COMPETING MECHANISMS

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Abstract. This paper gives a brief explanation of models used in the literature on competing mechanisms. It explains how auctions with no reserve prices emerge as equilibrium mechanisms in a broad variety of environments, including environments in which buyers valuations are correlated. The basic theme is that competition promotes simple mechanisms.

There are many ways to sell goods to people whose values you don't know. Auctions are one way, but there are many others. If you buy a car or house, the seller will often engage in a complicated negotiation process that typically involves auction like tricks designed to sort the low and high value buyers. For example, a car dealer is happy to match lower prices you find at other dealers, but will warn you that his cars are in high demand, the car you want may no longer be available when you get back. Whether you believe the assertion or not, the seller learns something about your value for the car when you are willing to take that chance. Houses are often sold at auction in Australia and New Zealand, however, in North America a very formalized offer-counteroffer process is more common.

In North America, fixed price sales are common (for example, in supermarkets). Yet outside North America, it isn't hard to find markets where haggling is the norm even when the commodity being sold is of known quality and has a relatively low (and commonly known) value. A visit to the night market on Temple Street in Hong Kong gives an idea. The night market provides many alternatives to buyers, so there is little reason for a seller to hold out for a high price (their response to this is often to offer goods of relatively low quality).

One of the interesting things about the night market is that the ability to haggle is an attraction of the market itself. Beyond creating a tourist attraction for North Americans who aren't used to bargaining, the market provides an opportunity for very low value buyers to buy

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stuff they otherwise might not want. Whether it is intended or not, bargaining provided a way of accommodating buyers with low values.

A more familiar selling technique is restaurant reservations. In principle, restaurants could auction their Saturday evening tables to the highest bidder. Instead, all they do is to require reservations, then uniformly raise the prices of all their meals. Presumably, the reservation system sorts out diners who particularly want to go to that restaurant from diners who simply want something to eat. Knowing that their reservation system is selecting buyers for them, they can set higher prices.

This proliferation of selling techniques can be partly explained by simple informational considerations - for example, if a good has a commonly known value which is the same for everyone, there is little point holding an auction in order to sell it. However, competitive considerations are also likely to determine how goods are sold. On eBay, sellers literally offer competing auctions. The impact of competition on eBay became very easy to see after the introduction of the "But it Now" option that allows a bidder to circumvent what would otherwise be a fairly straightforward second price auction by accepting a take it or leave it price offer. In data collected in 2005 on camera auctions at eBay, around 75% of all auctions were resolved using the buy it now option (there were dozens of simultaneous auctions for each model).

Many internet sellers also experiment with selling techniques. For example, domain name resellers like flippa.com resell domain names bundled with websites and software. Their auction site offers sellers a variety of different techniques, including eBay like second price auctions, second price auctions augmented with buy it now, and a variant of the buy it now in which the seller essentially conducts a first price sealed bid auction. The site itself attributes some of its success to the fact that it allows sellers to auction domain names. This encourages sellers to bundle the names with websites which suggest ways that the domains can be used.

These examples suggest that selling methods may be as important as prices in attracting buyers. The theory of *mechanism design* was created to address exactly this issue, except that as originally formulated, the theory only allows one mechanism designer. This paper reviews a couple of models that explicitly model competition in mechanisms.

Competition makes it possible to address a perplexing theoretical issue as well. The theory of mechanism design with a single designer makes an implausible prediction. The theorem due to (Cremer and McLean 1988), says that if buyers' valuations are even slightly correlated, then the seller can design a selling mechanism that provides the same expected revenue that he would have earned had he known the buyers' values. Of particular interest is the implication this theorem has for the properties of the seller's best selling mechanism. The technique that (Cremer and McLean 1988) used to extract buyer surplus was to ask each buyer to commit to pay a fee after the auction finishes. This fee depends on the bids that were submitted by all of the other buyers. Since the fee depends only on the other buyers' bids, a buyer could not manipulate this fee at all. They then showed that the correlation in valuations could be used to design the fee so that the expected payment associated with the fee is exactly equal to the surplus the bidder receives by participating in a second price auction.

Whether or not conditions in existing auction markets exactly mimic the conditions required for their theorem to hold, the result suggests that sellers' most profitable selling mechanisms should involve fees that depend on what other bidders do. No one has yet come up with a 'real life' example in which sellers actually use these fees. The conclusion - either sellers aren't designing selling mechanisms to maximize their revenues, or something else is going on.

In this chapter we discuss one possible resolution to this problem - competition. If buyers have better alternatives, they simply won't participate in a selling mechanism that takes away all their surplus. Yet it isn't the surplus extraction that presents a problem - it is the fees contingent on others bids. Competition readily explains why surplus extraction doesn't happen, but it doesn't seem inconsistent with these fees.

1. Directed Search

Competition in prices is pretty straightforward, because every buyer is attracted to a low price. The night market example illustrates that selling mechanisms are quite different. Haggling is attractive to low value buyers who can demonstrate their values by walking away, but unattractive to high value buyers who need to find a good deal quickly.[1] A change in selling mechanism may not be attractive to all buyers.[2]

[1]One of the interesting things about the Temple Street night market in Hong Kong, is that the stalls where haggling occurs often obscure entrances to shops where more serious buyers can buy. Perhaps this is the device the night market uses to keep its high value customers.

[2]One of the big changes that has occurred on eBay is the emergence of the buy-it-now feature. Rather than bidding in the auction, there is now a fixed (typically high) take it or leave it price that any buyer can agree to pay immediately, thus ending the auction. In data collected in 2005 for camera auctions, about 3/4 of all the auctions ended with some buyer clicking the buy it now price. Presumably, this

One way to model competition in mechanisms is to borrow a technique originally designed to model competition in labor markets where search frictions were significant. We detour a bit to explain this method before returning to competing mechanisms.

Suppose there are two firms trying to attract workers. They offer wages w_1 and w_2 with w_1 being larger. There are two workers, each of who applies to one and only one of the two firms. If a firm receives a single applicant, they hire him or her. If a firm receives two applicants, they hire one of them at random. If a firm doesn't receive an application, it does without a worker. If a worker applies and isn't hired, then that worker does without a job. A firm who hires a worker produces revenue of \$1. Workers who aren't hired, and firms who don't hire both earn nothing.

The theory of directed search is based on the assumption that both workers apply to firm 1 with the same probability, say π. When firm 1's wage offer rises, this probability should increase. The firm trades off the higher wage that it offers against the higher probability that it will hire some worker in order to determine its wage. Eventually, we want to apply this idea to selling mechanisms. For example, it is reasonable to expect that if a firm lowers the reserve price that it sets in an auction, then all bidder types will be more likely to bid in that firm's auction.

In the job application problem, it is straightforward to tie down this bidding probability. Each worker is going to apply to the firm where it has the highest expected payoff. If we think a worker is going to apply to two different firms with positive probability, it better be the case that the worker receives the same expected payoff from both. In particular, if the other worker is applying to firm 1 with probability π, then the expected payoff to the worker if he applies there is

$$\pi\frac{w_1}{2} + (1-\pi)\,w_1.$$

The explanation is that if the other worker also applies to firm 1, then there is half a chance that the worker will be hired. If the other worker applies to firm 2, then the worker is hired for sure.

Using the same reasoning to compute the expected payoff associated with an application to firm 2, the probability with which the worker expects the other worker to apply to firm 1 had better satisfy

$$\pi\frac{w_1}{2} + (1-\pi)\,w_1 = \pi w_2 + (1-\pi)\,\frac{w_2}{2}$$

new feature is great for high value buyers who are anxious to get their cameras, but bad for low value buyers who have to work a lot harder to find a bargain.

or

$$\pi = \frac{2w_1 - w_2}{w_1 + w_2}.$$

What this algebra shows is that is worker 1 expects worker 2 to apply to firm 1 with probability $\frac{2w_1 - w_2}{w_1 + w_2}$, then worker 1 will be just indifferent about whether he applies to firm 1 or firm 2. Of course, if he is just indifferent, then it wouldn't be unreasonable to expect him to apply to firm 1 with probability $\frac{2w_1 - w_2}{w_1 + w_2}$, so that worker 2 would also be indifferent about which firm he applies to.

The application strategy $\pi = \frac{2w_1 - w_2}{w_1 + w_2}$ constitutes a Nash equilibrium for the application game that is played by the workers. This Nash equilibrium gives a very nice description of how workers go about choosing between different firms. As is apparent from the formula, as firm 1 raises its wage, both workers are more likely to apply to firm 1. It is exactly that logic that we want to apply when we think about competing mechanisms.

2. Competing Mechanisms

The logic we want to develop is that when a firm alters a characteristic of its selling mechanism, this change will increase the probability with which bidders participate in the mechanism whenever this change increases the surplus they expect to earn. To illustrate we can focus on auctions and assume that two firms compete in reserve prices. Then our logic suggests that raising the reserve price (which lowers all buyers surplus ceteris paribus) will reduce participation probability.

To see the argument, suppose there are two firms each of whom possesses a single unit of output which they hope to sell to one of two buyers. We will imagine that each of the firms uses a second price auction with a reserve price. Firms don't value their goods at all, apart from what they think they can sell them for. So we imagine the sellers' valuations are both 0. However, the sellers are explicitly concerned with how their reserve prices will affect buyer participation since this affects the revenue they expect to earn from their auctions.

There are two buyers who both feel that the goods offered by the sellers are perfect substitutes for one another. However, the buyers differ in their valuations for the goods. We will suppose each buyer's valuation is independently drawn from a common probability distribution F.

As in the labor market story presented above, we imagine that the firms begin by describing their auctions, then each of the buyers chooses which of the two auctions he wants to participate in. If only one buyer bids in an auction, the good is sold for its reserve price, if two bidders bid, then the good is sold to the high bidder at price equal to the second highest bid. The probabilities with which the different types of bidder participate in seller 1's auction depend on the two reserve prices. Let $\pi(v)$ be the probability that a buyer with valuation v chooses to bid in seller 1's auction.

To see how to find the equilibrium, we use two ideas. One is a nice insight from (McAfee 1993), the other a standard argument in mechanism design. Lets start with the mechanism design argument. When a bidder participates in an auction his expected payoff is equal to his probability of winning the auction when he participates multiplied by his value, less the price he expects to pay to the firm. The winning probability and expected price both depend on his value as well as participation probabilities of the other bidder, and the reserve price he faces. For the moment, lets ignore the participation probabilities and reserve price and write this out as

$$Q(v)\,v - P(v)$$

where $Q(v)$ is the probability that a seller of type v wins the auction and $P(v)$ is the price he expects to pay. In the equilibrium of the second stage of the game, the bidders will adopt some participation strategies. We have no idea at the moment what they are, but the equilibrium participation strategy of a bidder with value v must be at least as good for him as the strategy that would be used by a bidder with a different value, say v'. If the bidder with value v were to adopt the participation strategy of the bidder with valuation v', then his payoff would be

$$Q(v')\,v - P(v').$$

In fact, since the payoff he gets by using his equilibrium strategy must be better than the payoff he could get by using *any* other bidder's strategy it must be that

$$Q'(v')\,v - P'(v') = 0$$

when $v' = v$. This tells us that in equilibrium $Q'(v)\,v = P'(v)$ for every v.

This is sort of helpful since the fundamental theorem of calculus tells us that

$$P(v) = \int_{\underline{v}}^{v} P'(v')\,dv' = \int_{\underline{v}}^{v} Q'(v')\,v'dv'. \tag{2.1}$$

We can use this information to simplify the equilibrium payoff function. If we integrate the right hand side of (2.1) by parts we get

$$P(v) = Q(v')\,v'|_{\underline{v}}^{v} - \int_{\underline{v}}^{v} Q(v')\,dv'.$$

If we assume that $\underline{v} = 0$ just to make things simple, then the equilibrium payoff to a buyer of type v is given by

$$Q(v)\,v - P(v) = \int_{\underline{v}}^{v} Q(v')\,dv'. \tag{2.2}$$

It isn't particularly intuitive that the equilibrium payoff should be equal to the integral of the trading probability, but as you will see, it is an analytically very useful result. It is very general in the sense that the same result will hold no matter how many buyers there are. We use this fact below. It is very special in the sense that it relies heavily on the assumption that buyer types are independent. To see this, simply observe that in (2.1), we treat Q' as if it were the marginal impact of a change in buyer type on trading probability. In fact, it is the marginal impact on trading probability when a buyer pretends to have a higher type, which is not the same. The reason is that a higher type buyer will have a different belief about the types of the other buyers when types are correlated, whereas a buyer pretending to have a higher type won't.

Before going over how to simplify this, we should explain the sellers' payoffs. Provided you recognize that the functions P and Q both depend on reserve prices the sellers set, seller's payoffs can be written in a straightforward way using the information given so far. There are two potential bidders, each of who makes expected payment $P(v)$ when their type is v. The distribution of types is given by $F(v)$, so the seller's payoff is just

$$2\int_{\underline{v}}^{\overline{v}} P(v)\,dF(v)$$

From (2.2) this is equal to

$$2\int_{\underline{v}}^{\overline{v}} \left\{ Q(v)\,v - \int_{\underline{v}}^{v} Q(v')\,dv' \right\} dF(v)\,. \tag{2.3}$$

Now we want to simplify the buyers' and sellers' payoffs as given by (2.2) and (2.3). We do this using the directed search logic explained above along with a very nice insight from (McAfee 1993). A buyer with valuation v will win the auction in two circumstances, the first is when the other bidder has a valuation below v, the second is when the other buyer chooses to participate in the other auction. This probability is very simple, and is given by

$$1-\int_{v}^{\overline{v}}\pi\left(v'\right)F'\left(v'\right)dv'.$$

[3]

Why might this simple expression be helpful? Well suppose we have a couple of auctions and the bidders choose their participation strategies. Now suppose that bidder v bids in both auctions with positive probability. Then as in the labor market example given above, the payoff that v gets in equilibrium from both auctions must be the same. This means that

$$\int_{\underline{v}}^{v}Q_1\left(v'\right)dv'=\int_{\underline{v}}^{v}Q_2\left(v'\right)dv'$$

where Q_1 and Q_2 are the equilibrium trading probabilities for the different buyer types at the two different auctions. The same equality should hold for all the higher types as well, so this expression is an identity. Then the derivatives of both sides with respect to v must also be equal, i.e.,

$$Q_1\left(v\right)=Q_2\left(v\right)$$

or, using McAfee's idea

$$\int_{v}^{\overline{v}}\pi_1\left(v'\right)F'\left(v'\right)dv'=\int_{v}^{\overline{v}}\pi_2\left(v'\right)F'\left(v'\right)dv'.$$

Now since the original expression is an identity, we can differentiate again to get

$$\pi_1\left(v\right)F'\left(v\right)=\pi_2\left(v\right)F'\left(v\right)$$

or $\pi_1\left(v\right)=\pi_2\left(v\right)=\frac{1}{2}$ for every buyer type who participates in both auctions.

This is quite different from from the labor market example above. What it implies is that when a seller adjusts his reserve price what he changes is not the participation probabilities, as in the labor market example, but the set of buyer valuations that apply. When a seller raises his reserve price, he chases away some of the lowest valuation buyers completely. This means that the high valuation buyers are less likely to face an opponent. It is this that keeps the higher valuation buyers indifferent.

[3]This is just 1 minus the probability that the other buyer both has a higher valuation and comes to the same auction.

The main lesson from this argument is that when sellers compete in auctions, they aren't competing directly for the high valuation buyers, it is only the low valuation buyers who change their behavior in response to changes in their mechanisms.

It is easy enough to find the rest of the equilibrium in the second stage where the buyer choose where to bid. If the reserve prices are $r_1 < r_2$, then it is pretty obvious that buyers whose valuations are below r_1 won't bother to bid at all. Buyers whose valuations are between r_1 and r_2 will bid with seller 1 for sure, since they can't afford (i.e don't want) to pay seller 2's reserve price. Even buyers whose valuation is slightly higher than r_2 are going to restrict their bidding to seller 1. Such buyer types aren't likely to win either auction. However, even if they are the only bidders at the auction, they only get a tiny surplus with seller 2. If they are the only bidder with seller 1, they get a much larger surplus because seller 1's reserve price is lower.

Suppose that $v^*$is the lowest bidder type who bids at seller 2's auction. Since buyers with lower valuations all go to seller 1 for sure, he will pay seller 2's reserve price if he wins, but he can only win if no other bidder participates. The probability that this happens, as described above is $1 - \frac{1}{2}\int_{v^*}^{\overline{v}} F'(v')\,dv' = 1 - \frac{1-F(v^*)}{2}$.

Putting all this together, makes it possible to determine the value for v^*. The payoff that the marginal buyer of type v^* receives when she bids with seller 2 is

$$(v^* - r_2)\left(1 - \frac{1-F(v^*)}{2}\right).$$

This should be just equal to the payoff she gets by bidding at seller 1 instead. As we explained above, this is the integral of the trading probability with seller 1 up to the value v^*. Buyers whose values are below r_1 never trade with seller 1. Buyers whose valuations are below v^* (but above r_1) trade as long as the other bidder either has a lower valuation, or chooses to bid with seller 2. This probability is given by

$$1 - (F(v^*) - F(v)) - \frac{1-F(v^*)}{2}.$$

So v^* is determined by the condition that

$$(v^* - r_2)\left(1 - \frac{1-F(v^*)}{2}\right) =$$

$$\int_{r_1}^{v}\left\{1 - (F(v^*) - F(v')) - \frac{1-F(v^*)}{2}\right\}. \tag{2.4}$$

All that is left is to write down the profit functions for the two sellers. For seller 2 who charges the high reserve price, it is most straightforward since the marginal buyer type is highest. From (2.3), the high reserve price seller's profits are

$$2\int_{\underline{v}}^{\overline{v}}\left\{Q(v)v-\int_{\underline{v}}^{v}Q(v')\,dv'\right\}dF(v)=$$

$$2\int_{v_2^*}^{\overline{v}}\left\{\left(1-\frac{1-F(v)}{2}\right)v-\int_{v_2^*}^{v}\left(1-\frac{1-F(v')}{2}\right)dv'\right\}dF(v).$$

Integrating the second term by parts gives

$$2\int_{v_2^*}^{\overline{v}}\left(1-\frac{1-F(v)}{2}\right)vdF(v)-$$

$$2\left\{\int_{v_2^*}^{v}\left(1-\frac{1-F(v')}{2}\right)dv'F(v)\Big|_{v_2^*}^{\overline{v}}-\int_{v_2^*}^{\overline{v}}F(v)\left(1-\frac{1-F(v)}{2}\right)dv\right\}=$$

$$2\left\{\int_{v_2^*}^{\overline{y}}\left(v-\frac{1-F(v)}{F'(v)}\right)\left(1-\frac{1-F(v)}{2}\right)F'(v)\,dv\right\}. \tag{2.5}$$

This is a complicated function of the high reserve price seller's reserve price r_2. Yet it is quite a simple function of the high reserve price seller's cutoff valuation v_2^*. Once the cutoff valuation is high enough such that $v^*>\frac{1-F(v^*)}{F'(v^*)}$, the seller will have no interest in raising his cutoff valuation by raising his reserve price. To understand equilibrium, it is then necessary to understand what happens to the seller who sets the low reserve price.

Following the same logic as above, the low reserve price seller has payoff

$$2\left\{\int_{r_1}^{y_2^*}\left(v-\frac{1-F(v)}{F'(v)}\right)F(y)\,F'(v)\,dv\right\}+$$

$$2\left\{\int_{v_2^*}^{\overline{y}}\left(v-\frac{1-F(v)}{F'(v)}\right)\left(1-\frac{1-F(v)}{2}\right)F'(v)\,dv\right\}. \tag{2.6}$$

Once again, this is a fairly simple function of the cutoff valuations of the two sellers. However, there is one significant complication. As the low reserve price seller cuts his reserve price r_1, he changes both the cutoff valuations v_2^* for the high reserve price seller. This is apparent from (2.4). In particular, when the low reserve price seller cuts his reserve price slightly, he raises the cutoff valuation at the high price seller. In particular, this causes some buyer types who are bidding with equal probability at both sellers, to decide to bid for sure at the low reserve price. So just as we wanted, cutting reserve price draws customers away from the high reserve price seller.

The monopoly reserve price in a single seller auction is the reserve price such that the buyer v^* whose type satisfies $v^* = \frac{1-F(v^*)}{F'(v^*)}$ is just indifferent to bidding. Notably, this 'optimal' reserve price is independent of the number of bidders. The argument above shows why it cannot be an equilibrium for both sellers to set this monopoly reserve price when there is competition. Each of the sellers has an incentive to cut reserve price slightly to steal some of the customers away from the other seller. Once again, the customers who are attracted to this are not the high value bidders. Instead, it is those bidders whose values are close to the reserve price anyway.

2.1. **Calculating Equilibrium Reserve Prices.** A Nash equilibrium for the game just described can be defined as a pair of reserve prices that are jointly best replies to one another. In the story above, a seller attracts bidders from the other seller by lowering reserve prices. The cost of this is that the seller is now selling to some buyer types for whom $v < \frac{1-F(v)}{F'(v)}$. From (2.6) it is apparent that the seller lowers his profits slightly by selling to these buyers. In a Nash equilibrium, the marginal cost of selling to these buyers must be just offset by the marginal gain of selling to the higher valuation buyers. (Burguet and Sakovics 1999) analyze this game and show that equilibrium is in mixed strategies. Reserve prices in this mixed equilibrium are strictly larger than the seller's cost, but strictly less than the monopoly reserve prices.

Some auction markets are much more competitive than this example suggests. For example, a search for digital camera auctions on eBay will turn up thousands of opportunities to bid. Even if the auctions involve very different kinds of camera, this still provides hundreds of opportunities to bid on a new version of any particular model. In such large markets, sellers have a very small chance of attracting any of the high value bidders. Instead of getting each high value bidder with probability $\frac{1}{2}$, as occurs in the example, the seller gets each of them with probability $\frac{1}{100}$. By itself this isn't a problem since there are typically a lot more bidders to compete over.

However, this makes a big difference when the seller cuts his reserve price relative to the other sellers. Then instead of raising the probability that higher valuation bidders will bid in his auction from $\frac{1}{2}$ to 1, he raise it from $\frac{1}{100}$ to 1, and this has a big and positive impact on his profits. The paper by (Peters and Severinov 1997) shows that as the number of bidders and sellers becomes very large, as it does in the eBay camera auction, it is profitable to cut reserve prices whenever they are positive. This result doesn't show that equilibrium reserve prices are zero when the number of buyers and sellers is large. When all sellers are identical and all reserve prices are zero, each seller has an incentive to raise his reserve price if there are enough buyers. A pair of papers establish the convergence of equilibrium in finite competing auction games. (Hernando-Veciana 2005) shows that when there are a finite number of reserve prices, then very generally there will be an equilibrium in a large finite competing auction game in which each seller sets his reserve price equal to his value. More recently (Virag 2008) shows that in finite competing auction games with a continuum of feasible reserve prices, if all sellers are identical, mixed strategy equilibrium exist among sellers. As the number of buyers and sellers become large, all reserve prices converge in distribution to sellers' value.

These are quite a significant results. Recall from the discussion above, that the *monopoly* reserve price ensures that a buyer whose type satisfies $v^* = \frac{1-F(v^*)}{F'(v^*)}$ is just indifferent about whether or not to bid. The odd thing about this result is that this 'optimal' reserve price is very sensitive to what the seller thinks the distribution of bids is. In this sense the optimal auction is like the Cremer McLean mechanism - to find the optimal reserve price requires a careful calculation involving information that is hard to get. The competitive results in (Peters and Severinov 1997) and the convergence results in (Hernando-Veciana 2005) and (Virag 2008) explain how competition among sellers eliminates this counterintuitive result. In a large enough market all the seller needs to know in order to set his reserve price is his own selling cost, more or less exactly what we assume he would do in a simple competitive market.

The competing auction game is complex because a seller has to explicitly calculate how a change in his own mechanism will affect the payoff that buyers get by going to some other mechanism - an effect that disappears as markets become large. It is possible to get around some of the complexity associated with the competing auction game by assuming that sellers behave 'competitively' in relatively small markets. Usually competitive sellers are price takers. Obviously, they can't literally be price takers, since price setting is an integral part of the mechanism that they offer. Instead, we might try to capture the competitive flavour of the limit results described above by assuming that sellers are 'payoff takers'. In other words, they believe that there is a market payoff that they have to provide buyers in order to attract them to their mechanisms.

The thing that makes this assumption nice is that, like price takers, they also believe that provided they offer buyers this market payoff, they can have any distribution of buyer types that they want. The trade-off that sellers have to work out when they design their mechanism is that the more buyers they plan to attract and the higher their types, the more competition buyers will face when they come. So sellers have to pick the types and participation probabilities that they want in such a way that all they types they expect to attract earn their market payoff. We turn now to this formulation in order to analyze.

3. Competitive Equilibrium in Mechanisms

In this section we return to the more general problem of equilibrium mechanisms. In particular, we want to add back correlation in valuations to create an environment like the one in Cremer McLean in which a monopoly seller can extract all buyer surplus. We want to show that there is a unique symmetric 'equilibrium' in which all sellers offer to run second price auctions with reserve price equal to their cost. The surprising thing about this result is that competition will prevent sellers from using Cremer-McLean type entry fees to extract surplus. Thus we have an argument that shows why competition keeps mechanisms simple.

There are a couple of remarks that need to be made before we proceed. First, we are going to restrict sellers to mechanisms that are 'direct' in the naive sense that allocations depend on buyers. payoff types. Buyers' types are complex objects in competing mechanism games. These types include market information that a seller would want to know. For example, sellers might want to ask bidders to tell them whether some other seller has deviated from some convention that is usually used in the industry.[4]

Second, we use a competitive market payoff taking assumption. Sellers will choose the distribution of types that they want conditional on participants all receiving their market payoff when this distribution is realized. In equilibrium, the distributions that sellers choose will coincide with the true distribution of types that they face. However, as in all competitive models, the same will not be true outside equilibrium. If a seller deviates, he will typically anticipate a distribution

[4] An appropriate formulation of bidder types is given in (Epstein and Peters 1999).

that does not coincide with the distribution associated with the continuation equilibrium among buyers that follows his deviation. This is analogous to the idea that in a Walrasian equilibrium a buyer can deviate from his equilibrium demand and imagine the payoff he would get from buying more of some good, even though he wouldn't be able to find more of this good at prevailing prices.

This description of a large sub market on eBay seems plausible. Camera sellers, for example, aren't likely to know much about all the different alternatives buyers consider before choosing to participate in their auctions. Certainly, other active sellers on eBay are observable. Yet buyers also purchase from standard retail outlets. Since sellers don't know where buyers live, they can't know much about these alternatives. On the other hand, many of the camera sellers have lots of experience selling on eBay. They are likely to know approximately what payoffs they need to offer buyers to keep them bidding. Generally, the eBay camera market is embedded in a much larger market that sellers may not fully understand, so that competitive assumptions about how this market works seem reasonable.

On the other hand, eBay itself is a significant player in this market. To capture this we are simply making the extreme assumption that the bidders do understand the market.

It should be mentioned at this point that a fully game theoretic treatment of the competing auction market runs up against a problem. Absent equilibrium refinements and restrictions feasible mechanisms, competing mechanisms can be used to support a large variety of equilibrium allocations.[5] So some stand has to be taken on how to restrict player in order to have any predictive theory at all.

In the market there are s sellers and n bidders. Each seller has a single unit of output to sell. He has no cost of offering this output for sale. Each bidder wants to acquire exactly one unit. Bidders have valuations x_i. A bidder who buys a unit of output at price p earns surplus $x_i - p$. The seller in this transaction earns p. Sellers offer direct mechanisms, bidder choose to participate in one and only one of the mechanisms.

A direct mechanism is a pair of functions $q : [0,1]^n \rightarrow [0,1]^n$ and $p : [0,1]^n \rightarrow \mathbb{R}^n$. The array $q(x)$ is a vector of probabilities with which objects are awarded to each of the different players depending on their types. The sum of these probabilities should be less than or equal to one. The function $p(x)$ specifies a vector of payments to or from each

[5]The most general folk theorem in this regard is (Peters and Troncoso-Valverde 2009), however, the basic idea is due to (Yamashita 2007).

bidder. These functions should specify probabilities and payments only for participating bidders. To compensate we will treat non-participants as if they had value $x_i = 0$. Feasible mechanisms are then required to assign $q_i(x_i, x_{-i}) = p_i(x_i, x_{-i})$ for any bidder i who has valuation 0.

The market is subject to an external shock y which is distributed $G(y)$ on some compact interval. Sellers face a joint distribution of types $Z(x_1, \ldots, x_n|y)$ which depends on the external shock. However, they also believe this distribution is related to the direct mechanism that they offer. Specifically, they believe the market provides a bidder of type x_i a payoff $\beta(x_i|y)$. Sellers don't think they have any impact at all on this market payoff. They also believe they can support any distribution of buyer types that they like, provided the mechanism they offer provides each buyer type with at least her expected payoff when buyers have the same belief about this distribution.

The surplus for a seller who offers mechanism (q, p) and faces distribution $Z(x|y)$ is given by

$$\int\int p(x)\, dZ(x|y)\, dG(y). \tag{3.1}$$

A bidder who participates in seller j's mechanism and shares the seller's beliefs earns surplus

$$\int\int [x_i q_i(x_i, x_{-i}) - p(x_i, x_{-i})]\, dZ(x_{-i}|x_i, y)\, dG(y).$$

Given the market payoff function β, the seller chooses his mechanism (q, p) to maximize (3.1) subject to the constraint that

(3.2)

$$\int\int [x_i q_i(x_i, x_{-i}) - p(x_i, x_{-i})]\, dZ(x_{-i}|x_i, y)\, dG(y) \geq \int \beta(x_i|y)\, dG(y)$$

for each x_i in the support of (the marginal distribution associated with) Z.

Bidders are more sophisticated. They share the belief that valuations are *conditionally independent* with distribution $F(x_i|y)$ on the interval $[0, 1]$. Implicit in this is the assumption that valuations lie between 0 and 1. The joint distribution of valuations that buyers face is then given by

$$\int \prod_{i=1}^{n} F(x_i|y)\, dG(y).$$

Bidders adopt a symmetric participation strategy $\pi_j(x_i)$ which gives the probability with which they will participate in the mechanism offered by seller j (they can only participate in one mechanism). Given a participation strategy π_j for seller j, the true distribution of types faced by seller j is given by

$$z_j(x_1,\ldots,x_n) = \int \left[\prod_i 1 - \int_x^1 \pi_j(x')\,dF(x'|y)\right] dG(y).$$

A participation strategy $\{\pi_1(\cdot),\ldots,\pi_s(\cdot)\}$ is a *continuation equilibrium* if the sum of these participation strategies across sellers is less than or equal to one and

$$\int \left[x_i q_i^j(x_i,x_{-i}) - p_i^j(x_i,x_{-i})\right] d\int \left[\prod_{i'\neq i} 1 - \int_{x_{i'}}^1 \pi_j(x')\,dF(x'|y)\right] dG(y|x_i) \geq$$

$$\int \left[x_i q_i^{j'}(x_i,x_{-i}) - p_i^{j'}(x_i,x_{-i})\right] d\int \left[\prod_{i'\neq i} 1 - \int_{x_{i'}}^1 \pi_{j'}(x')\,dF(x'|y)\right] dG(y|x_i)$$

for each j for which $\pi_j(x_i) > 0$.

A *symmetric equilibrium in mechanisms* is a common mechanism (q,p) to be used by each firm, a common conditional distribution function $Z(x|y)$, and a market payoff function β having the property that (q,p) and Z jointly maximize (3.1) subject to (3.2), and such that

$$\int \left[x_i q_i(x_i,x_{-i}) - p_i(x_i,x_{-i})\right] d\int \left[\prod_{i'\neq i} 1 - \int_{x_{i'}}^1 \pi_j(x')\,dF(x'|y)\right] dG(y|x_i) =$$

(3.3)

$$\int\int \left[x_i q_i(x_i,x_{-i}) - p(x_i,x_{-i})\right] dZ(x_{-i}|x_i,y)\,dG(y) = \int \beta(x_i|y)\,dG(y)$$

for each x_i in the support of Z, where $\pi_1 = \pi_2 = \cdots = \pi_s = \pi$ is a continuation equilibrium.

When all sellers offer an equilibrium mechanism, the distribution of types they expect to face is equal to the distribution of types associated with a continuation equilibrium. What makes this non-standard is what happens when there is a deviation. If a seller unilaterally chooses some other mechanism, he will expect a new distribution which provides each bidder the expected payoff they had before the deviation. In a subgame perfect equilibrium, the payoff associated with the continuation equilibrium following a deviation would be different from what it was before the deviation. So generally sellers expectations are incorrect out of equilibrium (as in true in every competitive model).

3.1. **Efficient Mechanisms.** We now want to use the Cremer-McLean idea to show that there is a unique symmetric equilibrium. The main part of this argument shows that equilibrium mechanisms have to be efficient.

The approach that Cremer McLean used was to imagine that some mechanism, say a second price auction, is being used by a seller, and that this mechanism generates a payoff $\beta(x)$ to participants. They suggested the seller augment the auction with a menu of fees, $\{\omega_\theta(x_2, \dots, x_n)\}$. The variables x_2 through x_n are intended to represent the 'bids' of the other participants. The variable θ simply indexes the fee schedule. In their story, there are a finite number of possible values for θ. In the new mechanism, participants would submit their bid as before, and, in addition, select one of the fee schedules. They provide a condition on the joint distribution of valuations such that for any continuous function $\beta : [0,1] \to [0,1]$, a menu of fees could be created such that

$$\beta(x) = \min_\theta \int \omega_\theta(x_2, \dots, x_n) \prod_{j=2}^{n} dF(x_j|y)\, dG(y|x) \tag{3.4}$$

for each x in the finite support of the distribution F. Each of a finite set of buyer types would choose one fee schedule (the one that minimized her expected fee conditional on her interim belief). This formulation has the property that the expected fee chosen by each bidder type is exactly the surplus they expect from the auction. Provided bidders bid their true values in the action, requiring them to select one of these fees ensures that their expected surplus from participation in the auction is zero. The second price auction allocates the good to the bidder with the highest value, so that the expected surplus the seller earns is the same as his surplus under complete information.

The exact formulation is in the original article by (Cremer and McLean 1988). The extension to the continuous case is difficult and is discussed in (McAfee, McMillan, and Reny 1989). They provide conditions under which (3.4) will hold approximately when the number of fees in the schedule is finite. As they assume that the distribution of type is absolutely continuous, their assumptions don't work in the competitive case since there is a strictly positive probability that buyers won't participate at all. (Peters 2001) shows that their assumptions also ensure that fees can be designed to satisfy (3.4) in a competitive market assuming that fees are based on bids of participating bidders.

All we are interested in here is how to use the Cremer McLean argument to understand the competitive case. So rather than dealing with these issues here, we will simply assume that the joint distribution of types has enough correlation to support these fees when bidders choose among mechanism with equal probability. In particular

Definition 1. The joint distribution of types has the Cremer-McLean property if for any continuous function β, there exists a family of fees $\{\omega_\theta\}$ mapping $[0,1]^{n-1} \to \mathbb{R}$ such that

$$\beta(x) = \inf_\theta \int \omega_\theta(x_2, \ldots, x_n) \prod_{j=2}^{n} d\left[1 - \int_{x_j}^{1} \frac{1}{s} dF(x'|y)\right] dG(y|x).$$

Our argument will then involve three parts. First, we are going to use the Cremer McLean idea to show why competitive mechanisms must always allocate the object to the bidder with the highest value. The argument is to suppose that this isn't true and that sellers are using mechanisms that inefficiently allocate. Sellers have the option of replacing their mechanism with one that efficiently allocates. The complication of doing this is that such a mechanism may not provide the buyers the seller wants to attract with their market payoff. Here we will apply the Cremer McLean idea and augment existing fees with a new set that will just compensate all buyer types for their lost surplus. and extract any extra surplus that the new mechanism might create. This allows the seller to extract all the surplus gains from switching to an efficient mechanism.

This much establishes that every equilibrium involves mechanisms that allocate the good to the participant who has the highest valuation. It is then immediate that if all other sellers are offering second price auctions with zero reserve price, then no seller can improve his expected surplus by doing otherwise.

Finally, we show that the payoff function associated with second price auctions is the only one that can satisfy our equilibrium conditions.

Theorem 2. *If the joint distribution of types has the Cremer-McLean property, then every competitive equilibrium in mechanisms has sellers using efficient mechanisms that award the good to the bidder with the highest value.*

Proof. Suppose $(q(\cdot), p(\cdot))$ is an equilibrium mechanism, that it is not efficient in the sense that there is an event E having strictly positive probability for which $t_i > t_j$ for all j and $q(t_i, t_{-i}) < 1$. Observe that since the bidders' payoff is always equal to the market payoff $\int \beta(x|y)\, dG(y)$, the seller's profit can be written as the total surplus, less what the seller expects to give to each participating bidder, i.e.,

(3.5)

$$\int\left\{\int\cdots\int\sum_{i=1}^{n}q\left(x_i,x_{-i}\right)x_i dZ\left(x_1,\ldots,x_n|y\right)-n\int\beta\left(x|y\right)dZ\left(x_1,\ldots,x_n|y\right)\right\}dG\left(y\right).$$

In a symmetric competitive equilibrium, sellers' expectations about the distribution of types they face must be correct, so this must be equal to

$$\int\left\{\int\cdots\int\sum_{i=1}^{n}q'\left(x_i,x_{-i}\right)x_i d\left[1-\int_{x_j}^{1}\frac{1}{s}dF\left(x'|y\right)\right]-n\int\beta\left(x|y\right)d\left[1-\int_{x_j}^{1}\frac{1}{s}dF\left(x'|y\right)\right]\right\}dG\left(y\right)$$

Now replace the mechanism (q,p) with a simple second price auction (q',p') (which, in particular, is an efficient and incentive compatible mechanism). Observe that

$$\int\left\{\int\cdots\int\sum_{i=1}^{n}q'\left(x_i,x_{-i}\right)x_i d\left[1-\int_{x_j}^{1}\frac{1}{s}dF\left(x'|y\right)\right]-n\int\beta\left(x|y\right)d\left[1-\int_{x_j}^{1}\frac{1}{s}dF\left(x'|y\right)\right]\right\}dG\left(y\right)$$

strictly exceeds (3.5). This comparison will be irrelevant if replacing the original mechanism with a second price auction leaves some buyer types with a payoff that is less than their market payoff. To ensure this doesn't happen, we can augment the second price auction with a fee. Let

$$\alpha\left(x\right)=$$

$$\int\left\{\beta\left(x|y\right)-\int\int\cdots\int\sum_{i=1}^{n}\left\{q'\left(x_i,x_{-i}\right)x_i-p'\left(x_i,x_{-i}\right)\right\}dZ\left(x_{-i}|x_i,y\right)\right\}dG\left(y\right).$$

By the Cremer-McLean property, there is a menu of fee schedules ω_θ such that

$$\alpha\left(x\right)=\min_{\theta}\int\int\cdots\int\omega_\theta\left(x_{-i}\right)d\left[1-\int_{x_j}^{1}\frac{1}{s}dF\left(x'|y\right)\right]^{n-1}dG\left(y\right).$$

Now if we augment the second price auction by requiring each player to choose any of the fee schedules that he likes, a bidder of type x will choose the fee indexed $\theta\left(x\right)$. If he does so, then he is participating in a mechanism $\left(q',p'\left(x_i,x_{-i}\right)+\omega_{\theta(x_i)}\left(x_{-i}\right)\right)$. By construction this scheme provides each participating player exactly his market payoff. No player has an incentive to misrepresent his type under this new mechanism since the fees don't depend on his type report and nothing else in the mechanism depends on which fee the bidder chooses. However, the mechanism does give the seller strictly higher profits. This contradiction shows that equilibrium mechanisms must be second price auctions augmented by Cremer-McLean like fees. □

This makes the equilibrium slightly simpler. However, it still admits the possibility that sellers might extract full surplus in equilibrium. To rule this out, what we need is a restriction that allows sellers to use the fees outside of equilibrium. (McAfee, McMillan, and Reny 1989) provide a condition under which the Cremer-McLean property holds approximately when joint distributions are absolutely continuous with respect to Lebesque measure. Their theorem doesn't apply to the competing mechanism case since the distribution associated with the symmetric equilibrium is not absolutely continuous. (Peters 2001) provides the extension of their theorem to allow for atoms in the joint distribution. We do not want to get into these mathematical issues here, so we provide a stronger restrictions

Definition 3. The joint distribution of types satisfies the extended Cremer-McLean property if there is some $\epsilon > 0$ such that for every family of conditional distribution functions $Z(x|y)$ satisfying

$$\int_B \left[Z(x|y) - \left[1 - 1 - \int_{x_j}^1 \frac{1}{s} dF(x'|y) \right] \right] \le \epsilon$$

on each measurable subset B of $[0,1]$, and every continuous function β there is a family of fees $\{\omega_\theta^Z\}$such that

$$\beta(x) = \inf_\theta \int \omega_\theta^Z (x_2, \ldots, x_n) \prod_{j=2}^n d[Z(x|y)]\, dG(y|x).$$

The extended Cremer-McLean property extends the surplus extraction property from the true distribution of types to a weakly open set of distributions around the true distribution. The implication of this property is that, at least for small changes in the distribution of types, the seller can always adjust fees so that bidders who participate earn their market payoff.

We can now finish the theorem

Theorem 4. *If the distribution of types given by the family of conditional distributions $F(x|y)$ satisfies the extended Cremer McLean property, then there is a unique competitive equilibrium in mechanisms in which all sellers offer second price auctions with zero reserve price.*

Proof. In equilibrium, sellers choose a distribution of types that maximizes their expected payoff conditional on buyers receiving their market payoff when this distribution of types is realized. By the extended Cremer McLean property, there is a weakly open neighborhood of the symmetric distribution in which sellers can design fees to compensate bidders for changes in their mechanisms. From the argument in Theorem 2, if the seller wants one of these distributions, he might as well use a second price auction augmented by fees to support it. So his payoff when he chooses one of these alternative distributions will be

$$\int \left\{ \int n x_i Z\left(x_i|y\right)^{n-1} dZ\left(x_i|y\right) - n \int \beta\left(x_i|y\right) dZ\left(x_i|y\right) \right\} dG\left(y\right).$$

The second term integrates by parts to

$$\int \left\{ \beta\left(1|y\right) - \int Z\left(x|y\right) \beta'\left(x|y\right) dx \right\} dG\left(y\right).$$

Similarly, the first term can be integrated by parts to

$$\int \left\{ 1 - \int Z\left(x|y\right)^{n} dx \right\} dG\left(y\right).$$

Putting them together gives the expression

(3.6)

$$\int \left\{ \left(1 - \beta\left(1|y\right)\right) - \int \left\{ Z\left(x|y\right)^{n} - nZ\left(x|y\right) \beta'\left(x|y\right) \right\} dx \right\} dG\left(y\right).$$

Since the sellers' payoff should be maximal in equilibrium, the conditional distribution $Z\left(y|y\right)$ should satisfy the necessary condition for point-wise maximization

$$Z\left(x|y\right)^{n-1} = \beta'\left(x|y\right).$$

If Z coincides with the distribution supported by the symmetric continuation equilibrium, this resolves to

$$\left[1 - \frac{1 - F\left(x|y\right)}{n}\right]^{n-1} = \beta'\left(x|y\right). \tag{3.7}$$

As we showed in (2.2) above, this means that apart from a constant, the conditional payoff function $\beta\left(x|y\right)$ must coincide with the payoff function associated with a second price auction in which each bidder participates in each auction with the same probability.

If $\int \beta\left(x|y\right) dG\left(x\right) = 0$ on some non-degenerate interval, then $\int \beta\left(x|y\right) dG\left(x\right) = 0$, which implies that $Z\left(x|y\right)^{n-1} = \beta'\left(x|y\right) = 0$ on this interval. Then $Z\left(x|y\right) = \left[1 - \frac{1-F(x|y)}{n}\right] = 0$, which can't be satisfied any by distribution $F\left(x|y\right)$. This ensures that $\int \beta\left(x|y\right) dG\left(y\right) > 0$ for all x.

Finally, $\int \beta\left(0|y\right) dG\left(y\right) = 0$, otherwise by continuity, sellers would not want to attract buyers whose types are close enough to 0. □

The argument that the second price auction without reserve price is an equilibrium is straightforward. The seller will want every buyer whose type is at least his market payoff to participate. If all other sellers are offering second price auctions, this will be so for all types.

4. Conclusion

We have reviewed the basic theory of competing mechanisms. A partial equilibrium analysis suggests that competition will ensure efficiency. In models in which types are assumed to be independent and sellers are restricted to auctions, it has been shown that the equilibrium in the partial equilibrium model approximates the equilibrium in large finite games. Hopefully the same property is true in the more general environment considered here, but a proof of this has not yet been established.

References

Burguet, R., and J. Sakovics (1999): "Imperfect Competition in Auction Designs," *International Economic Review*, 40(1), 231–47.

Cremer, J., and R. McLean (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Economtrica*, 56(6), 1247–1259.

Epstein, L., and M. Peters (1999): "A Revelation Principle for Competing Mechanisms," *Journal of Economic Theory*, 88(1), 119–161.

Hernando-Veciana, A. (2005): "Competition Among Auctioneers in a Large Market," *Journal of Economic Theory*, 121(1), 107–127.

McAfee, P. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281–1312.

McAfee, R. P., J. McMillan, and P. J. Reny (1989): "Extracting the Surplus in a Common Value Auction," *Econometrica*, 57(6), 1451–1459.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 55–73.

Peters, M. (2001): "Surplus Extraction and Competition," *Review of Economic Studies*, 68(3), 613–633, University of Toronto, manuscript.

Peters, M., and S. Severinov (1997): "Competition Among Sellers who offer Auctions Instead of Prices," *Journal of Economic Theory*, 75(1), 141–179.

Peters, M., and C. Troncoso-Valverde (2009): "A Folk Theorem for Competing Mechanisms," manuscript, University of British Columbia.

Riley, J., and W. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 71, 381–392.

Virag, G. (2008): "Competing AUctions:Finite Markets and Convergence," Working paper, University of Rochester, to appear in Theoretical Economics.

Yamashita, T. (2007): "A Revelation Principle and A Folk Theorem without Repitition in Games with Multiple Principles and Agents," manuscript, Stanford University.