

# UNDERSTANDING TRANSITIONS USING DIRECTED SEARCH?

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**ABSTRACT.** This paper explains how a directed search model can be used to understand worker transition data in labor markets. The basic theory provides a dynamic extension of the model in Peters (2010) in which workers have privately known types that are observable to firms once workers apply. It shows how the wage offer distribution can be derived from the accepted wage distribution and the employment distribution by solving a differential equation. This relationship is used to derive a search outcome distribution that can be used to study transitions between jobs. The paper then illustrates how to use the results to test the idea by using transition data from the well known DADS data on French workers.

This paper explains how a directed search model can be used to understand worker transition data in labor markets. The basic theory provides a dynamic extension of the model in (Peters 2010) in which workers have privately known types that are observable to firms once workers apply. The purpose is primarily to derive the search outcome distribution and to show how the wage distribution and search outcome distribution are related, and how this relationship yields a very testable theory of worker transitions between jobs. We suggest a way to test this relationship.

The basic idea is that type information is incorporated into workers' search decisions, so that the wage at which worker is currently employed reveals something about his or her type. In particular, workers who are currently employed at high wage firms are more likely to have high types. As a consequence, if they are forced to move to new jobs, or

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move as a result of on the job search, they are more likely to receive higher wages at these new jobs as well.

This idea is not new. The main purpose of the exercise here is to try to establish whether directed search can impose additional structure on this relationship that might ultimately make it possible to distinguish between directed search and other models like random matching that might yield a similar result. The main results in the paper consist of a series of theorems relating the wage offer distribution to the the properties of the relationship between workers wages before and after a job transition.

For example, suppose that the wage offer distribution is given by  $G(w)$  on some compact interval  $[\underline{w}, \bar{w}]$  of wages. We show that the search outcome distribution for any worker type is given by

$$\tilde{G}_{w_0}(w) = \frac{\int_{w_0}^w \frac{dG(\tilde{w})}{\tilde{w}}}{\int_{w_0}^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}}$$

for some wage  $w_0$ .

This fact can be used to suggest various empirical tests. For example, we show that if the variance of this truncated distribution is a decreasing function of  $w_0$ , then the variance of a worker's future wage should be a decreasing function of the wage at the job the worker currently holds. Similarly, we can show conditions on this derived distribution in which the relationship between the workers old and new wage is convex or concave.

The model we provide here is richer in empirical content than most models of directed search. In its most basic variant, directed search assumes all firms are identical and offer the same wage in equilibrium. Models that do allow for heterogeneity among firms (for example, (Peters 2000)), still assume workers are identical but use mixed application strategies when they apply to firms, applying with highest probability at the firms who offer the highest prices. In the steady state of such a model, if workers use the same mixed strategy in every period, their wages will be variable over time. However, there will be no correlation at all between the wage they receive in the match that they leave and the match that they move to.

At the other extreme, models that support pure assortative matching (for example (Shi 2001) or (Eeckhout and Kircher 2010)) will predict that workers who land high wage jobs in one period will do so again in future periods. Theoretically, outcomes are perfectly correlated over time. The same kind of outcome could be expected from wage-ladder like models (e.g., (Delacroix and Shi 2006)) in which homogeneous

workers search on the job and implicitly use the current wage as a way of coordinating applications. Workers who are employed at some wage will apply to firms offering slightly higher wages until they are successful at finding a new job. This provides a high correlation between the wages of workers who move between jobs without an unemployment spell. This correlation is broken when workers' matches are terminated exogenously and they experience unemployment. They then fall to the bottom of the wage ladder. The basic prediction is very high correlation in wages for job to job transitions, and no correlation (or perhaps a negative correlation) for movements between jobs that involve an unemployment spell.

Older models of directed search then, either seem to predict no correlation of an individual workers' wages over time, or a nearly perfect correlation. The model we develop here produces a correlation in between that is more in line with empirical evidence (some of which we show below).

Random matching models with distributions of worker types can also be used to produce an intermediate correlation between wages across transitions, though we are not aware of a model that has explicitly studied this. For example, (?) studies a model with wage posting in which firms are identical but workers are described by an atomless distribution of supply prices. Workers who need high wages to convince them to work will tend to be paid high wages in each of their jobs, tending to support high, but not perfect, correlation in wages over time. However, they do not explicitly study transitions.

Our approach differs from their in two ways. First, we do not assume that firms are identical. This generalization means that our model can be consistent with a larger set of wage distributions. Second, our distribution of supply prices for workers is endogenously determined by the wage offer distribution. It is this relationship between supply prices and the wage offer distribution that supports all of the empirical implications in our model.

(Postel-Vinay and Robin 2002) study a model with random matching and bargaining with heterogeneity in worker (and firm) types. After being randomly matched with some worker, a firm makes the worker a take it or leave it offer. The firm is assumed to observe the worker's productivity type, which is the same assumption we make here. Workers with higher types have better outside options. As a consequence their wages tend to be high in all their matches. However, their purpose is not to study these correlations, so they do not derive the outcome distribution for workers as we do here.

There are two major differences between our model and theirs. First, from the theoretical perspective, we do not use discounting. Instead we assume that workers maximize the limit of their average expected wage payments. Random matching models need discounting to support match frictions. Without it, workers and firms simply wait around until they are assortatively matched. Directed search doesn't require the same device because frictions are built directly into the matching process because of workers inability to coordinate their search strategies. The advantage of our approach is that it allows us to draw a much closer connection between the wage distribution and workers search outcomes than is possible with discounted payoffs.

One of the implications of our assumption is that search strategies no longer depend on the wage at which a worker is currently working. In typical wage ladder models, workers' search decisions collapse to what are effectively pure strategies in which workers with higher wages only apply to higher wages. This supports too much autocorrelation in wages of workers who search on the job. As this effect disappears in our model, wage variation after on the job transitions is instead attributable to workers' unobservable types.

Second, of course, is the fact that we use directed search. This assumption makes it possible to directly tie the search outcome distribution to the wage offer distribution in a way that is not possible when the two are indirectly connected through a value function. This approach makes it possible for us to provide a non-parametric test of the model.

## 1. FUNDAMENTALS

A labor market consists of measurable sets of positions and workers. It will be assumed that all the workers in this market are identical in terms of observable qualifications, including education, experience, past performance, etc. So all workers are acceptable employees at all firms. However, workers also possess observable but non-verifiable types that are potentially valuable to firms.

Types might be things like the potential employee's charm and articulation, or references that the employee gets from outsiders. An example of what we have in mind may be the academic market for newly graduated phd's where employers need to see reference letters and conduct interviews before they make offers. As in the academic market, the 'market' doesn't know workers' types in the sense that firms don't know workers' types before they apply, and workers don't know each others' types. However, we assume that firms can identify

workers' types and rank them once they contact them with an application. So rather than randomly selecting among applications as is typical in models of directed search, firms choose among workers on the basis of these types. Firms rank these types the same way, so a type moves between jobs with a worker. This is the property of the model that connects the outcomes for workers as they move between positions.

Workers types are contained in a compact subset  $Y$  of  $\mathbb{R}_+$ . The measure of the set of searching workers with types less than or equal to  $y$  is given by  $F(y)$ , where  $F$  is a monotonically increasing and differentiable function defined on the interval  $Y = [y, \bar{y}]$ . Types have nothing to do with worker preferences, which are assumed to be the same for all workers. The measure of the set of workers will be normalized to 1.

We'll model jobs as renewable short term contracts offering fixed wage payments. Workers compete for these contracts by applying for them. Following the random matching literature, we'll assume that when a worker's application to a new job is successful, she will give up the lowest paying job that she currently holds. We'll differ from the random matching literature by assuming that she will have to fulfill an existing contract, unless it is terminated by her employer. So a worker who searches on the job and wins a new contract will temporarily hold two jobs, one of which she will relinquish at the end of the period. This is convenient, since multiple job holders do exist in the French labor market data we use.

The assumption that workers will accept a second job when they already have one is reasonable - though it is far from the norm, it seems to occur with some frequency in our data. Contract workers will typically seek new work while working on an existing job simply because existing contracts are subject to termination. This might well involve a period of overlap between the old job and the new one. This is the behavior we are trying to capture with this assumption. Of course, employees moonlight in order to augment their income. However, an implication of our assumption is that workers will give up their lower paying contract even though it pays a wage that is higher than what they can expect to earn by searching. This assumption reflects possible restrictions imposed by employers preventing employees from working at other firms, or employee capacity constraints that prevent them from working multiple contracts for long periods of time.

So we assume that a worker begins each period either unemployed, or in a match. In either case, workers apply to one and only one position. We view the 'single application' assumption as a modelling device used

to approximate a frictional matching process. As such it is no better or worse than assuming that a worker is randomly contracted by a firm. So we do not discuss it further.

If the worker is already employed under some contract, we call this search on the job. If the worker is offered the job to which he or she applied, they accept it and earn the corresponding wage in that period. After the outcome of the worker's search decision is made, the worker's existing match, if he has one, may be exogenously terminated. This occurs with probability  $\gamma$ . If termination occurs, the worker is either unemployed, or earns the wage associated with any new job the worker managed to get during the period. If the original job does not terminate, the worker is paid by both firms, then resigns from the lower wage job at the end of the period.

We use the limit of the average of expected payoffs as the objective for workers. Since we will focus entirely on steady state equilibrium, this simply means maximizing expected payments, period by period.

At the end of each period, firms either have an unfilled position caused by exogenous termination, an unfilled position caused by the fact that their existing worker has resigned to move to a higher paying firm, or have a continuing employee. If the position is unfilled, the firm enters the market at the beginning of the next period, and advertises an opening for their position.

Positions are parameterized by some characteristic  $x \in X$ , where  $X$  is a compact subset of  $\mathbb{R}$ . Associated with each position is an optimal wage offer. As mentioned above, we aren't much interested in the firm's optimization problem. So we'll characterize the population of firms with the distribution of wages these firms offer. We'll use  $E$  to represent the accepted wage distribution and  $G$  to represent the wage offer distribution. Generally, both distributions will be assumed monotonically increasing and differentiable. The notation  $dE$  and  $dG$  will be used to refer to the densities of these distributions.

Firms offer contracts that specify the expected wage payment  $w$  a worker will receive in each period during which he or she is employed in the position. Firms set wages to maximize the expected profit they earn from whichever worker they hire. A position of type  $x$  filled by a worker of type  $y$  under a contract that provides expected payment  $w$  to a worker, generates an expected per period profit to the firm of  $v(w, x, y)$ .

## 2. THE MARKET

We model the labour market as a large game in which the payoffs that players receive depend on their own actions, and on the distributions of actions taken by the other players and focus on a steady state equilibrium in which the distribution of wages on offer from open positions supports a distribution of expected payments that does not change over time.

The market begins each period with a set of matched workers and firms, and a set of vacant positions. The presumption in the paper is that each of the worker firm matches consists of a wage type pair. the notation  $e(w, y)$  will be used to denote the density of the joint distribution of matched workers and firms. The unknown distribution of types employed at wage  $w$  given by  $\psi(\cdot|w)$ , so  $E(w) = \int_{\underline{w}}^w \int_{\underline{y}}^{\bar{y}} e(\tilde{w}, y) d\psi(y|\tilde{w}) d\tilde{w}$ .

Firms have to decide what wage to offer, while workers have to decide where to apply. We'll focus on symmetric equilibrium, so we can write the firm's strategy rule as  $\rho : X \rightarrow \mathbb{R}$ . As in any directed search model, we expect the workers to use mixed application strategies. So write  $\pi : [\underline{w}, \bar{w}] \times [\underline{y}, \bar{y}] \rightarrow [0, 1]$  to be the probability that a worker of type  $y$  applies to a firm offering a wage less than or equal to  $w$ . We assume that for each  $y$ ,  $\int_{\underline{w}}^{\bar{w}} d\pi(w, y) \leq 1$ .

A symmetric worker application strategy  $\pi$  gives rise to a distribution  $P$  of applications, where  $P(w, y)$  is the measure of the set of applications made to firms whose wages are not higher than  $w$  by workers whose types are no higher than  $y$ . This distribution is given by

$$(2.1) \quad P(w, y) = \int_{\underline{y}}^y \int_{\underline{w}}^w d\pi(\tilde{w}, \tilde{y}) dF(\tilde{y}).$$

Since  $P$  is absolutely continuous with respect to Lebesgue measure on  $[\underline{y}, \bar{y}]$  and with respect to  $G$  on the interval  $[\underline{w}, \bar{w}]$ , we can write

$$P(w, y) = \int_{\underline{w}}^w \int_{\underline{y}}^y p_{\tilde{w}}(\tilde{y}) d\tilde{y} dG(\tilde{w})$$

for some (Lebesgue) measurable function  $p_{\tilde{w}}(\tilde{y})$ <sup>1</sup>.

Heuristically, the function  $p_w(y)$  is the ratio of the measure of the set of workers of type  $y$  who apply to firms offering wage  $w$  to the measure of firms offering wage  $w$ . In other words, it is a variant of the 'queue size' that is so commonly used in directed search. In an urn ball matching model, the probability that a worker is hired when he or she

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<sup>1</sup>This follows from the Radon-Nikodym theorem

applies at a firm offering the wage  $w$  is the exponential of the negative of the queue size.

An analogous formula applies when workers have unverifiable types that are used to determine who is hired. The difference is that in the standard model with identical workers all the other workers who apply at the same wage are potential competitors. In the model here, if a worker has type  $y$ , only workers who apply at the same wage and have higher types are competitors.<sup>2</sup> So the appropriate queue size is the ratio of the measure of the set of workers who apply at wage  $w$  and have types higher than  $y$  to the measure of the set of firms offering wage  $w$ . In other words

$$\int_y^{\bar{y}} dp_w(\tilde{y}),$$

is the appropriate queue size. So we use the familiar formula  $e^{-\int_y^{\bar{y}} dp_w(\tilde{y})}$  to give the probability that the worker will be hired if he applies at wage  $w$ .<sup>3</sup>

Since workers maximize average expected wages, the payoff to a worker of type  $y$  who is employed at wage  $w$  and applies at a firm offering wage  $w'$  as

$$(2.2) \quad w' e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} + (1 - \gamma) w.$$

Of course, if the worker is unemployed, the  $w$  term is just 0. Since any wage that maximizes the first term maximizes this expression for any  $w$ , a worker will maximize his expected payments from firms by maximizing the first term in every period.

Given the measure  $P$ , we can write down the probability that a worker leaves his current position at the end of any period. It is the probability the worker applies to and is hired by a firm paying a wage that is higher than his current wage, plus the probability that the match is exogenously terminated. This formula is

$$(2.3) \quad Q(w, y) = \gamma + (1 - \gamma) \int_w^{\bar{w}} e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} d\pi(w', y).$$

A firm who hires a worker retains him or her until the match terminates. Firms who have multiple applications hire the highest type

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<sup>2</sup>In (Peters 2010) it is shown that the formulas that follow coincide with limits of the payoffs that workers receive in finite markets.

<sup>3</sup>A formal derivation of this probability as the limit of the probability of being hired in a large finite game is given in (Peters 2010).

worker who applies and set wages to maximize the expected profit generated by whatever worker they hire. So an unfilled position has value

$$(2.4) \quad \max_w \left\{ \int_{\underline{y}}^{\bar{y}} \frac{v(w, x, y) e^{-\int_y^{\bar{y}} dp_w(\tilde{y})}}{Q(w, y)} dp_w(y) \right\}$$

where  $Q(w, y)$  is defined by (2.3).

Finally, in a steady state, a firm who has hired a worker at wage  $w$  and loses the worker, either because the worker leaves for higher pay, or because the match is terminated for some exogenous reason, should post a new offer with same wage  $w$  that it offered before. The reason is simply that the wage that the firm pays its existing worker is the one that maximized the expression in (2.4) when the firm attracted that worker in the first place.

The steady state condition is then given in a manner similar to the other formulas above. Firms who offer wage  $w$  and employed a worker of type  $y$  in the last period enter the market looking for a new hire if their worker decided to move during the last period. The 'measure' of firms in this position is  $dE(w) \int_{\underline{y}}^{\bar{y}} Q(w, y) d\psi(y|w)$ . Also joining the market is a set of firms whose employee left two periods ago, but who were unable to hire a new worker last period. The measure of this set is

$$dE(w) \int_{\underline{y}}^{\bar{y}} Q(w, y) d\psi(y|w) \left( 1 - \int_{\underline{y}}^{\bar{y}} e^{-\int_x^{\bar{y}} dp_w(\tilde{y})} dp_w(x) \right).$$

Similarly, there are firms who lost their worker three periods ago, but failed to hire in the previous two periods, and so on. Adding all these gives the measure of firms who are looking for new workers and offering wages less than or equal to  $w$  as

$$(2.5) \quad G(w) = \int_{\underline{w}}^w \frac{\int_{\underline{y}}^{\bar{y}} Q(\tilde{w}, y) d\psi(y|\tilde{w})}{\int_{\underline{y}}^{\bar{y}} e^{-\int_x^{\bar{y}} dp_{\tilde{w}}(\tilde{y})} dp_{\tilde{w}}(x)} dE(\tilde{w}).$$

In this formulation, both  $Q(w, y)$ ,  $\psi(\cdot|\cdot)$  and  $p_w$  depend on the wage offer distribution  $G$ , so that the steady state wage offer distribution is a fixed point. This fixed point is developed formally below.

An equilibrium for this model is a collection  $\{E, G, \pi\}$  satisfying three conditions

- (optimality of search strategies) for every  $y$   $w$  maximizes (2.2) for every  $w$  in the support of  $\pi(\cdot, y)$ ;
- (optimality of wage offers) for every  $x$ ,  $W(x)$  maximizes (2.4); and

- (steady state condition) The relation (2.5) holds almost everywhere for  $G$ .

### 3. CONTINUATION STRATEGIES

The approach we are going to take here is somewhat unusual. Rather than starting with a fixed distribution of firm types, then deriving the equilibrium distributions  $E$  and  $G$ , we will instead take the wage distribution  $E$  to be exogenously given. The search strategies needed to support that distribution can then be derived by solving a fixed point problem to find  $G$ . These strategies will, in turn determine firms' profit functions. At this point, we just imagine that firms profit functions are distributed in a way that supports the observable wage distribution.

To find the wage offer distribution, we begin by assuming that we know it, then work out the search strategies that satisfy (2). These strategies determine the transition function (2.5) which provides a fixed point problem whose solution identifies the wage offer distribution.

What the following theorem says is that if the wage offer distribution is given by  $G$ , workers apply at every wage above a type dependent reservation wage  $\omega(y)$  with *equal* probability. This reservation wage is increasing in type. In this sense, the model resembles random search and matching models with worker types in which higher type workers hold out for higher wages in the future because they know they can get them. The logic here differs in that workers trade off wage against trading probability as they do in all directed search models.

**Theorem 1.** *For any differentiable wage offer distribution  $G$ , there is a continuation equilibrium characterized by a monotonically increasing reservation wage strategy  $\omega(y)$  in which each worker applies with equal probability at every wage at or above  $\max[\underline{w}, \omega(y)]$ . Formally, for every  $y$*

$$\int_{\underline{w}}^w d\pi(\tilde{w}, y) = \int_{\omega(y)}^w d\pi(\tilde{w}, y) = \int_{\omega(y)}^w \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))}.$$

*The reservation wage is characterized by the solution to the differential equation*

$$(3.1) \quad \omega'(y) = \frac{\omega(y) F'(y)}{G(\bar{w}) - G(\omega(y))}$$

*through the point  $(\bar{y}, \bar{w})$ . Finally for every wage  $w$  in the support of  $G$ , the queue size faced by a worker of type  $y$  who applies for a position offering wage  $w$  is*

$$(3.2) \quad \int_y^{\bar{y}} dp_w(\tilde{y}) = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')$$

The proof of this theorem (which follows the logic in (Peters 2010)) is given in the appendix. What gives this theorem most of its power is the fact that the only way that a worker's search strategy depends on  $y$  is through the reservation wage. We exploit this property extensively in what follows.

### FULL EQUILIBRIUM

The wage that firms offer determines the quality of their applicants as well as how long an applicant stays in a job. The results in the previous section provide a useful way to view this trade off.

**Lemma 2.** *In a symmetric steady state equilibrium, the function  $Q(w, y)$  is equal to*

$$(3.3) \quad \gamma + (1 - \gamma) \frac{\omega'(y)}{F'(y)} \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}.$$

*Proof.* From Theorem 1, the function  $Q(w, y)$  can be written as

$$(3.4) \quad \gamma + (1 - \gamma) \int_w^{\bar{w}} e^{-\int_y^{\tilde{y}} dp_{\tilde{w}}(\tilde{y})} \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))} = \\ \gamma + (1 - \gamma) \int_w^{\bar{w}} e^{-\int_y^{\omega^{-1}(\tilde{w})} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')} \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))}.$$

Since workers used mixed application strategies, they must receive the same payoff from every seller. So

$$e^{-\int_y^{\tilde{y}} dp_{\tilde{w}}(\tilde{y})} = \frac{\omega(y)}{w}.$$

Substituting this into (3.4) gives

$$Q(w, y) = \gamma + (1 - \gamma) \int_w^{\bar{w}} \frac{\omega(y)}{\tilde{w}} \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))}.$$

From (3.1) this becomes

$$Q(w, y) = \gamma + (1 - \gamma) \int_w^{\bar{w}} \frac{\omega'(y)}{F'(y)} \frac{dG(\tilde{w})}{\tilde{w}} = \\ \gamma + (1 - \gamma) \frac{\omega'(y)}{F'(y)} \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}.$$

□

An immediate corollary of this Lemma gives the first useful result:

**Theorem 3.** *In a symmetric steady state equilibrium, a worker is more likely to leave a job the higher is his or her type.*

*Proof.* By (3.1) and the fact that  $\omega$  is increasing,  $\frac{\omega'(y)}{F'(y)}$  is an increasing function of  $y$ . The theorem then follows immediately from (3.3).  $\square$

To put this another way, job duration is a declining function of type.

This result is an implication of on the job search. Whether on the job search is important in any particular labor market is an empirical issue. In fact, the work below, we use this result. Since the wage at which a worker is employed is informative about his or her type. It follows that in markets where duration doesn't vary with the wage, transitions are likely occurring as the result of exogenous termination. As we show, the wage offer distribution is readily identified in this case.

To end this section, we add an additional useful result.

**Lemma 4.** *Almost everywhere*  $dp_w(\tilde{y}) = \frac{\omega'(\tilde{y})}{\omega(\tilde{y})} d\tilde{y}$

*Proof.* Using the fact that workers are indifferent about applying at all wages above their reservation wage, we have

$$we^{-\int_y^{\tilde{y}} dp_w(\tilde{y})} = \omega(y).$$

Taking logs gives for every  $w > \omega(y)$

$$\log w - \log \omega(y) = \int_y^{\omega^{-1}(w)} dp_w(\tilde{y}).$$

Writing the difference between the logs as the integral of the derivative

$$\int_y^{\omega^{-1}(w)} dp_w(\tilde{y}) = \int_y^{\omega^{-1}(w)} \frac{\omega'(\tilde{y})}{\omega(\tilde{y})} d\tilde{y}$$

from which the result follows.  $\square$

For those who are interested, we explain in the appendix how firms wages can be modelled. As a part of this, we explain why it is without loss to assume that workers types are uniformly distributed. The gist of the argument is that any observed behavior can be rationalized for any distribution of worker types by modify the profit function.

## EMPLOYMENT HISTORIES

For the rest of the paper, we'll focus on workers who move between firms with an intervening unemployment spell. The formulas can be adapted for workers who transit directly from one job to another because of on the job search. However, the formulas are more complex and do not change the basic logic.

The logic developed above is simply that worker types are unobservable, but outcomes provide some information about type. In particular

the wage at which a worker is currently employed will say something about the worker's type provided type actually matters to firms. If types matter, high type workers will be more likely to get jobs with high type firms. The equilibrium conditions allow us to derive the distribution of search outcomes for workers who leave jobs at different wages.

Our particular interest is to establish conditions on the wage offer distribution that determine the shape of the relationship between the workers current wage and his expected future wage, as well as conditions under which the variance of the worker's future wage will be declining with the wage at which he or she is currently employed.

The model has other implications about transitions. Of course, workers who are employed at high wage firms are less likely to apply to and be hired with firms making higher offers, no matter what their type. So duration of employment will be longer at high wage firms.

At any given wage, the highest type workers who are employed at that wage are more likely to apply to and receive offers from higher wage firms. So high type workers at any wage will move more frequently than low type workers. This effect should be reflected in wages after a job transition. In particular, the longer the duration of a worker's employment with a firm, the lower his wage after transition is likely to be. One reason is that a low type worker is more likely to suffer a wage cut after an exogenous termination. The other is simply that the lower type worker is just less likely to be hired at the higher wage firms.

**Relationship between current wage and type.** The core argument used above is that the wage at which a worker is currently employed is positively (but not perfectly positively) correlated with his type. To see why, note that by Theorem 1, workers make applications to every firm whose wage is above their reservation wage with, so to speak, equal probability. This means that when a worker moves from one job to another (in other words, conditional on moving), the wage of a worker of type  $y$  moves to is a random variable. Our first task is to compute this distribution.

The calculation is slightly different for workers who have unemployment spells than it is for those who make direct transitions on the job. So we'll start with unemployed workers.

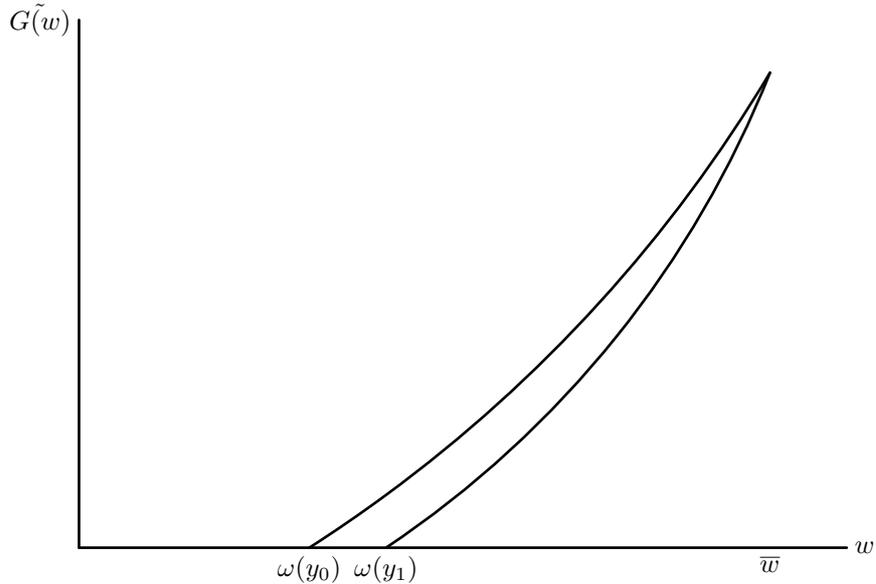
According to Theorem 1, a worker of type  $y$  could end up being hired at a lot of different wages, since the support of his equilibrium mixed application strategy includes all wages above his reservation wage  $\omega(y)$ . Since the worker applies to a firm offering wage  $w$  with density  $\frac{dG(w)}{G(\bar{w})-G(\omega(y))}$  and is then hired with probability  $\frac{\omega(y)}{w}$  (because

the expected payoff at each wage must be the same as the payoff he gets from applying at his reservation wage and being hired for sure), the probability density with which the worker is hired at a wage  $w$  (his outcome distribution) when he eventually leaves unemployment is given by

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{dG(w)}{G(\bar{w}) - G(\omega(y))} \frac{\omega(y)}{w} \left[ \int_{\omega(y)}^{\bar{w}} \left( 1 - \frac{\omega(y)}{\tilde{w}} \right) \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))} \right]^t = \\ \frac{\frac{dG(w)}{G(\bar{w}) - G(\omega(y))} \frac{\omega(y)}{w}}{\int_{\omega(y)}^{\bar{w}} \frac{\omega(y)}{\tilde{w}} \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y))}} = \\ \frac{dG(w) \frac{1}{w}}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})}. \end{aligned}$$

Notice that the numerator in this term is independent of the workers type,  $y$ . Since  $\omega$  is a strictly increasing function by Theorem 1, this density is higher at every wage the higher the worker's type. Since the cumulative distribution function is monotonically increasing and reaches a value 1 at  $\bar{w}$ , the distribution function for a higher type worker must first order stochastically dominate that of a lower type worker.

Then if we compare the probability distributions over future wages for two workers of types  $y_1 < y_2$ , they look like the distributions given in the following figure.



It is straightforward to calculate the mean wage received by a worker of type  $y$  when he or she eventually moves to a new job. It is given by

$$(3.5) \quad \frac{\int_{\omega(y)}^{\bar{w}} w \cdot \frac{1}{w} dG(w)}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} = \frac{G(\bar{w}) - G(\omega(y))}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})}$$

while the variance of this distribution of future wages is

$$(3.6) \quad \frac{\int_{\omega(y)}^{\bar{w}} w^2 \cdot \frac{1}{w} dG(w)}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} - \left[ \frac{G(\bar{w}) - G(\omega(y))}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} \right]^2.$$

The advantage of these two formulas is that they give a simple relationship between the wage offer distribution and unemployed workers' experience when they return to the workforce. Of course, we don't observe the type directly, but do get some information about it from the wage at which the worker was previously employed.

The next step then, is to find the distribution of types  $\psi(y|w)$  that are hired at each different wage.

**Lemma 5.** *The conditional distribution function  $\psi(y|w)$  satisfies*

$$\psi(y|w) = \frac{\int_{\underline{y}}^y \omega'(\tilde{y}) d\tilde{y}}{\int_{\underline{y}}^{\omega^{-1}(w)} \omega'(\tilde{y}) d\tilde{y}} = \frac{\omega(y) - \underline{w}}{w - \underline{w}}.$$

if  $y \leq \omega^{-1}(w)$ , and is equal to 1 otherwise.

*Proof.* The probability with which a firm offering a wage  $w$  hires a worker of type  $y$  can be derived using the help of Theorem 1. When a firm has vacancy, the probability that it immediately hires a worker whose type is  $y$  or less is given by

$$\int_{\underline{y}}^y e^{-\int_{\underline{y}}^{\tilde{y}} dp_w(\tilde{y})} dp_w(y)$$

when  $y$  is less than or equal to  $\omega^{-1}(w)$ , and is 1 otherwise. Using Theorem 1 and the fact that the probability with which a worker of type  $y$  gets a job at wage  $w$  is  $\frac{\omega(y)}{w}$ , this can be written as

$$\int_{\underline{y}}^y \frac{\omega(\tilde{y})}{w} \frac{\omega'(\tilde{y})}{\omega(\tilde{y})} d\tilde{y} = \int_{\underline{y}}^y \frac{\omega'(\tilde{y})}{w} d\tilde{y}.$$

The probability with which the firm fails to hire initially is

$$1 - \int_{\underline{y}}^{\omega^{-1}(y)} \frac{\omega'(\tilde{y})}{w} d\tilde{y} = 1 - \frac{w - \omega(y)}{w} = \frac{\omega(y)}{w}.$$

Of course, if the firm fails to hire a worker on its initial try, it will continue to try in future. Using the steady state reasoning given above, the probability that a worker who is working at wage  $w$  has type less than or equal to  $y$  is then

$$\sum_{t=0}^{\infty} \int_{\underline{y}}^y \frac{\omega'(\tilde{y})}{w} d\tilde{y} \left[ \frac{\omega(\underline{y})}{w} \right]^t = \frac{\int_{\underline{y}}^y \frac{\omega'(\tilde{y})}{w} d\tilde{y}}{\int_{\underline{y}}^{\omega^{-1}(w)} \frac{\omega'(\tilde{y})}{w} d\tilde{y}}$$

if  $y \leq \omega^{-1}(w)$ , and is equal to 1 otherwise.  $\square$

Since the wage appears as a constant in both the numerator and denominator, they can be canceled. Giving the first result - if  $w_1 > w_0$ , then the probability distribution over types employed at  $w_1$  first order stochastically dominates the corresponding distribution at  $w_0$ .

However, the main purpose of this result is to compute the expected future wage of an unemployed worker whose previous employer paid a wage  $w_1$ . This is given by

$$\int_{\underline{y}}^{\omega^{-1}(w_1)} \frac{G(\bar{w}) - G(\omega(y))}{\int_{\omega(y)}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} \frac{\omega'(y) dy}{\int_{\underline{y}}^{\omega^{-1}(w_1)} \omega'(\tilde{y}) d\tilde{y}} = \frac{1}{(w_1 - \omega(\underline{y}))} \int_{\underline{y}}^{\omega^{-1}(w_1)} \left( \frac{G(\bar{w}) - G(\omega(\tilde{y})) \omega'(\tilde{y})}{\int_{\omega(\tilde{y})}^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} \right) d\tilde{y}.$$

A change of variable in the integration gives the following:

**Theorem 6.** *The expected future wage of an unemployed worker who was previously employed at wage  $w_1$  is*

$$(3.7) \quad \frac{1}{(w_1 - \underline{w})} \int_{\underline{w}}^{w_1} \left( \frac{G(\bar{w}) - G(w)}{\int_w^{\bar{w}} \frac{1}{\tilde{w}} dG(\tilde{w})} \right) dw.$$

This is the main theorem in the paper. It says that the expected future wage can be calculated from knowledge of the wage offer distribution alone. Apart from the fact that the expected future wage is a monotonically increasing function of the current wage  $w_1$ , this formula suggests that the relationship between current and future wage is highly non-linear.

The function given by (3.7) can be estimated directly from the accepted wage distribution as we explain below. Conceptually, the way to test this is to estimate the relationship between the wage a worker

received at his last job, and his current wage. For example, in the dataset considered below, if we estimate the regression

$$w = \alpha_0 + \alpha_1 w_1 + \alpha_2 w_1^2$$

the coefficient  $\alpha_0$  is estimated to be somewhere around .5 (depending on which market is considered) while  $\alpha_2$  is small but positive. One could try to compare this regression with the estimated relationship given by (3.7). Of course, (3.7) suggests a highly non-linear relationship, so ideally the relationship between  $w$  and  $w_1$  should be estimated non-parametrically, then compared with (3.7).

Tedious calculations provide some special cases. When the wage offer distribution is uniform (on  $[0, 1]$ ) the relationship between current and future wage is slightly concave. When the wage offer distribution has cdf  $x^2$  on  $[0, 1]$ , the relationship is linear, while the wage offer distribution  $x^3$  gives a slightly convex relationship. If  $G(x) = x^2$ , for example, (3.7) reduces to

$$\begin{aligned} \frac{1}{w} \int_0^w \frac{1 - \tilde{w}^2}{2(1 - \tilde{w})} d\tilde{w} &= \frac{1}{2w} \int_0^w (1 + \tilde{w}) d\tilde{w} = \\ &= \frac{1}{2} + \frac{1}{2}w. \end{aligned}$$

A similar analysis can be applied to the variance. The variance of the worker's future wage, when his or her previous wage was  $w_1$  is (following the argument associated (6))

$$(3.8) \quad \frac{1}{(w_1 - \underline{w})} \frac{\int_{w_1}^{\bar{w}} w dG(w)}{\int_{w_1}^{\bar{w}} \frac{1}{w} dG(\tilde{w})} - \left[ \frac{1}{(w_1 - \underline{w})} \frac{G(\bar{w}) - G(w_1)}{\int_{w_1}^{\bar{w}} \frac{1}{w} dG(\tilde{w})} \right]^2$$

Again this relationship suggests a fairly complicated relationship between wage and variance. Heuristically, regressing  $w$  on  $w_1$  will lead to a relationship that exhibits a lot of heteroskedasticity.

The formulas given above require information about the wage offer distribution  $G$ , while available data only provides information on the accepted wage distribution  $E$ .

**Theorem 7.** *The wage offer distribution is the solution to*

$$(3.9) \quad G(w) = \int_{\underline{w}}^w \left( \gamma + (1 - \gamma) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) \frac{\tilde{w}}{(\tilde{w} - \underline{w})} dE(\tilde{w}).$$

The proof is in the appendix.

Finally, in the case where the distribution  $E(w)$  is differentiable, the wage offer distribution can be found by solving a differential equation.

**Theorem 8.** *Let*

$$(3.10) \quad h(w) \equiv \int_w^{\bar{w}} \frac{dG(w')}{w'}.$$

*The wage offer distribution can be found by solving the differential equation*

$$-h'(w) = \frac{(\gamma + (1 - \gamma)h(w))}{w - \underline{w}} e(w)$$

*then integrating the solution  $h'(w)w$ .*

*Proof.* Differentiate (3.9) to get

$$g(w) = \left( \gamma + (1 - \gamma) \int_w^{\bar{w}} \frac{dG(w')}{w'} \right) \frac{w}{w - \underline{w}} e(w).$$

Now substituting (3.10) gives

$$-h'(w) = \frac{(\gamma + (1 - \gamma)h(w))}{w - \underline{w}} e(w),$$

which is the equation in the statement of the theorem.  $\square$

This is the method we follow below. Notice that this leaves an unidentified variable  $\gamma$  which is the probability a match is exogenously terminated in any period. This can be calibrated with external data. Alternatively, it can be estimated by checking the number of job transitions that result in an unemployment spell. In our theory at least, each such transition is the result of an exogenous termination.

#### 4. COMPARISON WITH OTHER MODELS

This prediction suggests a way to compare three different models of directed search. Each of these three models can be thought of as special cases of the model discussed here.

For example, one special case of the model above occurs when the type of an employee determines whether or not he is hired just as in the model above, but where type is not retained from one period to another. For example, each employees type could be redrawn each period by selecting randomly from the distribution  $F$ . This is nothing more than a restatement of a standard model of directed search in which workers use mixed application strategies.<sup>4</sup> In particular, since type is not persistent, this means the wage at which a worker is currently employed should have no relationship at all with the wage a worker gets when he or she moves on to a new job.

<sup>4</sup>For example (Peters 2000).

Conversely, worker types might again determine the probability of being hired, but these types may be public observable to other workers. If everyone knows who the highest type worker is, they will also know where he or she will apply given any distribution of wages. As a result, mixing will break down, and workers will match assortatively, as in special cases discussed by (Shi 2001) or (Eeckhout and Kircher 2010). Wages received by workers as they move between jobs will be very highly correlated in this case.

The following picture may help make the results in the empirical section, and the connections between the various search models clearer.

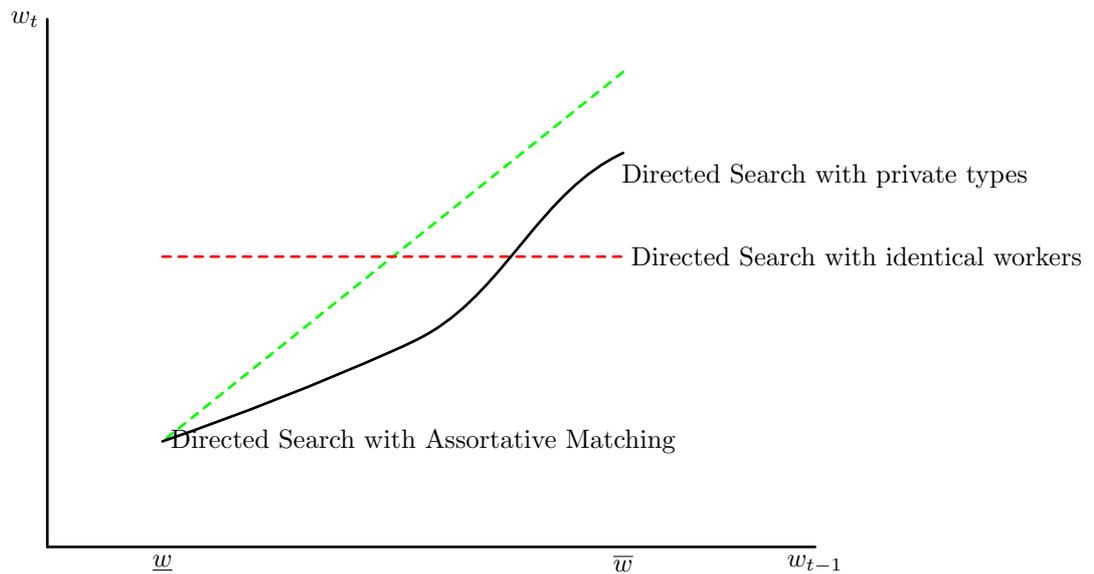


FIGURE 4.1. Comparison of Models

The horizontal axis in the picture represents the current wage of a worker, while the vertical axis represents the wage in the job that the worker moves to after a match is terminated. All the predictive consequences of the model we consider here emerge from studying the predictive content of current wage. The most basic directed search model in which workers are all identical, but firms offer different wages is represented by the dashed red line in the picture above. It is flat because a lucky draw in one period will not last - when a worker moves to a new job, he or she will receive the same wage on average no matter what their current wage happens to be.

On the other hand, pure assortative matching leads to the relationship indicated by the green line in the picture - whatever wage a worker

gets today, he will also get tomorrow, simply because assortative matching will continue to place him or her at the same point in the wage distribution.

Finally, the model described above will lead to a correlation between current and future wage, but this correlation will be much weaker than that predicted by assortative matching. The reason for the weaker correlation is that the wage is a very imperfect signal of worker type because of the mixed application strategies that workers use. On the other hand, higher type workers are more likely to be employed at higher wage firms.

In general, the model says the relationship between past and future wage is non-linear. The shape of the relationship depends on the shape of the wage offer distribution, as we described above.

## 5. EMPIRICAL APPLICATION

**5.1. Data (very preliminary).** In order to examine transitions in applications, we use a data set on the French labor market. In this section, we'll describe the data, and give some preliminary calculations to illustrate. Our main data source is the DADS (*Declarations Annuelles de Donnees Sociales*), a large scale administrative database of matched employer-employee information collected by INSEE (the French National Institute of Statistics and Economic Studies). The data are collected in accordance with the mandatory employer reports of the employment status and gross earnings, for the payroll tax purposes.<sup>5</sup>

Each observation in the DADS file corresponds to a unique individual-year-establishment combination. The observation includes an identifier for each employee, an identifier for each establishment, and an identifier for the parent enterprise of the establishment. For each data entry, we observe the number of days during the calendar year the employee worked in the establishment and whether or not the employment was full-time or part-time. Furthermore, we observe the employee's gender, age, occupation, total income (both before and after tax), as well as the location and industry of the employing establishment.

Such a database is ideal for our purpose, as one can easily trace workers' movements between jobs. Another advantage of this data is that the employment information is at the plant (establishment) level. Indeed, identify different labor markets using plant level information.

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<sup>5</sup>Same data source and similar data construction have been used in literature. Refer to Abowd et al (1999), Postel-Vinay and Robin (2002), Cahuc et al (2006) for more detailed elaboration of data source.

Plants within a firm are assumed to be independent in terms of technological diffusion and production choices, and no internal labor markets are supposed to exist.

We choose the data from DADS for financial year of 2007. Our base sample thus contains individual employment information and income data. For each worker and firm match in 2007, the data also includes match information from the previous year (2006) and the following year (2008). So the transition data is taken from a three year period. Below, we will explain how we separate the match data into different sub-markets. More technical details and discussion are deferred into data Appendix.

**Worker Transitions.** We start our data work by looking for worker transitions in the sample. For each matched worker we trace her employment history by looking for earlier and later matches (i.e., the same worker identifier but different firm identifiers). We regard a pair of jobs to involve a transition if they satisfy the following conditions:

- the difference of dates between the end of previous job and the begin of current employment is less than 30 days;
- the end of previous job occurs before the end of current employment;
- the start of previous job occurs before the starts of current employment.

Since workers often hold more than one job at the same time, there are sometimes multiple jobs that satisfy the definition above. In other words, there may be more than one other jobs where the employee worked while holding his current job. In that case, we use the job that consumed the largest number of working hours while the worker held his current position.

Using this approach, each employee's work history can be reduced to a series of transitions from one job to another during the period 2006 to 2008.<sup>6</sup>

**Defining Labor Markets.** In the theory, a labor market consists of a set of workers who are all equally qualified for a certain set of jobs. Rather than breaking the data into markets in an ad hoc way, we employ a flexible and data-driven approach to identify markets. Basically a market is defined by mobility - set of jobs and workers is defined to be a market when there is evidence that a certain group

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<sup>6</sup>Firm transitions can be defined in an analogous way. We aren't too interested in firm transitions in what follows so we leave out details.

of workers are mobile between the various positions offered in these markets. More technical notes are provided in an Appendix.

**Wages.** The wage variable we shall use in the analysis is the hourly wage rate, which is defined by total wage bill received by the worker divided by the effective working hours.

## 6. EMPIRICAL RESULTS

The theory suggests a number of empirical relationships. The main idea is that wage is a signal of worker type and that the information contained in this signal should be reflected in the relationship between wages across transitions. We'll focus on one particular connection here because it is a bit unexpected.

One complication in testing this is estimating the wage offer distribution from the accepted wage distribution. One approach would be to estimate  $\gamma$  from observed transitions, then solve for the fixed point defined in Proposition 7. Since this will generally give a very poor estimate of  $G$ , we take another approach. First we use Proposition 3 which says that the length of time an employee stays in any job is a declining function of his or her type. The reason for this is on the job search - employees who have higher types are more likely to find better jobs no matter what wage they are currently working at. It should then follow that employees working at any wage should remain at that job for a shorter period the more they earned in their previous jobs.

We then try to identify labor markets in which this duration effect is small. The effect of wage on duration can be small for one of two reasons - either worker types don't matter to firms, so all workers are equally likely to move between jobs, or on the job search is uncommon. If the former explanation is true, then the transitional effects we have talked about above should all disappear. However, if the latter explanation is right, many of the effects we discussed should still be evident.

**Regression.** To do this used observations involving worker transitions for the regression analysis. In doing so, we first normalize the wage observed in the data to its quantile and carry out our empirical analysis based on the wage rank. Our goal here is to check whether a worker's current wage rank is correlated with his type. If it is, then we expect to see this information preserved in the wage a worker receives after a job transition. For example, if wage ranks were perfectly correlated with type, then we would expect normalized wage ranks to remain constant as a worker moves from firm to firm.

To try to capture this, we first regressed the wage rank of a worker's wage after a transition ( $w_{i,t}$ ) on the wage rank at his last position ( $w_{i,t-1}$ ) as in the following equation:

$$(6.1) \quad w_{i,t} = \alpha_0 + \alpha_1 w_{i,t-1} + \alpha_2 w_{i,t-1}^2 + \gamma_1' X_{i,t} + \mu_{i,t}$$

Here,  $X$  is the vector of observed characteristics of the worker. The squared term appears here because of the possibility that the strength of the correlation between wage ranks across transitions can vary with the wage rank the worker had before the transition.

The results of this regression for occupations 62 and 67 (skilled and unskilled manufacturing workers) appear in the first column of Table 1. The coefficient on past wage is around .5 in both cases (.541 and .465 respectively). This correlation is strong enough to suggest that type information is important to firms, but is hardly strong enough to indicate either assortative matching, or a simple wage ladder.

The interaction term  $w_{i,t-1}^2$  suggests that the relationship is convex, suggesting that more information is conveyed about type in high wage jobs.

This leads to the main predictions of the theory here. When a worker moves between jobs there is no guarantee about what his wage will be at the new job. In the theory this is because the worker's search strategy involves some randomization, and because his success at being hired is random. Recall that our theory suggests that the wage rank that the worker holds at his current position should be negatively related to the *variance* of wage rank he should expect after a move. To try to measure this, we took the absolute values of residuals from the regression (6.1) as a measure of the variation in the outcome of search, and regressed them on the wage rank held by workers before a transition. That is,

$$(6.2) \quad |\mu_{i,t}| = \beta_0 + \beta_1 w_{i,t-1} + \gamma_2' X_{i,t} + \epsilon_{i,t}$$

Again, the results of this regression are reported in Table 1, this time in the second column. For skilled jobs in manufacturing (occupation 62) the declining variance effect appears and is significant. For unskilled manufacturing workers, the declining variance effect disappears. These estimates are consistent with the interpretation that types don't matter much for unskilled workers. We don't read too much into the coefficients associated with  $w_{i,t-1}$  and its square terms because of possible presence of heteroscedasticity. We then regressed the absolute value of residuals from (6.1) on past wages, Column (2) gives the results of regression (6.2). As can be seen, the coefficient is negative and significant, indicating that the variance of outcomes is declining with past wages,  $w_{i,t-1}$ .

To reaffirm the presence and monotonic pattern of heteroscedasticity, we conducted several formal tests, whose results are reported in the table 2. We first implemented White’s general test for heteroscedasticity. Then, we did Goldfeldt-Quant (GQ) test, which can be used when it is strongly believed that the variance of the error term decreases (or increases) consistently as one of the regressors increases. The F-statistic for our GQ test rejects the null at 10% significance level, suggesting a decreasing pattern of variances with respect to  $w_{i,t-1}$  only in skill-intensive occupation.

The significant coefficient of  $\beta_1$  suggests the presence of heteroscedasticity. Interpreting the significance of estimates on  $\alpha_1$  and  $\alpha_2$ , we had to adjust the estimation procedure taking into account the fact of shrinking conditional variance. To this end, we further assume  $Var(\mu_{i,t}|w_{i,t-1}) = \sigma^2/w_{i,t-1}^2$ . This specification enables us to conduct a feasible GLS. The results are reported in Column (3) of table 1.

As is apparent current wages rise with past wages. Furthermore, the squared past wage term in the regression illustrates that this effect gets stronger as the wage rises.

## 7. CONCLUSION

The results reported here are consistent with the theory of mismatch we have presented, but not consistent with either of the other three models of directed search. The model we use is about as simple as it could possibly be. Nonetheless it seems to get some of the basic empirical properties right.

On the theoretical side, there are at least three dimensions in which the model seems to be going too far. First, it assumes that workers’ types don’t change over the course of their life. This may be the least objectionable assumption. Workers will obviously acquire new skills as they age. Yet these skills are more often than not contractible. For example, a worker who acquires an MBA will probably be compensated for it. We don’t interpret this as an improvement in the worker’s type. In our regressions, we capture this by adding experience (measured by worker age) and assuming that this will explain much of the rise in income that workers experience as they move between jobs.

We also assume that match termination is independent of worker and firm type. Notice that this is different from the assumption that match termination doesn’t depend on duration. It surely does. However, in our theory no one cares about duration per se, and wages represent expected income of the life of the match. The only important assumption

is that matches are terminated in a way that maintains the distribution of worker and position types available on the market in any period.

Lastly, we do not allow firms to refuse to hire. If we did, high type firms would refuse to employ very low type workers making current wage a much better signal of worker quality. Pursuing this modification goes well beyond the scope of the current paper.

This brings us to one of the implications of the results here. The results are consistent with a model in which firms use uncontractible information to screen candidates when they hire. A natural empirical question this suggests is whether firms actually value this uncontractible information, or whether it is simply a way of coordinating search. These are independent questions. For example, it could well be that a firm hires a worker because the worker knows one of the boss's relatives. That isn't the same as saying that the firm is willing to pay more to hire someone who knows a relative, nor that the firm is more profitable when it hires someone who knows a relative instead of someone who doesn't. The model here seems to fit well enough to move to a structural approach which tries to estimate the distribution of firm types.

TABLE 1. Regression Results

Occupation	PCS:62			PCS:63		
	(1)	(2)	(3)	(1)	(2)	(3)
Variables	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$
$w_{i,t-1}$	0.541*** (28.861)	-0.018*** (-2.584)	0.540*** (27.489)	0.365*** (7.806)	-0.011 (-1.311)	0.362*** (7.592)
$w_{i,t-1}^2$	0.161*** (8.907)		0.162*** (8.899)	0.177*** (4.017)		0.180*** (4.028)
$dur_{i,t-1}$	-0.814*** (-7.436)		-0.723*** (-6.713)	0.144** (2.200)		0.198*** (3.001)
$dur_{i,t-1}^2$	-23.873*** (-6.026)		-24.395*** (-6.004)	-7.633* (-1.665)		-7.814* (-1.690)
Constant	0.124*** (7.589)	0.077*** (5.538)	0.110*** (6.608)	-0.147*** (-5.600)	0.131*** (5.033)	-0.150*** (-5.715)
Observations	418,018	418,018	418,018	311,370	311,370	311,370
$R^2$	0.551	0.024	0.553	0.391	0.017	0.392
	OLS	OLS	GLS	OLS	OLS	GLS
Occupation	PCS:64			PCS:65		
	(1)	(2)	(3)	(1)	(2)	(3)
Variables	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$
$w_{i,t-1}$	0.306*** (18.229)	-0.011*** (-4.565)	0.297*** (17.483)	0.499*** (12.967)	-0.011* (-1.677)	0.494*** (12.721)
$w_{i,t-1}^2$	0.204*** (7.246)		0.212*** (7.555)	0.143*** (4.228)		0.146*** (4.288)
$dur_{i,t-1}$	0.202** (2.116)		0.285*** (3.023)	-0.646*** (-7.214)		-0.545*** (6.406)
$dur_{i,t-1}^2$	-13.156*** (-4.911)		-13.867*** (-5.095)	-12.155*** (-4.347)		-12.951*** (-4.679)
Constant	-0.025*** (2.520)	0.225*** (25.612)	-0.026** (-2.648)	-0.083* (-1.822)	0.173*** (5.836)	-0.092** (-2.077)
Observations	267,866	267,866	267,866	123,990	123,990	123,990
$R^2$	0.326	0.015	0.330	0.491	0.023	0.494
	OLS	OLS	GLS	OLS	OLS	GLS
Occupation	PCS:67			PCS:68		
	(1)	(2)	(3)	(1)	(2)	(3)
Variables	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$	$w_{i,t}$	$ \mu_{i,t} $	$w_{i,t}$
$w_{i,t-1}$	0.465*** (14.510)	0.005 (0.650)	0.476*** (15.306)	0.055 (1.243)	-0.007 (-1.284)	0.051 (1.073)
$w_{i,t-1}^2$	0.176*** (4.747)		0.165*** (4.507)	0.454*** (11.023)		0.461*** (10.444)
$dur_{i,t-1}$	-1.191*** (-11.665)		-1.339*** (-12.927)	-0.360* (1.918)		-0.432** (2.418)
$dur_{i,t-1}^2$	-29.454*** (-7.180)		-28.426*** (-7.284)	-8.980*** (-5.311)		-11.040*** (-6.230)
Constant	0.052* (1.708)	0.220*** (16.354)	0.059** (2.002)	0.240** (2.229)	0.324*** (4.548)	0.211** (2.097)
Observations	148,732	148,732	148,732	204,411	204,411	204,411
$R^2$	0.562	0.071	0.568	0.332	0.035	0.340
	OLS	OLS	GLS	OLS	OLS	GLS

TABLE 2. Heterogeneity Test Results

Occupation	White	Goldfeld-Quandt
<b>PCS:62</b>	1642.65***	1.34*
<b>PCS:63</b>	718.51***	1.26
<b>PCS:64</b>	689.21***	1.24
<b>PCS:65</b>	438.17***	1.38
<b>PCS:67</b>	602.23***	0.92
<b>PCS:68</b>	1424.46***	1.34

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## 8. APPENDIX

## 8.1. (Proof of Theorem 1).

*Proof.* Fix a continuous non-decreasing rule  $\omega : Y \rightarrow W$ . Notice that  $\omega$  is not required in this definition to have range contained in  $\bar{G}$ , so the proper interpretation is that  $\omega(y)$  is the wage that yields the worker his market payoff if he is hired for sure at that wage. If all searching workers apply to all wages at or above their reservation wage, then

$$P(w, y) = \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \frac{G(w) - G(\omega(y'))}{G(\bar{w}) - G(\omega(y'))} dF(y').$$

The ‘queue size’  $p_w(y)$  has to satisfy (2.1), so

$$p_w(y) = \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y').$$

To see this observe that for any  $w$ ,

$$\begin{aligned} \int_{\underline{w}}^w p_{\tilde{w}}(y) dG(w) &= \int_{\underline{w}}^w \int_y^{\min[\omega^{-1}(\tilde{w}), y]} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y') dG(\tilde{w}) \\ &= \int_y^{\min[\omega^{-1}(w), y]} \int_{\omega(y')}^w \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y'))} dF(y') = \\ &\quad \int_y^{\min[\omega^{-1}(w), y]} \frac{G(w) - G(\omega(y'))}{G(\bar{w}) - G(\omega(y'))} dF(y'). \end{aligned}$$

This implies that

$$(8.1) \quad \int_y^{\bar{y}} dp_w(\tilde{y}) = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y').$$

So hiring probabilities will be given by (3.2) provided that workers all use the application strategy described. Given this matching probability we can now describe the condition that  $\omega(y)$  has to satisfy in order for them to be willing to follow this strategy. In order for a searching worker of type  $\omega(y) > \underline{w}$  to be indifferent between all wages above his reservation wage, it should be that for each  $w' > \omega(y)$

$$\begin{aligned} (w' + \gamma U(y)) e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} + \left(1 - e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})}\right) \gamma U(y) \\ = \omega(y) + \gamma U(y), \end{aligned}$$

or

$$w' e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} = \omega(y).$$

Taking logs yields

$$\int_y^{\bar{y}} dp_{w'}(\tilde{y}) = \log(w') - \log(\omega(y)).$$

By the fundamental theorem of calculus this implies

$$(8.2) \quad \int_{\omega(y)}^{w'} \frac{1}{\tilde{w}} d\tilde{w} = \int_y^{\bar{y}} dp_{w'}(\tilde{y}).$$

Substituting (8.1), then gives the identity

$$\int_{\omega(y)}^w \frac{1}{\tilde{w}} d\tilde{w} = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')$$

is satisfied for all  $y$ . Differentiating both sides with respect to  $w$  gives the differential equation

$$(8.3) \quad \omega'(y) = \frac{\omega(y) F'(y)}{G(\bar{w}) - G(\omega(y))}.$$

The reservation wage function  $\omega$  will support the continuation equilibrium if it has a solution with  $\omega(\bar{y}) = \bar{w}$ . This is not immediate since the right hand side does not have a continuous derivative around the point  $(\bar{y}, \bar{w})$ .

However it does have a solution through the point  $(\bar{y}, \bar{w} - \epsilon)$  for any  $\epsilon > 0$ . Denote the solution for  $\epsilon > 0$  as  $\omega^\epsilon(y)$ . Observe that each  $\omega^\epsilon$  is strictly increasing and that  $\omega^\epsilon$  and  $\omega^{\epsilon'}$  cannot cross, therefore the sequence  $\{\omega^\epsilon\}_{\epsilon \rightarrow 0}$  is an increasing sequence of increasing functions. As the sequence  $\omega^\epsilon(y)$  is a bounded increasing sequence of real numbers,  $\omega^\epsilon$  converges point-wise, therefore uniformly (Dini's Theorem) to some function  $\omega$ . If (8.3) fails at some point  $y$ , then by uniform convergence, it must fail for small  $\epsilon$ . So  $\omega$  is a solution to (8.3).

The remaining bits of the theorem then follow by using (2.2) along with the reservation wage.  $\square$

**Analyzing the firms' problem.** Though we pay little attention to the firms' problem in what follows, we can use the previous results to describe what firms do. Readers who are only interested in the implications for wage data can skip this section.

Using Theorem 1, we get the following characterization:

**Lemma 9.** *In a symmetric steady state equilibrium, the firm's payoff function can be written as*

$$(8.4) \quad \tilde{V}(w) = \int_{\underline{y}}^{\omega^{-1}(w)} \left\{ \frac{\frac{v(w,x,y)}{w} \omega'(y)}{\gamma + (1-\gamma) \frac{\omega'(y)}{F'(y)} \int_w^{\bar{w}} \frac{dG(\bar{w})}{\bar{w}}} \right\} dy.$$

*Proof.* Substituting this into  $\tilde{V}(w)$  and using (3.4) and Lemma 4, we can write the firm's payoff function as

$$\begin{aligned} \tilde{V}(w) &= \int_{\underline{y}}^{\omega^{-1}(w)} \frac{v(w,x,y) e^{-\int_y^{\bar{y}} dp_w(\bar{y})}}{Q(w,y)} dp_w(y) = \\ &= \int_{\underline{y}}^{\omega^{-1}(w)} \frac{v(w,x,y) \frac{\omega(y)}{w} \omega'(y)}{Q(w,y) \omega(y)} dy = \\ &= \int_{\underline{y}}^{\omega^{-1}(w)} \left\{ \frac{\frac{v(w,x,y)}{w} \omega'(y)}{\gamma + (1-\gamma) \frac{\omega'(y)}{F'(y)} \int_w^{\bar{w}} \frac{dG(\bar{w})}{\bar{w}}} \right\} dy. \end{aligned}$$

$\square$

One way to view the formula in (8.4) is that the firm trades off the wage it pays against the highest quality worker who applies. With this interpretation the firm's maximization problem could be expressed as

$$\max_{w,y} \int_{\underline{y}}^y \left\{ \frac{\frac{v(w,x,\tilde{y})}{w} \omega'(\tilde{y})}{\gamma + (1-\gamma) \frac{\omega'(\tilde{y})}{F'(\tilde{y})} \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}} \right\} d\tilde{y}$$

subject to the constraint that  $\omega(y) = w$ .

This is a pretty standard directed search problem. The formula above is somewhat complex, but it illustrates a fundamental identification problem. Fix the (steady state) wage offer distribution  $G$ . Theorem 1 ensures the existence of a reservation wage strategy  $\omega$ . Define

$$\phi(w, x, y) = \frac{\frac{v(w,x,\tilde{y})}{w} \omega'(\tilde{y})}{\gamma + (1-\gamma) \frac{\omega'(\tilde{y})}{F'(\tilde{y})} \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}}.$$

Now suppose that we change the distribution  $F$  so that it is uniform. Again using Theorem 1, there will be a new reservation wage rule, say  $\tilde{\omega}(y)$ . The equation

$$\frac{\frac{\tilde{v}}{w} \tilde{\omega}'(\tilde{y})}{\gamma + (1-\gamma) \tilde{\omega}'(\tilde{y}) \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}}} = \phi(w, x, y)$$

with unknown  $\tilde{v}$  has a positive solution for each pair  $(w, y)$  given by

$$\tilde{v}(w, x, y) = \frac{w\phi(w, x, y) \left( \gamma + (1-\gamma) \tilde{\omega}'(\tilde{y}) \int_w^{\bar{w}} \frac{dG(\tilde{w})}{\tilde{w}} \right)}{\tilde{\omega}'(y)}.$$

This means that when  $F$  is replaced with a uniform distribution function and the profit function is replaced by  $\tilde{v}$ , the expected profit function for every firm type  $x$  is uniformly the same as the old one. As a consequence, the distribution of best replies  $G$  will remain unchanged.

As the result in the previous section shows, we might as well assume from now on that the distribution of worker types is uniform while imagining that the profit function describes the profit to the firm association with hiring workers at different quantiles of the type distribution.

## 8.2. Proof of Theorem 7.

*Proof.* The steady state condition is

$$G(w) = \int_w^w \frac{\int_{\underline{y}}^{\bar{y}} Q(\tilde{w}, y) d\psi(y|\tilde{w})}{\int_{\underline{y}}^{\bar{y}} e^{-\int_x^{\bar{y}} dp_{\tilde{w}}(\tilde{y})} dp_{\tilde{w}}(x)} dE(\tilde{w}).$$

By (2)

$$G(w) = \int_{\underline{w}}^w \frac{\int_{\underline{y}}^{\bar{y}} \left( \gamma + (1 - \gamma) \omega'(y) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) d\psi(y|\tilde{w})}{\int_{\underline{y}}^{\omega^{-1}(\tilde{w})} e^{-\int_x^{\bar{y}} dp_{\tilde{w}}(\tilde{y})} dp_{\tilde{w}}(x)} dE(\tilde{w}).$$

By (1) and (4), the right hand side of this equation equals

$$\int_{\underline{w}}^w \frac{\int_{\underline{y}}^{\bar{y}} \left( \gamma + (1 - \gamma) \omega'(y) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) d\psi(y|\tilde{w})}{\int_{\underline{y}}^{\omega^{-1}(\tilde{w})} \frac{\omega(y)}{\tilde{w}} \frac{\omega'(y)}{\omega(y)} dy} dE(\tilde{w}).$$

Simplifying the denominator gives

$$\int_{\underline{w}}^w \frac{\int_{\underline{y}}^{\omega^{-1}(\tilde{w})} \left( \gamma + (1 - \gamma) \omega'(y) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) d\psi(y|\tilde{w})}{\frac{1}{\tilde{w}} (\tilde{w} - \underline{w})} dE(\tilde{w})$$

By (5), this is equal to

$$\int_{\underline{w}}^w \frac{\int_{\underline{y}}^{\omega^{-1}(\tilde{w})} \left( \gamma + (1 - \gamma) \omega'(y) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) \frac{\omega'(y)}{(\tilde{w} - \underline{w})} dy}{\frac{1}{\tilde{w}} \int_{\underline{y}}^{\omega^{-1}(\tilde{w})} \omega'(y) dy} dE(\tilde{w})$$

Changing variable in the integration then gives the equality

$$\int_{\underline{w}}^w \frac{\int_{\underline{w}}^{\tilde{w}} \left( \gamma + (1 - \gamma) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w'')}{w''} \right) \frac{1}{(\tilde{w} - \underline{w})} dw'}{\frac{1}{\tilde{w}} \int_{\underline{y}}^{\omega^{-1}(\tilde{w})} \omega'(y) dy} dE(\tilde{w}).$$

$$G(w) = \int_{\underline{w}}^w \frac{\left( \gamma + (1 - \gamma) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right)}{\frac{1}{\tilde{w}} (\tilde{w} - \underline{w})} dE(\tilde{w})$$

$$\int_{\underline{w}}^w \left( \gamma + (1 - \gamma) \int_{\tilde{w}}^{\bar{w}} \frac{dG(w')}{w'} \right) \frac{\tilde{w}}{(\tilde{w} - \underline{w})} dE(\tilde{w}).$$

This gives the fixed point stated in the Theorem.  $\square$