand what he has committed to.

The fact that buyers don't notice seller commitments has been widely documented in the marketing literature. (Dickson and Sawyer 1990), for example, asked buyers in supermarkets about their price knowledge as they were shopping. Only 50% of all respondents claimed to know the price of the object they had just just put in their basket. Even when the item being placed in the basket had been specially marked down and heavily advertised, 25% of consumer didn't even realize the good was on special.

Supermarket prices are staring consumers in the face as they look at the shelves. It would seem much harder for consumers to be aware of selling techniques on the internet. There is a pricing technique used by many websites called *click stream pricing.* The idea behind it is that the price offer that is made to a customer can be made to depend on exactly what the buyer does before he or she gets to the offer page (i.e., the stream of clicks that leads them to the website). An example might be offering a higher price to a consumer who searches directly for a deluxe model of a product than the price offered to a consumer who searches for the standard model then asks how much it would cost to upgrade.

This propensity for being uninformed may be one reason why airlines, for example, make so little effort to explain the details of their pricing algorithms to buyers.

The point of this paper is to consider the implications of this possibility for the behavior and payoffs of informed bidders in the best known mechanism design environment of all - independent private value auctions.[1]

Our seller will realize that some buyers are uninformed, but won't be sure how many uninformed buyers there are, or who they are. Our buyers will be in a similar position. Those who are uninformed will understand that they don't understand the seller's mechanism. Informed buyers will know their own values and understand that there are uninformed buyers (but again, won't know the identities of the uninformed buyers).

We emphasize that our uninformed bidders aren't behavioral - they have rational expectations in the equilibria we construct. Instead, we model them as randomly attentive in the fashion of (Masatloglu 2015) or (Masatloglu, Nakajima, and Ozbay 2012). Yet they understand their own inattention. Formally, we treat the mechanism design process as a game of imperfect information in which inattentive buyers don't see part of the history.

[1]See reference in, for example, (Krishna 2010).

Our main results come in two parts. First we show that standard auction formats simply aren't robust to the possibility that there are uninformed buyers. The key to this result is that when a seller 'deviates' and changes his mechanism, informed buyers will understand the change and respond to it in the usual way. Uninformed buyers simply won't respond at all.

What this does is to give the seller the opportunity to extract surplus from the uninformed without losing any surplus from the informed. To see this consider a second price auction. Suppose the uninformed believe that the seller is holding a second price auction. In the usual way, a best response is for them to bid their values. The informed bidders know whether or not the seller is using a second price auction, so, of course, they also bid their values.

The seller can then deviate, explaining to the (informed) buyers that if they want to bid in a second price auction, then they have to provide a certain password along with their bid.[2] If the high bidder in the auction has given the password, his payment will be the second highest bid. If the high bidder does not give the password, his payment will be whatever he bid.

The password analogy is one we'll use again below. Examples of passwords abound. For example, with simple click stream pricing, the password is just the stream of clicks needed to find the low price offer. Meet the competition (match lower prices elsewhere) require buyers to provide information uninformed buyers don't have. Super-saver clubs can literally ask consumers for password to get discounts. Coupons have to be collected to be used in future transactions.

When some bidders are uninformed, buyers are treated asymmetrically. For this reason, we'll sometimes refer to the mechanisms we construct below as discriminatory auctions. Discrimination is similar to separation in the 'separating equilibrium' sense but different in that informed bidders have to verify that they are informed before they are given special treatment.

Informed bidders can verify their type by providing a password, but they don't have to unless there is an advantage to doing so. This fact is the core argument in much of what follows below. In the example above, it is straightforward that an informed bidder can't gain by pretending to be uninformed. The uninformed pay more (because they

[2]The term 'password' is just used by way of analogy. In practice, this is where the 'click stream pricing' method, and other things like it would be used. Burying lower prices beneath a complex web of click streams is a way to provide a password since the informed bidders will discover the correct stream, while the uninformed won't.

have to pay their bid instead of the second highest bid) for the same probability of winning. Alternatively, if the informed bidder pretends to be uninformed and submits a bid equal to what he expects to pay as an informed bidder, he will win a lot less often. So in this simple deviation, there is no incentive for informed bidders to pretend to be uninformed.

In fact, we show that they are discriminatory in a special way. Uninformed bidders will always receive a take it or leave it offer. From their perspective ex ante, the expected value of this offer must be independent of their valuation. This result suggests an explanation for the 'Buy it Now' option on eBay. In general, this offer might depend on how many uninformed bidders are actually participating in an auction - something the seller can see and condition on ex post. We provide a reasonably realistic condition under which the take it or leave offer is independent of the number of informed and uninformed bidders. In that sense, looks like eBay's *Buy it Now.*

Our model does suggest that reserve prices will vary with the number of uninformed bidders, which doesn't happen in eBay auctions. We analyze a variant of our model in which sellers are required to set a uniform reserve price and show that in this case the reserve price for informed bidders and the take it or leave offer to the uninformed must be the same.

We show that uninformed bidders always reduce the seller's revenues relative to what they would be with full observability. Uninformed bidders are offered higher prices than they would be if there were no commitment at all. Low value informed bidders generally benefit from uninformed bidders presence, higher value bidders may or may not.

1.1. **Literature.** As mentioned above, the idea that consumers might not notice prices in an old one in the marketing literature, as in (Dickson and Sawyer 1990) and references there in. The approach had been used earlier in economics, as in, say (Butters 1977), in which buyers randomly observe price offers in a competitive environment.[3] In that literature, firms advertise only prices which some buyers see, while others don't.[4] These papers considered the same problem that we do, which is how this unobservability would affect the prices that firms offer. The difference here is that we are interested in mechanisms, not prices.

[3]See also (Varian 1980), (Stahl 1989), or (Stahl 1994).

[4]Varian calls buyers informed if they see prices of all firms, and uninformed if they don't.

To our knowledge, the only other paper that uses this approach with mechanisms is (Peters 2014) which uses the idea in a competitive setting in which the mechanism design problem is much simpler. The problem we consider here is not competitive. We are interested in the more basic issue of how unobservability affects mechanism design.

As for mechanism design, our basic problem involves a standard mechanism design problem in which buyers have outside options which vary with their type. Such a problem was studied by (McAfee 1993). His model had buyers whose outside option involved waiting until next period to purchase. Here, though the outside option does vary with type, it is endogenous in the sense that the seller can change it by modifying the offer he makes to uninformed bidders. Outside options were endogenous, at least with respect to the seller's mechanism design problem, in (McAfee 1993).

2. The model

There are n potential buyers of a single homogenous good. Each bidder has a privately known valuation that is independently drawn from the interval $[0, 1]$. Assume for the moment, all values are distributed according to some differentiable distribution F. The seller is assumed to have zero cost of giving up the good. There is a *common* message space $\mathcal{M}$ which is used by all buyers to communicate with the seller. A message from bidder i will be denoted b_i.

In modern digital markets, b_i represents some kind of observable browsing history that sellers use to detect a bidder's type. We'll assume that the set of bidder values can be embedded in the set of messages and that the set of feasible messages is common knowledge.

The point of the seller's mechanism is to generate an *offer.* It is important to note that even bidders who are uninformed can understand a direct take it or leave it offer. This is the one commitment that is feasible with every buyer. The seller's indirect mechanism converts profiles of messages into the identity of the 'winning' bidder, that is, the bidder who receives the offer, and offer itself.

We assume that if this offer is refused, the game ends without trade. It would be possible, of course, to allow the seller to make a series of offers until one is accepted, or they are all rejected. We don't consider this case here.

In a game theoretic sense, what differentiates this paper from the previous literature is that we model the mechanism design process is a game of *imperfect* information in the sense that some buyers cannot observe the mechanism that is chosen by the seller. We assume each buyer independently sees the seller's mechanism with probability $(1-\mu)$. Our solution concept is just perfect Bayesian equilibrium, so buyer who don't observe the mechanism will none the less correctly guess what it is in equilibrium.

With this preamble, we can describe sellers' mechanisms more formally. There are n buyers each of whom wants to buy one unit of a good from a seller who has exactly one unit. Buyers' values are identically and independently distributed F on the interval $[0,1]$. Buyers' payoff when they buy at price p is given by $v-p$. The seller's cost is zero, so his profit if he sells at price p is just p.

A mechanism γ for the seller is a mapping from $\mathcal{M}^N$ into $[0,1] \times \triangle(N)$ where N is the set of bidders, and $\triangle(N)$ means the set of probability distributions on N. Γ is the set of mechanisms. The interpretation is that $(b_1, \ldots, b_n) \rightarrow (p_i, q_i)$ means that the seller makes a take it or leave it offer p_i to buyer i with probability q_i when the profile of messages is b. The seller has full commitment power, which every bidder knows. However, only the informed bidders see the commitments. Uniformed bidders need to guess them.

A strategy rule for a bidder is a function $\sigma_i : [0,1] \times \{0,1\} \times \Gamma \rightarrow \mathcal{M}$ that specifies what message the bidder will send for each of his types conditional on whatever he or she knows about the seller's mechanism. In what follows the bidders type is a pair (v, ι) where ι takes value 0 when the bidder in uninformed, and 1 when the bidder is informed. Since an uniformed bidder never sees the mechanism a seller offers, we have the informational constraint

$$\sigma(v_i, 1, \gamma) = \sigma(v_i, 1, \gamma') = \sigma(v_i, 1)$$

for all γ and γ'.

Notice that this definition restricts buyers to pure strategies. We retain this assumption throughout the paper.

As mentioned above, informed bidders can pretend to be uninformed, but not conversely. This allows the seller to identify informed bidders by, for example, asking for a password along with a bid. To prevent the uninformed bidders from guessing this password, it has to be random. One way to understand this is to imagine that the seller randomizes over mechanisms in order to make this password unpredictable. Refer to this mixture as $\psi \in \triangle(\Gamma)$. Let $R(\gamma, \sigma(\cdot, 1), \sigma(\cdot, 0, \gamma))$ be the expected revenue for the seller when informed and uninformed bidders use strategy rules $\sigma(\cdot, 0, \gamma)$ and $\sigma(\cdot, 1)$ respectively.

A perfect Bayesian equilibrium for this problem is a mixture ψ for the seller, and a pair of strategy rules $(\sigma(\cdot, 1), \sigma(\cdot, 0, \gamma))$ for uninformed and informed bidders respectively, which satisfy

$$\mathbb{E}_{v_{-i}}\left\{q_i\left(\sigma_i\left(v,0,\gamma\right),\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\max\left[\left(v_i-p_i\left(\sigma_i\left(v,0,\gamma\right),\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\right),0\right]\right\}\geq$$

$$\mathbb{E}_{-i}\left\{q_i\left(b_i,\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\max\left[\left(v_i-p_i\left(b,\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\right),0\right]\right\} \tag{2.1}$$

for all b and γ;

$$\mathbb{E}_{v_{-i},\gamma}\left\{q_i\left(\sigma_i\left(v,1\right),\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\max\left[\left(v_i-p_i\left(\sigma_i\left(v,1\right),\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\right),0\right]|\psi\right\}\geq$$

$$\mathbb{E}_{-i,\gamma}\left\{\max q_i\left(b,\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\left[\left(v_i-p_i\left(b,\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)\right)\right),0\right]|\psi\right\} \tag{2.2}$$

and

$$\mathbb{E}_{\gamma}\left\{R\left(\gamma,\sigma\left(\cdot,1\right),\sigma\left(\cdot,0,\gamma\right)\right)|\psi\right\}\geq$$

$$R\left(\gamma',\sigma\left(\cdot,1\right),\sigma\left(\cdot,0,\gamma'\right)\right) \tag{2.3}$$

for all γ'.

In the expressions (2.1) and (2.2), the terms $\sigma_{-i}\left(v_{-i},\iota_{-i},\gamma\right)$ should be understood to mean that whenever one of the bidders other than i is uninformed, they use the strategy $\sigma\left(\cdot,1\right)$. The reason that the max operation appears when taking expectations is because a mechanism generates an offer instead of an outcome. It will be assumed throughout that this offer is take it or leave it. As mentioned above, if it is refused there is no trade at all.

Relationship to Standard mechanism design. Once the outcome of the seller's randomization is realized, the equilibrium of the continuation game is simple Bayesian equilibrium. So we could try to analyze it using direct mechanisms in which buyers' types consist of both their value and whether or not they are informed. However, there some important issues to bear in mind.

The first is that information about whether or not a buyer is informed is quite different from information about his type because the buyer can verify to the seller that he is informed.[5] Being informed in this problem is similar to 'being able to speak French', or being able to play piano. We explained above how the seller could give the buyer the opportunity to reveal this part of his or her type. The seller randomizes over mechanisms, and includes in each mechanism a password - for example a real number selected in the interval $[0,1]$ using a uniform distribution. The seller would publish this password as part of the mechanism. The informed buyers see it, the uninformed don't.

[5]Verifiable information is discussed in (Green and Laffort 1986) or (Strauss 2016) among others.

The second issue comes from the fact that a mechanism is just a move in an imperfect information game. Since uninformed bidders can't see deviations by the seller, uninformed buyers with different types will receive the same offer in our equilibrium. Of course, it is possible to interpret a mechanism like this as a direct mechanism in which the seller commits to make an outcome that is independent of buyer type. This doesn't quite work here since if uninformed buyers do report their types, the seller can no longer commit not to use the information contained in those reports. As a consequence we need to retain, at least initially, much of the structure of the indirect mechanism.

Third, the outcomes that are produced by the seller in response to messages aren't like outcomes in a standard auction. Here, an outcome is just an offer. It is possible that buyers in equilibrium will receive offers that they won't want to accept. As will be discussed, the standard reduced form functions have to be reinterpreted in order to make them work.

Example. As an example to illustrate how the imperfect information part of the story affects the analysis, suppose that the seller is using a second price auction in which bids are submitted in US dollars. The high bidder wins and pays the second highest US dollar bid. We would argue that this can never be an equilibrium mechanism.

To see why, suppose the seller deviates to a new mechanism in which bids can be either in Canadian or US dollars. The bidder who submits the highest bid (exchange rate adjusted) wins the auction. If the winning bid was submitted in Canadian dollars, the winner pays the second highest bid whether it is in US dollars or in Canadian dollars. If, on the other hand, the winning bid is submitted in US dollars, the winner pays his bid.

The logic is that only informed bidders realize they can submit bids in Canadian dollars. If they choose to do so, they know that their bid will be treated in the usual second price way, so they will prefer to bid their value expressed in Canadian dollars. Uninformed bidders will not observe the deviation and erroneously believe that their US dollar bids will be treated as values in a second price auction. When they win they will be offered a trade at the value that they bid. This allows the deviating seller to extract all the surplus of the uninformed bidder when they win the auction.

The only possible complication in this story comes from the fact that the informed bidders don't have to bid in Canadian dollars. The reason they won't want to is simply that any bid they submit in US dollars will result in a strictly higher payment than the corresponding bid in Canadian dollars. So bidding in Canadian dollars is a dominant strategy for any informed bidder who wants to trade.

One way to express this is all equilibrium mechanisms will be discriminatory. Standard auction forms as they are usually studied can never be equilibrium mechanisms when there are uninformed bidders.

Revelation Principle. For the remainder of the paper, we'll focus on mechanisms that are symmetric with respect to bidder's identities and assume that continuation equilibrium are fully symmetric and involve pure strategies. A profile of types $\{(v_1, \iota_1), \ldots, (v_n, \iota_n)\}$ then results in trader i receiving an offer with probability

$$q\left(\sigma_i\left(v_i, \iota_i\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right).$$

The offer made in this case will be

$$p\left(\sigma_i\left(v_i, \iota_i, \gamma\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}, \gamma\right)\right).$$

Without loss, we can assume that $q\left(\sigma_i\left(v_i, \iota_i\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right) > 0$ if and only if there is a v' such that

$$\sigma_i\left(v', \iota_i\right) = \sigma_i\left(v_i, \iota_i\right)$$

and

$$p\left(\sigma_i\left(v', \iota\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right) \leq v'$$

. Any outcome function that doesn't satisfy this property is payoff equivalent to one that does. So we'll think of $q\left(\sigma_i\left(v_i, \iota_i\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right)$ as the probability with which i receives a 'serious' offer.

Similarly, using payoff equivalence, if there is a price

$$p\left(\sigma_i\left(v', \iota\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right)$$

such that

$$q\left(\sigma_i\left(v', \iota\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right) = 0,$$

we can assume

$$p\left(\sigma_i\left(v', \iota\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right) = 1.$$

Now define a reduced form probability as follows;

$$Q\left(v_i, \iota_i\right) \equiv \mathbb{E}_{v_{-i}, \iota_{-i}}\left\{q\left(\sigma_i\left(v_i, \iota_i\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right)\right\}.$$

Define

$$\mathcal{P}\left(v', \tilde{p}, \iota\right) \equiv$$
$$\Pr\left\{p\left(\sigma_i\left(v', \iota\right), \sigma_{-i}\left(v_{-i}, \iota_{-i}\right)\right) \leq \tilde{p} | Q\left(v', \iota\right) > 0\right\}.$$

Finally,

$$P\left(v, \iota\right) =$$

$$\begin{cases} \int \tilde{p} d\mathcal{P}\left(v, \tilde{p}, \iota\right) & \text{if } Q\left(v, \iota\right) > 0 \\ \inf_{v': Q(v', \iota) > 0} \int \tilde{p} d\mathcal{P}\left(v', \tilde{p}, \iota\right) & \text{otherwise.} \end{cases} \tag{2.4}$$

This definition forces $P(v,\iota)$ to be equal to or close to an offer that is made with positive probability.

As in standard mechanism design, the payoff to a buyer of type v_i in the continuation equilibrium is

$$V(v_i,\iota_i) \equiv$$
$$Q(v_i,\iota_i)\cdot\int\{\max[v_i-\tilde{p},0]\}\,d\mathcal{P}(v_i,\tilde{p},\iota).$$

By standard arguments, the function $V(v_i,\iota_i)$ is continuous in v_i.

The following Proposition helps simplify the argument.

Proposition 1. *In any symmetric equilibrium $P(v,1)$ is independent of v.*

Proof. Suppose the theorem is false and that there is a pair $v'$and v such that $P(v',1) > P(v,1)$. We can assume $Q(v',1)$ and $Q(v,1)$ are both strictly positive. This is because the definition (2.4) forces $P(v,1)$ for buyer types who don't receive offers to be either the same, or arbitrarily close to the expected price for buyers who do receive offers.

By construction, all offers in the support of $\mathcal{P}(v',\cdot,1)$ are accepted by some buyer type v'' for whom $\sigma_i(v'',1)=\sigma(y',1)$, so we can choose v'' such that $V(v'',1)>0$. Let

$$\underline{v}(v') \equiv \inf\{v'':\sigma(v'')=\sigma(v');V(v'',1)>0\}.$$

We claim that $\tilde{p}\geq\underline{v}(v')$ for almost all $\tilde{p}$ in the support of $\mathcal{P}(v',\cdot,1)$. If this were false, the seller could raise revenue by increasing his price offer on a set on a set of positive measure. Since uninformed buyers would not observe this change in the price offer, their reporting behavior would not be affected. Uninformed buyers whose values are at least $\underline{v}(v')$ would continue to accept these offers.

The implication of this is that buyers whose values are close to $\underline{v}(v')$ will have expected payoffs arbitrarily close to zero.

If $P(v,1)<P(v',1)$, then

$$\int_0^{\underline{v}(v')} d\mathcal{P}(v,\tilde{p},1)>0$$

which violates incentive compatibility since buyers whose values are close enough to $\underline{v}(v')$ could profitably deviate by adopting the strategy $\sigma_i(v,1)$. □

Say that a mechanism is *non-discriminatory* if $(Q(v,0),P(v,0)) = (Q(v,1),P(v,1))$ for all v. Proposition 1 implies that informed bidders must also receive a single take it or leave it offer if the mechanism is non-discriminatory. We already know this can't be an equilibrium with informed bidders since the seller can deviate and offer the informed bidders a second price auction without affecting the uninformed and without violating incentive compatibility.

The only situation in which a non-disciminatory mechanism can be supported is in the trivial case in which there are no informed bidders.

Proposition 2. *If all bidders are uninformed for sure, then there is a unique equilibrium in which the seller randomly selects a buyer then makes him or her an offer that maximizes* $(1 - F(p))\,p$.

Except in the case where all bidders are uninformed with probability 1, this can never be an equilibrium. The reason is that the seller can deviate to a mechanism that offers the optimal auction to informed bidders with a reserve price equal to the take it or leave it price for the uninformed. This must always increase seller's surplus.

The proof of Proposition 1 also illustrates why uninformed bidders are bad for sellers. The seller's best revenue occurs when he or she has full commitment power. Uninformed bidders can't see the commitment made by the seller directly. This creates a situation in which the seller can no longer commit not to use information revealed by the uninformed during communication. Even though sellers have full commitment power in a technological sense, uninformed bidders take that commitment power away from them.

Informed Bidders. In what follows, we make the usual assumption that virtual valuation is increasing. The virtual valuation is given by the function

$$\phi(v) = v - \frac{1 - F(v)}{f(v)}.$$

Recall that this implies that there is a single value r^* where $\phi(r^*) = 0$. This value r^* represents the optimal reserve price in a standard auction independent of the number of bidders. The reserve price is chosen so that the virtual valuation of the buyer whose value is just equal to the reserve price is equal to the value of the outside option to the seller. Since this optimal reserve price is independent of the number of bidders in the auction, $r^*$is also the optimal take it or leave if offer when the seller is dealing with a single buyer.

When $\mu = 1$, we have the result that there is no informative communication from the n bidders, and the seller randomly chooses one of them to make the offer of r^*.

More generally, anonymous outcome functions will depend on the number of informed and uninformed bidders, but not on their identities.

So we can write $q_m((v_i, \iota_i), (v_{-i}, \iota_{-i}))$ as the probability with which i can expect to receive a serious offer when m of the bidders other than i are uninformed.

Payments by informed bidders can be determined in the usual way using incentive conditions. However, if the seller decides to make an offer to an uninformed bidder he must decide what that offer will be. This offer can't depend on the uninformed bidder's type, but it can depend on how many bidders are informed. Let $\{p_1, \ldots, p_n\}$ be a series of take it or leave it offers to the uninformed. The prices are indexed by the number of uninformed bidders that participate.

Definition. A mechanisms is said to have *pooling of uninformed buyers* if $q((v, 0), (v_{-i}, \iota_{-i})) = q((v', 0), (v_{-i}, \iota_{-i}))$ for all v and v' .

We will focus on mechanisms that pool uninformed buyers in what follows. This amounts to an equilibrium refinement. When buyers are indifferent among a number of different messages only because all messages they send yield the same zero payoff, they could send the seller valuable information. We'll assume they don't.

Let B_m^μ be the probability that there are exactly m uninformed bidders among $n-1$ bidders. This is given by the binomial distribution:

$$B_m^\mu = \binom{n-1}{m} (1-\mu)^{n-1-m} \mu^m.$$

Then, the truth-telling payoff of an informed bidder with value v_i is given by

$$U(v_i) =$$

(2.5)
$$\sum_{m=0}^{n-1} B_m^\mu \mathbb{E}\left(q_i^m((v_i, 0), (v_{-i}, \iota_{-i})) \max\left[v_i - p_m((v_i, 0), (v_{-i}, \iota_{-i}))\right]\right).$$

Proposition 3. *If $p(1 - F(p))$ is a concave function and the virtual valuation function is strictly increasing, then in any equilibrium mechanism that involves pooling of uninformed buyers, the seller's mechanism will consist of a take it or leave it price p' and a series of n reserve prices $\{r_0, \ldots, r_{n-1}\}$ that maximize*

$$\mu p'(1 - F(p')) \sum_{m=0}^{n-1} B_m^\mu \left(F^{n-m-1}(r_{m+1})\right) +$$

$$\mu \sum_{m=0}^{n-1} B_m^\mu \int_{r_{m+1}}^1 (n-1-m)\left(F^{n-m-2}(v_i)\left\{v_i - \frac{1-F(v_i)}{f(v_i)}\right\}\right) dF(v_i) +$$

$$(1-\mu)\,p'\,(1-F(p'))\sum_{m=1}^{n-1}B_m^{\mu}\left(F^{n-m}(r_{m+1})\right)+$$

$$(1-\mu)\sum_{m=0}^{n-1}B_m^{\mu}\int_{r_m}^{1}(n-m)\left(F^{n-m-1}(v_i)\left\{v_i-\frac{1-F(v_i)}{f(v_i)}\right\}\right)dF(v_i)$$

subject to the bidding constraint

$$\sum_{m=0}^{n-1}B_m^{\mu}\left\{\int_{r_m}^{v_i}F^{n-1-m}(\tilde{v})\,d\tilde{v}\right\}\geq$$

$$\max\left[v_i-p',0\right]\sum_{m=0}^{n-1}B_m^{\mu}\frac{1}{m+1}F^{n-1-m}(r_{m+1}) \tag{2.6}$$

for every v_i.

The significance of this Proposition is to reduce the construction of the equilibrium so that it can be stated as the solution to a constrained optimization problem. The decision variables are $n+1$ constants - the take it or leave it offer to the uninformed, p', and the n reserve prices $\{r_0,\ldots,r_n\}$.

The objective function is derived using fairly standard arguments in mechanism design. It is basically an adjustment of the usual argument that is used to derive the optimal reserve price in an auction. Indeed, notice that absent the bidding constraint (4.3), then the point wise solution to the problem of maximizing (4.1) is given by

$$q_i^m(v_i,0)=$$

$$\begin{cases}1 & m>0;\left\{v_i-\frac{1-F(v_i)}{f(v_i)}\right\}\geq\max\left[p'(1-F(p')),\max_{j>m}\left\{v_j-\frac{1-F(v_j)}{f(v_j)}\right\}\right]\\ 1 & m=0;\left\{v_i-\frac{1-F(v_i)}{f(v_i)}\right\}\geq\left\{v_j-\frac{1-F(v_j)}{f(v_j)}\right\}\geq 0\\ 0 & \text{otherwise.}\end{cases}$$

because of the linearity of payoffs in the allocation rule.

What this expression says is that the seller should hold an auction with a reserve price that satisfies

$$r-\frac{1-F(r)}{f(r)}=p'(1-F(p'))$$

when there are uninformed bidders, and with a reserve price such that

$$r-\frac{1-F(r)}{f(r)}=0$$

otherwise.

This allocation rule won't typically be feasible. To see why, it might help to consider Figure 1.

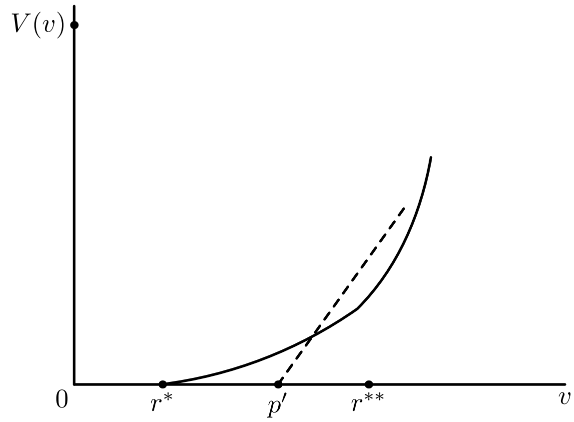

Figure 1 describes the payoffs to buyers in the case where there are just two of them, and the seller tries to implement a scheme like the one described above. When there is one uninformed bidder, the seller decides to offer one of the uninformed buyers (if there is one) a take it or leave it offer equal to p' unless an informed bidder submits a value which supports a virtual surplus at least $p'(1 - F(p'))$. This value is given by r^{**} in the diagram.

If both bidders are informed, the seller follows the strategy above and holds an auction with reserve price r^*

The curved line gives the payoff to informed buyers of different types. An informed buyer of type r^*receives an offer below his value if both buyers are informed, and the other buyer has a lower value.[6]

The dashed line represents the payoff that the informed bidder could attain by pretending to be uninformed. He'll receive the offer p' if the other buyer is also uninformed. Even if the other buyer is informed, he'll still receive an offer if the other buyer has a value below r^{**}. The picture indicates he'll do better by pretending to be uninformed, so the allocation isn't incentive compatible.

Lemma 4. *In any solution,* $r_0 < r^*$, $p' > r^*$ *for all* $m = 0, \ldots, n-1$, $r_m - \frac{1-F(r_m)}{f(r_m)} < p'(1 - F(p'))$.

Proof. If $r_0 > r_*$, then it is higher than the optimal reserve price in an auction with n informed bidders. This means revenue would be increased by reducing it. Since the lower reserve price will make the auction more attractive to the informed, a reduction in r_0 will relax the

[6]Notice the kink in this payoff to reflect the fact that higher value buyers will also receive a profitable offer if the other buyer is uninformed.

bidding constraint. So any profile of reserve prices for which $r_0 > r^*$ cannot be part of an equilibrium.

Similarly, if $p' < r^*$, then by the definition of r^*, $p'(1 - F(p')) < r^*(1 - F(r^*))$. Since raising p' can only relax the bidding constraint, $p' < r^*$ cannot be part of any equilibrium.

If $r_m - \frac{1-F(r_m)}{f(r_m)} > (1 - F(p'))p'$ for some m, then by marginally decreasing r_m, the seller increases the revenue while relaxing the bidding condition. Again, any profile for which this is true cannot be part of an equilibrium.

Finally, all the inequalities in the statement of the lemma are strict. To see why, observe that either

$$r_m - \frac{1 - F(r_m)}{f(r_m)} < p'(1 - F(p'))$$

or $p' > r^*$, for otherwise the bidding constraint is violated. This is because for any value v for an informed bidder that is close to r^*,

$$\sum_{m=0}^{n-1} B_m^\mu \left\{ \left(F^{n-1-m}(v)\, v - \int_{r^*}^{v} F^{n-1-m}(\tilde{v})\, d\tilde{v} \right) \right\} \leq$$

$$\sum_{m=0}^{n-1} B_m^\mu F^{n-1-m}(v)(v - r^*) \leq$$

$$\sum_{m=0}^{n-1} B_m^\mu \frac{1}{m+1} F^{n-1-m}(r_{m+1})(v - r^*)$$

for v close enough to r^*, so the bidding constraint is violated. Then if, for example,

$$r_m - \frac{1 - F(r_m)}{f(r_m)} = p'(1 - F(p'))$$

for any m, the first order impact of lowering the reserve price below r_m on profits conditional on m uninformed bidders is zero. Since this relaxes the incentive constraint, the first order impact of adjusting the reserve price (or the offer p') back toward r^* will be positive, and expected revenue will rise. □

Lemma 4 makes it possible to see who gains and loses from the presence of uninformed bidders. As we have already explained, the sellers' profits fall relative to what they might have been when all bidders are informed. The reason is that the seller faces an additional constraint when dealing with uninformed bidders - whatever offer he makes them must be sequentially rational.

From the lemma, the take it or leave it offer p' is strictly larger than r^*, which is the offer uninformed buyers would receive in the case where all bidders are uninformed (or equivalently, the seller has no commitment power).

What remains is the impact on informed bidders.

By Lemma 4, the reserve price that is set when all bidders are informed is strictly lower than the 'optimal' reserve price in a standard auction. This means that there is a set of bidder values below r^* whose payoffs are strictly higher than they are in an auction where all bidders are informed. This is simply because they have a chance to trade when there are uninformed bidders, but don't when all bidders are informed.

Now take a value $v > r^*$. The derivative of the informed bidder's payoff is

$$\sum_{m=0}^{n-1} B_m^\mu F^{n-1-m}(v)\,\phi(v, r_m)$$

where

$$\phi(v, r_m) = \begin{cases} 1 & v \geq r_m \\ 0 & \text{otherwise.} \end{cases}$$

When all bidders are informed, the corresponding derivative is just $F^{n-1}(v)$. This means that the payoff function is unambiguously rising faster with uninformed bidders provided the informed bidders value is larger than all the reserve prices r_m. However for values between the minimum and maximum reserve prices, the relationship between the payoffs cannot be determined in general. So the relationship between the payoffs of informed bidders when other bidders are uninformed with some probability and when they aren't is ambiguous.

3. Restrictions on Communication

The argument above takes the traditional approach to mechanism design in that it assumes that the seller communicates first with all the buyers, then decides who to make the offer to. Some features of the equilibrium mechanism seem similar to eBay, with the offer p' being analogous to eBay's *Buy it Now* offer, while the good is auctioned otherwise to the informed bidders.

However, there are some important differences. First, eBay's reserve prices don't vary with the number of bidders who participate in the auction. The way informed bidders are treated here is more similar to *Priceline* where buyers bid for hotel rooms, then hotels decide whether or not to accept one of the bids after they have seen them all. In this way, the hotels who use Priceline can see the number of informed bidders and their bids before they decide whether or not to hold the room to be sold in some other way.

In some ways, this kind of mechanism isn't like a traditional auction at all. We can make it more like a traditional auction, and at the same time provide a model more relevant to eBay, by assuming that the seller is constrained to offer only a single reserve price to informed buyers (that doesn't vary with the number of informed buyers).

If reserve prices must always be the same, the equilibrium mechanism is very different, but quite a bit simpler to describe. Let p^* be the solution to

$$v - \frac{1 - F(v)}{f(v)} = v(1 - F(v)).$$

The solution to this equation always exists (at least when revenue is concave) because $r^*(1 - F(r^*))$ is maximal and decreasing to the right of r^*, while $r^* - \frac{1-F(r^*)}{f(r^*)}$ is 0 and increasing to the right of r^*.

Lemma 5. *If sellers are constrained to use the same reserve price, the unique equilibrium has the seller offering an auction to the informed with reserve price $r^* < p' < p^*$, and making a take it or leave it offer p' to the uninformed if the informed are unwilling to pay.*

Proof. Imposing the constraint that the reserve prices be equal gives the payoff

$$\mu p'(1 - F(p')) \sum_{m=0}^{n-1} B_m^\mu \left(F^{n-m-1}(r')\right) +$$

$$\mu \sum_{m=0}^{n-1} B_m^\mu \int_{r'}^1 (n-1-m) \left(F^{n-m-2}(v_i) \left\{v_i - \frac{1 - F(v_i)}{f(v_i)}\right\}\right) dF(v_i) +$$

$$(1-\mu) p'(1 - F(p')) \sum_{m=1}^{n-1} B_m^\mu \left(F^{n-m}(r')\right) +$$

$$(1-\mu) \sum_{m=0}^{n-1} B_m^\mu \int_{r'}^1 (n-m) \left(F^{n-m-1}(v_i) \left\{v_i - \frac{1 - F(v_i)}{f(v_i)}\right\}\right) dF(v_i)$$

The payoff function for the informed buyer of value r' when the reserve price is r' is given by

$$\sum_{m=0}^{n-1} B_m^\mu \left\{\int_{r'}^{v_i} F^{n-1-m}(\tilde{v})\, d\tilde{v}\right\}\bigg|_{v_i = r'} = 0.$$

It has derivative at p^* equal to

$$\sum_{m=0}^{n-1} B_m^\mu F^{n-1-m}\left(r'\right). \tag{3.1}$$

The payoff function for the uninformed buyer of value p' is zero and has derivative at p' equal to

$$\sum_{m=0}^{n-1} B_m^\mu \frac{1}{m+1} F^{n-1-m}\left(p'\right). \tag{3.2}$$

If $r' = p'$ then (3.1) is larger than (3.2), so the bidding constraint is not binding when $r' = p'$. Suppose $p' > p^*$. Then since the virtual valuation function is increasing, while revenue is decreasing,

$$p' - \frac{1-F\left(p'\right)}{f\left(p'\right)} > p'\left(1-F\left(p'\right)\right)$$

and the revenue maximizing action is to set a reserve price below p', and the bidding constraint cannot be binding. Then since revenue is decreasing, the seller can increase profits by cutting the take it or leave it offer. We conclude $p' \leq p^*$and so

$$p' - \frac{1-F\left(p'\right)}{f\left(p'\right)} \leq p'\left(1-F\left(p'\right)\right).$$

Then the optimal reserve price $r' \leq p'$. Strict inequality would yield a profitable deviation for the seller to reduce p since that raises profits without violating the incentive constraint. We conclude that in every equilibrium $r' = p' < p^*$.

$$p'\left(1-F\left(p'\right)\right)F^{n-m-1}\left(p'\right)+$$
$$\int_{p'}^{1}(n-1-m)\left(F^{n-m-2}\left(v_i\right)\left\{v_i-\frac{1-F\left(v_i\right)}{f\left(v_i\right)}\right\}\right)dF\left(v_i\right)$$

that appears in the expansion of profit when $p' = r'$. If we differentiate with respect to p' we get

$$p'\left(1-F\left(p'\right)\right)(n-m-1)F^{n-m-2}\left(p'\right)f\left(p'\right)+$$
$$F^{n-m-1}\left(p'\right)\frac{d\left(p'\left(1-F\left(p'\right)\right)\right)}{dp'}-$$
$$(n-1-m)\left(F^{n-m-2}\left(p'\right)\left\{p'-\frac{1-F\left(p'\right)}{f\left(p'\right)}\right\}\right)f\left(p'\right)$$

$\square$

There

Lemma 6. *In any equilibrium of the problem in which sellers are constrained to use the same reserve price no matter how many bidders are informed,* $r_0 < r^*$*,* $p' > r^*$ *so* $r_0 - \frac{1-F(r_0)}{f(r_0)} < 0 < p'(1 - F(p'))$*.*

As before, sellers are worse off than they would have been if all buyers had been informed - we've just added another constraint on their mechanisms. Since p' continues to be above r^*, uninformed bidders are worse off than they would have been had all bidders been uninformed. However, for informed bidders we have

Proposition 7. *When sellers are constrained to offer a reserve price that is independent of the number of informed bidders, all informed buyers are weakly better off than they are when all bidders are informed.*

Proof. Since $r_0 < r^*$, buyers whose values are in the interval $[r_0, r^*]$ gain relative to their payoffs when all are informed. Now for any bidder who value is larger than r^*, the derivative of the informed seller's payoff is

$$\sum_{m=0}^{n-1} B_m^\mu F^{n-1-m}(v) \geq F^{n-1}(v)$$

where the value on the right hand side is the probability of trading (or the derivative of the payoff function) when all traders are informed. Hence the payoff function of the informed can never cross the payoff of the uninformed. □

To interpret this theorem with respect to eBay data, notice a couple of things. First, buy it now transactions are informative of a couple of things. The probability that trade occurs at the But it Now price is given by

$$\sum_{m=0}^{n-1} B_m^\mu F^{n-1-m}(r_0)(1 - F(p')).$$

It isn't hard to verify that this varies between 0 (when all buyers are informed) and $(1 - F(r^*))$ when all buyers are uninformed. So generally, the proportion of transactions that occur at the buy it now price will indicate how likely it is that there are uninformed bidders.

Second, because $r_0 < r^*$, ignoring the Buy it Now auctions, and treating all auctions as if they were optimal will result in an unnecessarily pessimistic view of the distribution of valuations.

4. Proofs

4.1. **Proof of Theorem 3.**

Proof. When dealing with informed bidders, we can restrict attention without loss to ex post offers that are no higher than the informed bidder's value.[7]Re-write the informed buyer's payoff in the usual way as

$$Q\left(v_i,0\right)v_i - P\left(v_i,0\right).$$

From standard arguments we can write

$$P\left(v_i,0\right) = Q\left(v_i,0\right)v_i - \int_0^{v_i} Q\left(\tilde{v},0\right)d\tilde{v}.$$

This decomposes as

$$P\left(v_i,0\right) = \sum_{m=1}^{n-1} B_m^{\mu}\left(Q_m\left(v_i,0\right)v_i - \int_0^{v_i} Q_m\left(\tilde{v},0\right)d\tilde{v}\right).$$

Using integrating by parts to eliminate the price function P gives a variant of the well known formula

$$\Pi =$$

$$\mu\sum_{m=0}^{n-1} B_m^{\mu}\left\{\left\{p_{m+1}\left(1-F\left(p_{m+1}\right)\right)\left(1-\int_0^1 Q_{m+1}\left(\tilde{v},0\right)dF\left(\tilde{v}\right)\right)^{n-m-1}\right.\right.$$

$$\left.\left.+\left(n-m-1\right)\int_0^1\left(Q_{m+1}\left(\tilde{v},0\right)\left\{\tilde{v}-\frac{1-F\left(\tilde{v}\right)}{f\left(\tilde{v}\right)}\right\}\right)dF\left(\tilde{v}\right)\right\}\right\}+$$

$$\left(1-\mu\right)\sum_{m=1}^{n-1} B_m^{\mu}\left\{p_m\left(1-F\left(p_m\right)\right)\left(1-\int_0^1 Q_m\left(\tilde{v},0\right)dF\left(\tilde{v}\right)\right)^{n-m-1}\right\}$$

$$+\left(1-\mu\right)\sum_{m=0}^{n-1} B_m^{\mu}\left(n-m\right)\int_0^1\left(Q_m\left(\tilde{v},0\right)\left\{\tilde{v}-\frac{1-F\left(\tilde{v}\right)}{f\left(\tilde{v}\right)}\right\}\right)dF\left(\tilde{v}\right) \tag{4.1}$$

Here we are using Proposition 1 and the assumption of pooling of uninformed buyers to conclude that if the seller doesn't make the offer to an informed bidder, then he must choose an uninformed bidder randomly and make them an offer. For the next step, we can decompose an arbitrary term. The others work the same way. In the case where there

[7]If for some profile, $p_m\left(\left(v_i,0\right),\left(v_{-i},\iota_{-i}\right)\right) > v_i$, then we can just reset the price offer to v_i and $q_m\left(\left(v_i,0\right),\left(v_{-i},\iota_{-i}\right)\right) = 0$ without affecting any player's payoff. We can't do this with uninformed bidders, because the offer can 't depend on their type.

are m uninformed bidders among the other players, and the bidder we are concerned with is also informed, the corresponding term is

$$\left\{ p_m \left(1 - F\left(p_m\right)\right) \left(1 - \int_0^1 Q_m\left(\tilde{v}, 0\right) dF\left(\tilde{v}\right)\right)^{n-m-1} \right.$$

$$\left. + \left(n - m\right) \int_0^1 \left(Q_m\left(\tilde{v}, 0\right) \left\{\tilde{v} - \frac{1 - F\left(\tilde{v}\right)}{f\left(\tilde{v}\right)}\right\}\right) dF\left(\tilde{v}\right) \right\} \Bigg\}$$

This can be rewritten as

$$p_m \left(1 - F\left(p_m\right)\right) \left(1 - \int_0^1 \cdots \int_0^1 q_m\left(\left(\tilde{v}, 0\right), \left(\tilde{v}_{-i}, \iota_{-i}\right)\right) dF\left(\tilde{v}\right) \ldots dF\left(\tilde{v}\right)\right)^{n-m-1}$$

$$+ \left(n - m\right) \int_0^1 \cdots \int_0^1 \left(q_m\left(\left(\tilde{v}, 0\right), \left(\tilde{v}_{-i}, \iota_{-i}\right)\right) \left\{v_i - \frac{1 - F\left(v_i\right)}{f\left(v_i\right)}\right\}\right) dF\left(\tilde{v}\right) \cdots dF\left(\tilde{v}\right)$$

In this formula, we are implicitly substituting in the appropriate values for ι_i for each of the other bidders. In the special case where $m = 0$, the first term becomes 0 and this is just the usual expression

$$n \int_0^1 \cdots \int_0^1 \left(q_n\left(\left(\tilde{v}, 0\right), \left(\tilde{v}_{-i}, \iota_{-i}\right)\right) \left\{v_i - \frac{1 - F\left(v_i\right)}{f\left(v_i\right)}\right\}\right) dF\left(\tilde{v}\right) \cdots dF\left(\tilde{v}\right)$$

The seller's overall profit is the expectation of a series of terms like this, each of which can be maximized point wise by setting $q_m\left(\cdot\right)$ to either 0 or 1 because of the linearity of payoffs in the q_m.

Unlike the usual auction, there is a bidding constraint to be satisfied. However, this decomposition makes it clear that *if* the seller wishes to make an offer, then profit maximization will require that he makes it to the bidder with the highest value. Since incentive compatibility requires trading probability to be increasing in type, if the seller awards the good with probability 1 to some value, then he must also award it with probability 1 for higher values.

This gives the usual characterization in which $Q_m\left(v, 0\right) = F^{n-m}\left(v\right)$ whenever $Q_m\left(v, 0\right) > 0$.

This observation reduces the seller's problem to a choice of $2n$ constants to be chosen to maximize

$$\mu \sum_{m=0}^{n-1} B_m^{\mu} \left\{p_{m+1}\left(1 - F\left(p_{m+1}\right)\right) \left(F^{n-m-1}\left(r_{m+1}\right)\right) + \right.$$

$$\left. \left(n - 1 - m\right) \int_{r_{m+1}}^1 \left(F^{n-m-2}\left(v_i\right) \left\{v_i - \frac{1 - F\left(v_i\right)}{f\left(v_i\right)}\right\}\right) dF\left(v_i\right) \right\} +$$

$$(1-\mu)\,p_m\,(1-F(p_m))\sum_{m=1}^{n-1}B_m^{\mu}\left(F^{n-m}(r_{m+1})\right)+$$

(4.2)

$$(1-\mu)\sum_{m=0}^{n-1}B_m^{\mu}\int_{r_m}^{1}(n-m)\left(F^{n-m-1}(v_i)\left\{v_i-\frac{1-F(v_i)}{f(v_i)}\right\}\right)dF(v_i)\,.$$

A few observations might make this last expression easier to follow. Since the mechanism is symmetric, what this expression does is to take a representative bidder, then condition on the number m of the other $n-1$ bidders who are uniformed. If the event that this representative bidder is uninformed, there will be $n-m-1$ informed bidders, so the probability the seller sells to one of them is $F^{n-m-1}(r_{m+1})$. The reserve price is the one that applies when there are $m+1$ uninformed bidders, since the representative bidder is uninformed in this case.

The revenue the seller gets from the informed bidders is $(n-m-1)$ times the revenue he gets from each informed bidder. This revenue comes from the well known formula

$$\int_{r_{m+1}}^{1}\left(F^{n-m-2}(v_i)\left\{v_i-\frac{1-F(v_i)}{f(v_i)}\right\}\right)dF(v_i)\,.$$

The term $F^{n-m-2}(v_i)$ is the probability the informed bidder has a higher value than each of the other informed bidders. There are $n-m-2$ other informed bidders in this case, not $n-m-1$ because we are conditioning on one of the bidders being uninformed.

The same reasoning explains the indices in the second term.

Notice that in the third term, the limits of the sum are different than they are in the fourth term. The reason is that when $m=0$ and we condition on an informed bidder, then all bidders are informed. In the case where no informed bidders meet the reserve price, there are no uninformed bidders to make an offer to.

To write down the bidding constraint, it follows from the argument above that the informed buyer's payoff in any equilibrium mechanism is

$$U(v_i)=\sum_{m=0}^{n-1}B_m^{\mu}\left\{\int_{r_m}^{v_i}F^{n-1-m}(\tilde{v})\,d\tilde{v}\right\}$$

which must be at least as large as the payoff the informed bidder gets from pretending to be uninformed. So the bidding constraint is given formally by

$$U(v_i)\geq\sum_{m=0}^{n-1}B_m^{\mu}\frac{1}{m+1}F^{n-1-m}(r_{m+1})\max\left[v_i-p_{m+1},0\right] \tag{4.3}$$

for every i and v_i.

Since $p(1-F(p))$is concave, (4.2) is the expectation of a concave function. On the other hand, the right hand side of the bidding constraint involves a convex function of p. So if some vector $\{p_1,\ldots,p_m\}$ satisfies the bidding constraint, replacing each p_m with the expected offer will still satisfy the constraint, and increase seller's payoff. So the n constants $\{p_1,\ldots,p_m\}$ can be reduced to 1, and this proves the theorem. $\square$

References

Butters, G. (1977): "Equilibrium Distributions of Sales and Advertizing Prices," *Review of Economic Studies*, 44, 465–491.

Dickson, P. R., and A. G. Sawyer (1990): "The Price Knowledge and Search of Supermarket Shoppers," *Journal of Marketing*, 54(3), 251–334.

Green, J., and J. J. Laffort (1986): "Mechanism Design with Partially Verifiable Information," *Review of Economic Studies*, 53(3), 447–456.

Krishna, V. (2010): *Auction Theory*. Science Direct.

Masatloglu, Y. (2015): "A Random Inattention Model," Discussion paper, University of Michigan.

Masatloglu, Y., D. Nakajima, and E. Ozbay (2012): "Revealed Attention," *American Economic Review*, 102(5).

McAfee, P. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281–1312.

Peters, M. (2014): "Can Mechanism Designers Exploit Market Information," Discussion paper, Vancouver School of Economics.

Stahl, D. (1989): "Ologopolistic Pricing with Sequential Consumer Search," *American Economic Review*, 79, 700–712.

——— (1994): "Ologopolistic Pricing and Advertising," *Journal of Economic Theory*, 64, 162–177.

Strauss, R. (2016): "Mechanism Design with Partially Verifiable Information," Discussion paper, Humboldt University.

Varian, H. (1980): "A model of Sales," *American Economic Review*, 70(4), 651–659.