CAN ECONJOBMARKET HELP CANADIAN UNIVERSITIES?

KIM NGUYEN, M. PETERS, AND AND M. POITEVIN

ABSTRACT. We analyze the academic matching market by considering a simple model in which applicants who face an application cost strategically choose portfolios of applications. Universities then play a decentralized offer game in which unaccepted offers result in failure to trade on both sides of the market. We characterize a basic equilibrium to illustrate the sorting role that application costs play. In a numeric example, we illustrate how reduced application costs can result in increased matching frictions.

1. INTRODUCTION

In 2016, computer operations for Econjobmarket.org, the official job market site of the Econometric Society, were moved onto Canadian servers managed jointly by the Vancouver School of Economics and the Economics Department at the University of Toronto. Since Econjobmarket.org began collecting applications in 2007, 105 different Canadian organizations have used its services. Econjobmarket.org is part of a recent trend toward digitizing applications. This digitization has obvious administrative benefits. One of the most basic questions one might like to ask about this trend is whether it is also improving matching outcomes by lowering the cost of applications.[1]

To get some idea of the size of the academic job market, there were 1453 ads on Econjobmarket and JOE during the calendar year 2016. During the same time period, 4365 applicants registered on Econjobmarket.org for the peak recruiting season from October through December. Crudely extrapolating from applications submitted to Econjobmarket, somewhere between 300 thousand and 400 thousand applications were submitted and processed by institutions throughout

University of British Columbia.
University of British Columbia.
CIREQ and Département de sciences économiques, Université de Montréal.

[1]Not everyone agrees that it even reduces administrative costs – for example remembering a password on your local site, or setting up accounts and remembering the passwords for the various sites where your students apply.

the recruiting season. If each of these applications has 3 letters of reference, between 900 thousand and 1.2 million reference letters were read during the year. This process involves a lot of work for both applicants and recruiters.

The output of all this work is a set of offers. Since this information gathering process is very imperfect and subject to idiosyncratic interpretation, many offers are made to individuals who don't accept them. Usually this is because the applicant receives a better offer. While the applicant holds the offer, many of the departments other alternatives also accept offers. All this happens because universities have little information about what other universities are doing at the time they make the offer. Offers that aren't accepted mean that other applicants who the department might have considered don't get offers (or end up with inferior offers). This is an example of the frictions that are the 'stuff' of a large literature on search.

No direct evidence is available on how successful departments are at filling their vacancies, or how many applicants either can't find jobs, or find jobs that are outside the fields for which they were trained. Yet hearsay evidence suggests that this is an issue. It might seem that by making applications cheap, digitization should help with market discovery, therefore lead to better outcomes. One argument against this is that it is costly for departments to process more applications.[2]

However, there are also strategic effects. Application costs can serve as a kind of self-selection device inducing applicants to target applications to departments where they would make a reasonable match. Closely related to this is the fact that once applications have been processed, departments engage in a very decentralized competition in offers. An offer to a candidate who gets an offer from a 'better' institution can be very costly since there is typically a significant delay between the time when an offer is made, and the time when the applicant rejects such an offer.

Application processing costs are straightforward, however the additional strategic costs associated with applications are very hard to measure. One way to try to measure these costs is to design a structural model of the interaction among departments, then use whatever outcome data is available to get a handle on these costs. We

[2]As many of us know from sending recommendation letters for grad schools, digitization by itself doesn't make all parts of the process less costly. Creating numerous computer accounts, remembering passwords you only use every two years, then filling out the same form over and over again in the same computer system for different schools is an extremely frustrating process.

provide an initial attempt to provide such a model here. Our model is just an example to illustrate some of the basic tradeoffs. However, we hope that it will provide some basis of discussion about this issue.

We assume that applicants face an application cost. A very basic sorting process occurs in which a Canadian university competes with an American university for applicants of different quality. In our usual Canadian self-effacing way, we'll assume that applicants always want to go to the American university. Idiosyncratic shocks or private signals mean that no one other than the American university knows, based on applicants observable characteristics, exactly which of two applicants the American university prefers. Furthermore, we'll assume, similar to [7], that applicants don't know the characteristics of other applicants at the time they make their application decisions.

Our equilibrium has all the properties that most of us are familiar with. Offers are strategic and departments won't always make an offer to their most preferred applicant. Applicants will divide themselves, through their application strategies into segments which target different parts of the market. Our main theorem provides a way for us to measure the frictions associated with equilibrium. At least conceptually, it provides a way to think about how application costs and frictions can be measured by looking at market outcomes.

Our basic model does not provide analytic solutions to some of the most interesting problems. So we do some numerical calculations to get some basic comparative statics. We provide some examples to show that reducing application costs will actually be bad for Canadian departments. Since reduced application costs result in more applicants applying to both universities, Canadian universities in particular find themselves more often making strategic offers which are less likely to be successful. In our numeric calculates a 1% reduction in application costs will actually increase the probability with which the Canadian university fails to hire by the same amount, 1%.

Ultimately, our goal is to use basic matching theory to contribute to the empirical literature on matching markets. The majority of this empirical literature has focused on a centralized mechanism, e.g. matches between resident doctors and hospitals and assignments of students to public schools (see, for instance, [4] or [8]). Empirical works that compare the efficiency of decentralized matching to that of centralized matching yield varying results across different markets. While [6] show that a medical student is more likely to find appropriate residency with no prior affiliation under centralized matching, [3] find that the decentralized marriage market shares comparable sorting pattern with the one generated by the Gale-Shapley algorithm. To our best knowledge, there is no empirical research that investigates the decentralized search and matching process of the economics job market where intermediaries such as EJM serve as a device to reduce application costs. Whether or not such reduction enhances efficiency in terms of matching outcomes is still an open question, both theoretically and empirically.

The closest in spirit to our model is by [2], which provides the optimal algorithm for solving the portfolio choice problem with exogenous payoff and acceptance chance. Applying the framework to the context of college admission problem, [5] develop equilibrium models of directed college choice where applicants can simultaneously apply to many colleges and focus on whether there is assortative matching in the presence of incomplete information. On the other hand, [1] formalize the problem for two ranked colleges with fixed capacities in order to study the effects of information frictions and application costs in equilibrium. In their model, colleges compete as if they were Bertrand duopolists.

2. THE MODEL

Our point here is not to give a useful structural model – we are far away from the point where we could do that. Instead we wish only to suggest the various tradeoffs in the hope of stimulating discussion. To do this, we'll focus on an example of an academic job market that consists of only two universities, an American one and a Canadian one, each having one position to fill. We'll imagine that there are two candidates who both prefer the American university, but are also happy to match with the Canadian university. Applicants know their own type, but not the type of the other applicant.

We'll assume it costs an applicant $c > 0$ to apply to a university. Once an applicant applies to a university, the university receives a noisy and private signal of the applicant's worthiness, then decides which applicant (if there are any) to make an offer to. We'll assume wages are fixed here and embedded in the value of the university. Each university is allowed to make one and only one offer. If an applicant receives an offer from the American university, they accept it. Otherwise, they accept an offer from the Canadian university, or go unmatched if they have no offers.

As for frictions, at the point where universities make offers, neither university knows the number of applicants at the other university, or how the other university ranks those applicants. Applicants types are independently drawn from some bounded interval according to some distribution F that has a strictly positive density everywhere. Let $u_A(t) > 0$ be the surplus an applicant of type t earns at the American university and $u_C(t)$ the surplus the applicant earns from the Canadian university. We assume $u_A(0) = u_c(0)$, and that $u_A(t) - u_C(t)$ is a strictly increasing function.

In our model of universities, we want to capture two things. First, universities typically focus on hiring the *best* applicant they can. The identity of the 'best' applicant is often easy enough to discover from information about the applicants type – her cv, reference letters etc, and from private signals that the universities often have about the different applicants. On the other hand, the exact value of an applicant of a particular type is often hard to determine with precision early in an applicant's career. So we'll simply assume that the applicant that the university feels is *best* yields a payoff to the university of v_b, while the worst applicant if hired yields a payoff v_w. The difference between v_b and v_w plays a central role in the theorem that follows. Define the *gap parameter*

$$\underline{g} = \frac{v_b - v_w}{v_b + v_w} \tag{2.1}$$

as a parametric measure of the value to universities of hiring the best applicant.

To capture the idea that universities care directly about the types of the applicants, we'll use two devices. First we'll assume that the best applicant for the university is idiosyncratically determined by a private signal or shock. After seeing the types of the applicants, the universities might disagree about which applicant is best for them. Secondly, we'll assume that universities have some endogenously determined reference type r having the property that if the types of applicants the university receives are below this reference value, then the university might decide that it is better not to hire any of them.

We then use the following rule to determine university payoffs. We'll assume a university that gets no applicants, or fails to hire because its offer is rejected receives payoff 0. A university who receives only a single applicant receives payoff v_b from hiring this applicant if the applicant's type is at least equal to its reference value. If the applicant's type is below this reference level, then the university gets payoff v_b with probability $1 - \frac{(r_i - t)}{2}$ and payoff $-v_\theta < 0$ with probability $\frac{r_i - t}{2}$ from hiring the applicant. If the latter event occurs, the university will feel it is better off not hiring. We'll refer to applicants whose type exceeds the reference type of a university as *acceptable* applicants for that university.

If the university has a pair of applicants with types t_1 and t_2, then the probability with which t_1 yields payoff v_b if hired is $g_i\left(t_1, t_2\right)$ where g_i is given by

$$g_i^b\left(t_1, t_2\right) = \frac{1 + t_1 - t_2 - \max\left[r_i - t_1, 0\right]}{2}.$$

Conversely, the applicant with type t_1 yields payoff v_w if hired with probability

$$g_i^w\left(t_1, t_2\right) = \frac{1 + t_2 - t_1 - \max\left[r_i - t_1, 0\right]}{2},$$

and payoff $-v_\theta$ with the complementary probability.

At this point, it should be apparent that there is no loss in assuming that F is uniform and embedding all our assumptions in the functions $u_A\left(t\right)$ and $u_C\left(t\right)$. This comes from a well known argument. If the applicant's type t is a random variable with distribution F, then the applicant's tile in this distribution, $F\left(t\right)$ is also a random variable with a uniform distribution on $\left[0, 1\right]$. If $u\left(t\right)$ is the payoff function payoff function for a worker of type t, then the payoff for a worker whose quantile is $F\left(t\right)$ is

$$u\left(F^{-1} \cdot F\left(t\right)\right) \equiv v\left(F\left(t\right)\right).$$

Here F^{-1} is well defined by our assumption that F has a strictly positive density. So we can work directly with quantiles by replacing the payoff function u with the payoff function $v = u \cdot F^{-1}$. We'll adopt the uniform distribution assumption in what follows, commenting only that restriction on u_A and u_c are joint restrictions on payoffs and distributions.

The following game is played.

(1) Candidates privately learn their type then choose where to apply.
(2) Universities observe the types of their applicant(s) and then make an offer to at most one candidate. A university does not directly observe whether or not an applicant has applied to another university or whether or not that university has made them an offer.

(3) Candidates accept an offer from the American university if they get one, otherwise they accept an offer from the Canadian university.

This model builds in frictions. If both departments make an offer to the same applicant, then one of the departments will be unable to hire because of the assumption that the university makes only a single offer. Furthermore, the applicant who doesn't receive the offer won't be hired. Again, this extreme assumption simplifies the complex algebra below without being too unrealistic. One characteristic of offers in the academic market is that they aren't accepted immediately. While a department is waiting for an applicant to reject, applicants who might be second in line will often accept other jobs. In this sense, the second offer a department makes will often be less valuable than the first. We take this to the extreme here by assuming the department can only make one offer.

Applicants enjoy positive surplus at both universities, so at stage 1, the candidates have three exogenously given pure strategies: they can apply only to A, or only to C, or they can apply to both A and C. At this point, we'll assume the application cost is low enough so that all applicants find it worthwhile to apply somewhere.

Instead of writing a full formal description of equilibrium, we can write down a few of the properties of equilibrium that are obvious, then restrict a more formal description to information sets where there is an actual decision to be made. For the American university, an offer will always be accepted. So:

- If it receives only one application which it ex post finds to be acceptable, it makes an offer to that applicant and it is accepted. The university earns v_b and the applicant earns $u_A(t)$.
- If it receives two applications, it just makes an offer to its favorite applicant if it has one. This offer is always accepted since $u_A(t) > u_C(t)$. So the university again earns v_b if it makes an offer.

Our formulation is intended to capture the idea that the American university might decide that the applicant with the lower type is actually the best applicant. The chances of this happening might be small if the difference between the applicants' types are large. From the Canadian university's perspective it is never sure whether the American university will hire the applicant with the higher type.

The problem for the Canadian university is the same as for the American university if it receives only one application – it makes an offer to that applicant if it finds the applicant acceptable. The offer will be accepted if the applicant doesn't have an offer from the American university.

The only real decision problem in this model is what happens when the Canadian department receives two acceptable applications. Of these it will have one that it strictly prefers. It will have to decide whether to make an offer to its preferred applicant (which might not be accepted), or make an offer to its less preferred applicant because it is more likely to be accepted.

An equilibrium now consists of an application rule for applicants (where to apply depending on their type), an offer rule for the Canadian university when it has two acceptable applicants, and a pair of reference types r_C and r_A for the two universities. These rules have the usual properties that neither applicant can improve their payoff ex post by doing something other than what their rule specifies. Similarly, the Canadian university must use an offer rule that provides a better payoff than any other offer in every information set they face. Finally both universities' reference type must coincide with the lowest type that sends them applications in equilibrium.

Equilibrium depends on application costs c and the gain that universities get from hiring the best applicant, parameterized by $\underline{g}$. Here we will focus on values for these parameters that support outcomes that seem closest to what happens in fact.

Proposition 1. *(Trifurcation Theorem) Suppose that* $u_C(1)\frac{g}{4} < c < u_A(1) - \frac{3}{4}u_C(1)$ *and* $1 + \underline{g} \leq \sqrt{2}$ *. Then there is a symmetric equilibrium for the game in which applicants whose type is below some critical value* t^* *apply only to the Canadian university; applicants whose type is above a critical value* $t^{**} > t^*$ *apply only to the American university, while applicants whose type is in between* t^* *and* t^{**} *apply to both universities. If the Canadian university has the choice between two applicants, then it will make an offer to the applicant with the lowest type if it prefers that applicant; if that applicant has a type below* t^**while the second applicant has a type above* t^**; or if the gap between the types of its two applicants is larger than* $\underline{g}$*.*

The proof of this theorem is in the appendix.
To understand the proof, it is enough to describe the basic logic. Given cutoffs t^* and t^{**}, the payoff to an applicant of type t who applies to only the Canadian university is described by some function $V(\{C\}, t|t^*)$. The payoff if he or she applies to both universities

is given by a function $V(\{A, C\}, t|t^*)$, while the payoff from applying only to the American university is $V(\{A\}, t|t^*)$. An applicant who has type t^* should be just indifferent about whether to apply as well to the American university. So t^* can be found by solving the equation

$$V(\{C\}, t^*|t^*) = V(\{C, A\}, t^*|t^*).$$

The proof shows that under the assumptions of the theorem, this has a unique solution.

Similarly, at the cutoff t^{**} an applicant should indifferent about adding an application to the Canadian university to his application to the American university. This happens when

$$V(\{C,, A\}, t^{**}|t^*) = V(\{A\}, t^{**}|t^*),$$

which again has a unique solution according to the proof.

From these two numbers, most of the important results follow. For example, with probability $2t^*(1-t^*)$ the outcome is something akin to assortative matching. One of the applicants applies to the Canadian university alone and receives an offer for sure. The other applicant who applies to the American university as well is hired there whether or not he or she also applies to the Canadian university. The only real loss in this event is the loss of the applicant who is hired at the American university who didn't actually have to bother to apply to the Canadian university.

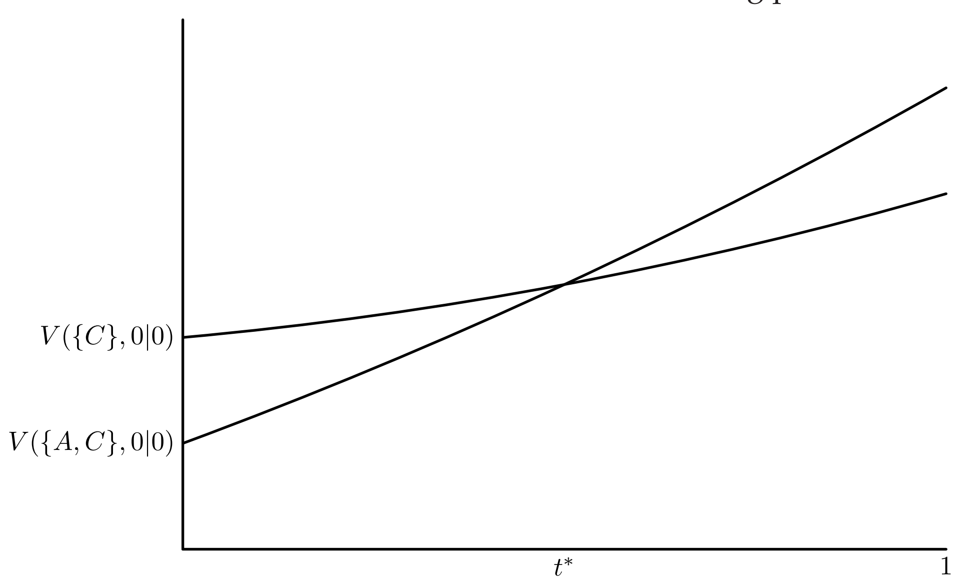
Things begin to break down when both applicants have types above t^*. If one of them has a type that exceeds the other by at least the gap $\underline{g}$, then the Canadian university will make an offer to the applicant with the lower type (independent of whether or not it actually prefers that applicant). Everything will be fine in this case, provided the American university doesn't also prefer the applicant with the lower type.

If the applicant types are close together in the sense that the difference between them is smaller than $\underline{g}$, then the Canadian university will make an offer to which ever applicant it likes best, and have this offer rejected with a probability not too far from $1/2$. The way the size of this inefficiency is affected by application costs is one of our main concerns, so we will try to quantify it later with an example.

3. Comparative Static

This equilibrium, simple as it is, captures many of the phenomena that we all experience every year. Almost every year there is a discussion about whether to make an offer the applicant we like best, or the one we think is more likely to accept. More often than not, we simply pay no attention to top applicants at all ('out of our league') because we don't expect them to accept and we don't want to waste an offer than might be rejected. In the equilibrium above, this is captured by the gap parameter $\underline{g}$. If the Canadian university receives a pair of applications from applicants whose observable types differ by more than the gap parameter, they will make the offer to the lower type applicant even if they prefer the applicant with the higher type.

The comparative statics with respect to application costs are also quite straightforward. To understand them, note that the construction involves finding a partition of the market at t^* such that applicants whose type is below t^* apply only to the Canadian university, while applicants whose type is above t^* apply to both universities. In the proof, the payoff an applicant of type t gets when the cutoff is at t^* is given by a function $V(\{C\}, t|t^*)$ for the Canadian university and $V(\{A,C\}, t|t^*)$ for applying to both. The calculation for finding a t^* involves solving the equation $V(\{C\}, t|t) = V(\{A,C\}, t|t)$ so that the applicant on the margin is just indifferent between adding the application to the American university or not. The proof verifies that these functions look like the ones in the following picture:



The functions $V(\{C\}, t|t)$ and $V(\{A,C\}, t|t)$ are both linear in c - the first one contains a term $-c$ for the cost of one application, the second one contains a term $-2c$ for the cost of two applications. Otherwise c does not occur in the rest of the functions. So reducing application costs (c) will cause $V(\{A,C\}, t|t)$ to rise at every point by twice as much as $V(\{C\}, t|t)$. So it is immediate that reducing application costs will cause the intersection of these two curves to move to the left – in other words, applicants are likely to make more applications – just as expected. This is a phenomena that is informally reported by many departments who use econjobmarket.org to take applications through their centralized application process.

Exactly the same kind of argument is true for the upper cutoff t^{**}. As the applicant's type rises, the gain to applying to the Canadian university once he applies to the American university is falling. Reducing application costs must then cause t^{**} to increase.

The impact of this change is quite different for American and Canadian universities. For the American university, they will receive applications for applicants with lower types because of the fact that t^* is falling. But they are very unlikely to hire these applicants anyway. From their perspective, lowering application costs primarily imposes additional processing costs. It is possible, but unlikely that the American university will hire an applicant who otherwise wouldn't have applied.

The impact on the Canadian university is quite different. The market works best in our equilibrium when applicants' types are very different. For example, when there is an applicant whose type is below t^* and another applicant whose type is above t^*, everyone is happy – both universities hire and both applicants get a job. The reduction in t^* that is caused by a reduction in application costs makes this outcome less likely.

The flip side of this is that the Canadian university will find itself more often in a position where it has to make a strategic decision. These situations can cause coordination failures where the Canadian department makes an offer to an applicant who receives a better offer from the American university. This is compounded by the fact that the highest type applicants will more often apply to the Canadian university. This comes from an increase in t^{**}, the point where the high type applicants stop applying to Canadian departments.[3]

4. Numerical Example

This section provides a numerical example where

$$u_i(t) = a_i t + k$$

[3]We don't give a proof, which is straightforward. The result is also completely intuitive.

with $a_A > a_C = 1$. We also assume that $k > c$ so that all types of the applicant have a positive net surplus of applying to some university. Note that $u_A(0) = u_C(0) = k > c$.

Our example shows that the equilibrium depends on the relative value for the candidates of the American university, a, the application cost c and the relative gain that universities get from hiring the best applicant, parameterized by $\underline{g}$. We focus on values for these parameters that support outcomes that seem closest to what happens in fact. Our proposition states that if $\underline{g} < \min\{\sqrt{2}-1, 2c\}$ – provided that $c < (4a + k - 3)/4$, there is a symmetric equilibrium in which applicants whose type is below some critical value t^* apply only to the Canadian University; applicants whose type is above a critical value $t^{**} > t^*$ apply only to the American university, while applicants whose type is in between t^* and t^{**} apply to both universities. Furthermore, if the Canadian university has the choice between two applicants, then it will make an offer to the applicant with the lower type if it prefers that applicant; if that applicant has a type below t^* while the second applicant has a type above t^*; or if the gap between the types of its two applicants is larger than $\underline{g}$.

We'll start by constructing the two cutoffs, t^* and t^{**} for our example.

We find the cutoff t^* such that

$$V\left(\{C\}, t^*|t^*\right) =$$

$$\frac{1}{4}\left(\left(t^*\right)^2 - 2t^* + 4\right)\left(k + t^*\right) - c \tag{4.1}$$

is equal to

$$V\left(\{C, A\}, t^*|t^*\right) =$$

$$\frac{1}{4}t^*\left(-a\left(t^*\right)^2 + 4at^* + a + \left(t^*\right)^2 - 4t^* + 3\right) + k - 2c, \tag{4.2}$$

where we use a in place of a_A to streamline the notation a bit.

From this the solution for t^* will be the positive root of the equation

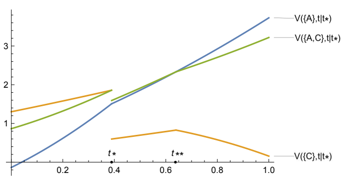
$$ax^3 + (k + 2 - 4a)x^2 + (1 - a - 2k)x + 4c = 0.$$

It is easy to show that an increase in the parameter a or k affects t^* in the same direction, while an increase in c affects t^* in the opposite direction, that is,

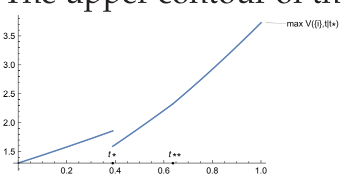
$$\operatorname{sign} \frac{dt^*}{da} = \operatorname{sign} \frac{dt^*}{dk} = -\operatorname{sign} \frac{dt^*}{dc}.$$

From the argument in the last section above, $dt^*/dc > 0$, from which the other results follow.

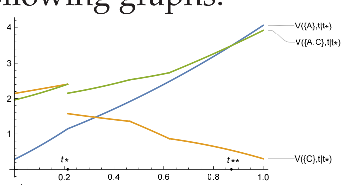
There is no analytical solution for t^{**} in our example, so we turn to numerical methods, and suppose that $a = 3$, $c = 1$, $k = 3$ and $\underline{g} = 1/4$. We can calculate that $t^* = 0.389$ and that $t^{**} = 0.637$, so that all types $t < 0.389$ apply only to the Canadian university, all types $0.389 \leq t \leq 0.637$ apply to both universities, and all types $t > 0.637$ apply only to the American university. We can graph the value of the three different strategies for an applicant of type t.

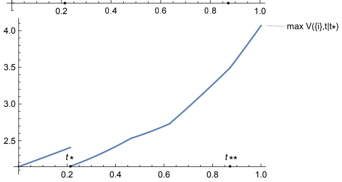


The upper contour of these payoffs function is illustrated here.

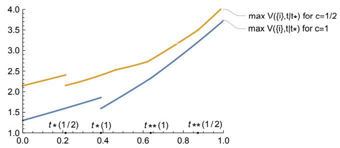


We can perform a similar exercise for $a = 3$, $c = 1/2$, $k = 3$ and $\underline{g} = 1/4$ to illustrate the impact of c on the strategy of the applicant. In this case, we get $t^* = 0.214$ and $t^{**} = 0.872$, so that many more types apply to both universities. Since t^{**} increases, the American university loses its exclusivity over high type applicants. We get the following graphs.



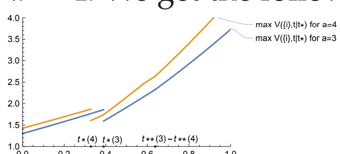


We can plot both upper contours on the same graph.



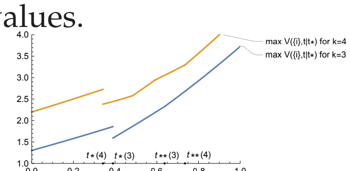
We clearly see that a decrease in c increases the applicant's payoff and that the set of applicants that apply to both universities expands. A decrease in c has two effects on the Canadian university. On the one hand, some of its lower type candidates (around $t^*(1)$ now also apply to the American university so that the Canadian university loses its exclusivity. This can be detrimental when these candidates get an offer from the American university. On the other hand, the Canadian university now attracts new higher type applicants around $t^{**}(1)$, which can be beneficial if some of these accept an offer from the Canadian university.

We can perform a similar comparative statics exercise on the parameter a. From our benchmark case with $a = 3$, $c = 1$, $k = 3$ and $\underline{g} = 1/4$, we increase the relative value of the American university to $a = 4$. We get the following upper contour values.

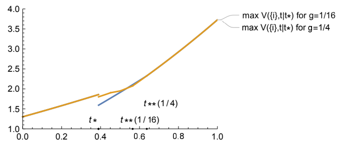


We see that the increase in a reduces t^*. It also increases slightly t^{**}, although it is not apparent on the graph. An interesting feature arises in this example. All types $t^*(4) < t < t^*(3)$ get a higher expected payoff when a is smaller. This seems paradoxical since $V(\{C\}, t|\, t^*)$ does not appear to depend on a, the relative value of the American university. But $V(\{C\}, t|t^*)$ does depend on a through t^*. In fact, $V(\{C\}, t|t^*)$ is decreasing in t^*. So, when a increases, t^* decreases and, hence, $V(\{C\}, t|t^*)$ increases as it is clear from the graph. Since there is a discontinuous drop in $V(\{A, C\}, t|t^*)$ at t^*, it is possible that some types maybe better off when a is smaller.

The parameter k has a similar effect as a. From our benchmark case with $a = 3$, $c = 1$, $k = 3$ and $\underline{g} = 1/4$, we increase the type independent payoff to $k = 4$. We get the following upper contour values.



A final parameter we can consider is the gap $\underline{g}$. It does not affect t^* (the polynomial expression that determines t^* does not depend on $\underline{g}$). It does, however, affect t^{**}. One should also note that the gap does not affect the expression $V(\{A\}, t|t^*)$. We therefore expect the gap to affect the expected payoff of an applicant only between t^* and t^{**}. This is indeed what the following graph reveals.



We see that a decrease in the gap $\underline{g}$ decreases t^{**}. The curve for $\underline{g} = 1/16$ is the orange one. From the expression for $V(\{A, C\}, t|t^*)$, we see that a decrease in $\underline{g}$ has an ambiguous effect on the probability that an applicant gets an offer from the Canadian university (when he or she applies to both universities). In fact, the effect of the gap depends on whether the min and/or the max are binding in the bounds for the integrals in $V(\{A, C\}, t|t^*)$. If both are binding, it is easy to show that the gap does not affect the expression for $V(\{A, C\}, t|t^*)$. If only the min is binding, then $V(\{A, C\}, t|t^*)$ decreases when the gap increases. If only the max is binding, then $V(\{A, C\}, t|t^*)$ increases when the gap increases. Since $V(\{A\}, t|t^*)$ is unaffected by the gap, this implies that the effect on t^{**} of a decrease in the gap is dictated only by its effect on $V(\{A, C\}, t|t^*)$. In our example, a decrease in the gap reduces t^{**}, so that the only the max is binding. This is more likely to occur when $\underline{g}$ is small.

Finally, we turn to the issue of the impact of application costs on frictions. Consider a decrease in c. Since we cannot derive explicit solutions, we compute examples to see the impact of c on the probability that the Canadian university fails to hire. Consider our previous benchmark case with $a = 3, c = 1, k = 3$ and $\underline{g} = 1/4$. For these values, we get the probability with which the Canadian university fails to hire as $P = 0.255$. When c decreases to $c = 1/2$, this probability rises to $P = 0.385$. In words, a 50% reduction in application costs causes about a 50% increase in the probability with which the Canadian university fails to hire at all. Comparable increases in frictions occur for other parameters. For example, if $a = 4, k = 3$ and $\underline{g} = 1/4$, P increases from 0.299 to 0.408 when c decreases from 1 to 1/2. If $a = 3, k = 4$ and $\underline{g} = 1/4$, P increases from 0.281 to 0.418 when c decreases from 1 to 1/2. Finally, if $a = 3, k = 3$ and $\underline{g} = 1/16$, P increases from 0.266 to 0.287 when c decreases from 1 to 1/2.

Perhaps surprisingly, a reduction in application costs increases frictions for the Canadian university. The reason is clear enough. Reduced application costs increase the probability with which the Canadian university will have to make a 'strategic' offer. Since strategic offers are quite likely to fail, frictions increase.

5. Conclusion

Obviously, this paper is an example, nothing more. However, it articulates many of the tradeoffs involved in matching markets like the academic job market. The first pressing problem is simply that existing matching theory isn't yet rich enough to provide guidance in this market. Most theoretical papers involve some variant of assortative matching with costless applications, or directed search with single applications. Structural estimation will require some method of characterizing application portfolios.

Second, for the most basic characteristic of the matching market, the frictions involved in the initial matching, there is little data available.

We hope this short example helps to illustrate some of these problems.

Appendix

Proof of the Trifurcation Theorem.

Proof. Fix t^* and t^{**} and imagine that both applicants are using the strategy described in the Proposition above. Notice that in this equilibrium, the Canadian university receives applications with positive probability from arbitrarily low types. So $r_c = 0$. Hence, when a Canadian university has two applicants, they will both be acceptable. If so, suppose the Canadian university decides that it likes the applicant with the higher type, and let $t_1 > t_2$ be the types of the two applicants. If $t_1 \leq t^*$ then the Canadian department should just make an offer to the higher type applicant, since it believes that neither of them have applied to the American university. If $t^{**} \geq t_1 > t^*$, then what the department does depends on t_2. If $t_2 < t^*$, then the Canadian department should believe that t_1 has applied to the American university as well, and that t_2 hasn't. In that case, t_1 is sure to accept an offer from the American department, the Canadian department should make an offer to t_2 which it expects to be accepted for sure. This is the first instance in which the Canadian university makes an application to its less preferred applicant.

If $t_2 \geq t^*$, on the other hand, then the Canadian department should believe that both applicants have applied to the American department. Since applicants with types below t^* are not expected to apply to the American department, $r_A = t^*$, so both applicants will be acceptable to the American department, though the Canadian department does not know which the American department prefers. In that case, the probability with which the American department will make an offer to t_1 is $g(t_1, t_2) = \frac{1+t_1-t_2}{2}$. The Canadian department should make an offer to t_1 if and only if

$$\left(1 - \frac{1+t_1-t_2}{2}\right) v_b \geq \frac{1+t_1-t_2}{2} v_w$$

or

$$\frac{v_b}{v_b+v_w} \geq \frac{1+(t_1-t_2)}{2},$$

which gives

$$\frac{v_b-v_w}{v_b+v_w} \geq (t_1-t_2).$$

In other words, there is a fixed gap

$$\underline{g} = \frac{v_b-v_w}{v_b+v_w} \tag{5.1}$$

having the property that the Canadian department will make an offer to t_1 if and only if the gap between their types is smaller than $\underline{g}$. This is the second instance in which the Canadian department will make an offer to its least preferred applicant.

From this logic, it is apparent that when the Canadian department prefers the applicant with the lower type, it will always make the offer to that applicant.

Since types above t^{**} are not supposed to apply to the Canadian department, we can just extend this strategy to include higher types by assuming that the Canadian department believes that off the equilibrium path, when an applicant of type $t > t^{**}$ applies, then they must also have applied to the American department.

Using this, we can construct the cutoff t^* by using the idea that a worker whose type is at the cutoff should be just indifferent about whether or not they apply to the American university. Consider an applicant with type $t < t^*$ who is making applications when other applicants are expected to apply to the American university only if their type exceeds t^*. If the other applicant's type is also lower than

t^* then the offer comes with probability $\frac{1+t-\tilde{t}}{2}$. His expected payoff is then

$$V\left(\{C\},t|t^*\right) =$$

$$u_C\left(t\right)\left\{\int_0^{t^*}\frac{1+t-\tilde{t}}{2}d\tilde{t}+\left(1-t^*\right)\right\}-c. \tag{5.2}$$

On the other hand if he applies to the American university as well his payoff is

$$V\left(\{C,A\},t|t^*\right) =$$

$$t^*\left(1-\frac{t^*-t}{2}\right)u_A\left(t\right)+\frac{t^*-t}{2}u_C\left(t\right)\int_0^{t^*}\frac{1+t-\tilde{t}}{2}d\tilde{t}+$$
$$\int_{t^*}^1\left\{\frac{1-t+\tilde{t}+\left(t^*-t\right)}{2}u_C\left(t\right)+\frac{1+t-\tilde{t}-\left(t^*-t\right)}{2}u_A\left(t\right)\right\}d\tilde{t}$$
$$-2c. \tag{5.3}$$

Of course, he also has the option to apply *only* to the American department, which gives

$$V\left(\{A\},t|t^*\right) =$$

$$t^*\left(1-\frac{t^*-t}{2}\right)u_A\left(t\right)+\int_{t^*}^1\left\{\frac{1+t-\tilde{t}-\left(t^*-t\right)}{2}u_A\left(t\right)\right\}d\tilde{t}-c. \tag{5.4}$$

To find the lower cutoff t^*, we evaluate (5.3) and (5.2) at $t = t^*$

$$V\left(\{C\},t^*|t^*\right) =$$

$$u_C\left(t^*\right)\int_0^{t^*}\frac{1+t^*-\tilde{t}}{2}d\tilde{t}+\left(1-t^*\right)u_C\left(t\right)-c$$

while

$$V\left(\{C,A\},t^*|t^*\right) =$$
$$t^*u_A\left(t^*\right)+\int_{t^*}^1\left\{\frac{1+\tilde{t}-t^*}{2}u_C\left(t^*\right)+\frac{1+t^*-\tilde{t}}{2}u_A\left(t^*\right)\right\}d\tilde{t}-2c.$$

If $t^* = 0$, then $V\left(\{C\},0|0\right) = u_C\left(0\right)-c$, while

$$V\left(\{C,A\},0|0\right) =$$
$$\int_0^1\left\{\frac{1+\tilde{t}}{2}u_C\left(0\right)+\frac{1-\tilde{t}}{2}u_A\left(0\right)\right\}d\tilde{t}-2c =$$

$$u_C(0) + \frac{1}{2}\left(u_A(0) - u_C(0)\right) - 2c = u_C(0) - 2c$$

because of the assumption that $u_C(0) = u_A(0)$. So $V(\{C\}, 0|0) > V(\{C, A\}, 0|0)$.

Conversely, it is straightforward that $V(\{C, A\}, 1|1) = u_A(1) - 2c > V(\{C\}, 1|1) = 3u_C(1)/4 - c$. Since $V(\{C\}, t|t)$ and $V(\{A, C\}, t)$ are both continuous in t it follows that there is some t^* at which they are equal. Note that $V(\{C\}, t^*|t^*) > 0$.

The first thing we need to do is to show that any applicant with type $t < t^*$ strictly prefers to apply only to the Canadian university. To do this we rewrite the payoff that a type $t < t^*$ applicant gets when applying only at the Canadian university as

$$u_C(t)\left\{\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t} + (1-t^*)\right\} - c =$$
$$u_C(t)\left\{\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t} + \int_{t^*}^1 \left\{\frac{1-t+\tilde{t}}{2} + \frac{1+t-\tilde{t}}{2}\right\} d\tilde{t}\right\} - c =$$
$$u_C(t)\int_0^1 \frac{1+t-\tilde{t}}{2} d\tilde{t} + u_C(t)\int_{t^*}^1 \left\{\frac{1-t+\tilde{t}}{2}\right\} d\tilde{t} - c.$$

While for the payoff to applying to both universities we have

$$t^*\left\{\left(1 - \frac{(t^*-t)}{2}\right) u_A(t)\right\} + u_C(t)\frac{(t^*-t)}{2}\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t} +$$
$$\int_{t^*}^1 \left\{\frac{1-t+\tilde{t}+(t^*-t)}{2} u_C(t) + \frac{1+t-\tilde{t}-(t^*-t)}{2} u_A(t)\right\} d\tilde{t} - 2c =$$
$$t^*\left\{\left(1 - \frac{(t^*-t)}{2}\right) u_A(t)\right\} + \frac{(t^*-t)}{2} u_C(t)\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t} +$$
$$(1-t^*)\, u_C(t) + \int_{t^*}^1 \left(\left(u_A(t) - u_C(t)\right)\frac{1+t-\tilde{t}-(t^*-t)}{2}\right) d\tilde{t} - 2c.$$

The difference is

$$t^*\left\{\left(1 - \frac{(t^*-t)}{2}\right) u_A(t)\right\} + \frac{(t^*-t)}{2} u_C(t)\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t}$$
$$-u_C(t)\int_0^{t^*} \frac{1+t-\tilde{t}}{2} d\tilde{t}$$
$$+\int_{t^*}^1 \left(\left(u_A(t) - u_C(t)\right)\frac{1+t-\tilde{t}-(t^*-t)}{2}\right) d\tilde{t} - c. \tag{5.5}$$

which is increasing in t by our assumption that the difference $(u_A(t) - u_C(t))$ is increasing in t. Since this last expression will be zero when $t = t^*$ by construction, it must be negative for $t < t^*$, which shows that applicants with low types don't want to apply to the American university.

To show that $V(\{A\}, t|t^*) < V(\{C\}, t|t^*)$ for $t < t^*$, observe that from (5.5)

$$V(\{A,C\}, t|t^*) =$$

$$V(\{A\}, t) - V(\{C\}, t) + c + \frac{(t^* - t)}{2} u_C(t) \int_0^{t^*} \frac{1 + t - \tilde{t}}{2} d\tilde{t}.$$

Since this expression is less than 0 for $t < t^*$, we get $V(\{A\}, t|t^*) < V(\{C\}, t|t^*)$.

The next step in the argument is to establish the upper cutoff, t^{**}, where the applicant's type is so large that he or she decides it is no longer worth it to apply to the Canadian university. The payoff to an applicant whose type is strictly larger that t^* (the argument above is based on the assumption that $t \leq t^*$) who applies *only* at the American university is given by (5.4). The payoff when he or she applies to both universities is different from what it was above, because the applicant will no longer be guaranteed an offer from a Canadian university. Recall from (5.1) that an applicant at the Canadian university whose type is $t > t^*$ will receive an offer from the Canadian university with positive probability only when his type does not exceed the other applicants type by more that $\underline{g}$. Using this fact gives the payoff function

$$\begin{aligned}
&V(\{A,C\}, t|t^*) = t^* u_A(t) + u_A(t) \int_{t^*}^{1} \frac{1 + t - \tilde{t}}{2} d\tilde{t} + \\
&u_C(t) \int_{\max[t-\underline{g}, t^*]}^{t^{**}} \frac{1 - t + \tilde{t}}{2} \left\{ \frac{1 + t - \tilde{t}}{2} \right\} dt + \\
&u_C(t) \int_{\min[t+\underline{g}, t^{**}]}^{t^{**}} \left\{ \frac{1 - t + \tilde{t}}{2} \right\}^2 d\tilde{t} + u_C(t) \int_{t^{**}}^{1} \frac{1 - t + \tilde{t}}{2} d\tilde{t} - 2c = \\
&V(\{A\}, t|t^*) + u_C(t) \int_{\max[t-\underline{g}, t^*]}^{t^{**}} \frac{1 - t + \tilde{t}}{2} \left\{ \frac{1 + t - \tilde{t}}{2} \right\} dt \\
&+ u_C(t) \int_{\min[t+\underline{g}, t^{**}]}^{t^{**}} \left\{ \frac{1 - t + \tilde{t}}{2} \right\}^2 d\tilde{t} \\
&+ u_C(t) \int_{t^{**}}^{1} \frac{1 - t + \tilde{t}}{2} d\tilde{t} - c.
\end{aligned} \tag{5.6}$$

Suppose that $t^{**} = t^*$ and evaluate equation (5.6) at $t = t^*$. We get

$$V(\{A,C\}, t^*|t^*) = V(\{A\}, t^*|t^*) + u_C(t^*) \int_{t^*}^{1} \frac{1 - t^* + \tilde{t}}{2} d\tilde{t} - c.$$

Since

$$u_C(t^*) \int_{t^*}^{1} \frac{1 - t^* + \tilde{t}}{2} d\tilde{t} > c,$$

it has to be the case that $V(\{A,C\}, t^*|t^*) > V(\{A\}, t^*|t^*)$, so that $t^{**} > t^*$.

Suppose now that $t^{**} = 1$ and evaluate equation (5.6) at $t = t^{**} = 1$. We get

$$V(\{A,C\}, 1|t^*) = V(\{A\}, 1|t^*) + u_C(1) \int_{1-\underline{g}}^{1} \frac{\tilde{t}}{2} \left(\frac{2 - \tilde{t}}{2}\right) d\tilde{t} - c,$$

when $\underline{g}$ is small enough. For an interior solution for t^{**}, we need that

$$u_C(1) \int_{1-\underline{g}}^{1} \frac{\tilde{t}}{2} \left(\frac{2 - \tilde{t}}{2}\right) d\tilde{t} - c < 0.$$

This condition reduces to $u_C(1) \frac{\underline{g}(3-\underline{g}^2)}{12} < c$. Since $u_C(1) \frac{\underline{g}(3-\underline{g}^2)}{12} < u_c(1) \frac{\underline{g}}{4} < c$ by the assumption in the statement of the theorem, we have an interior solution for t^{**}. □

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