- there are two goods, one public (y) and one private (x)
- there are two consumers
- the public good is produced using the private good
- for example, the two consumers belong to a club - they can spend time x to organize events at the club (y).

- the payoffs to the consumers are given by

$$u_1\left(x_1, y\right)$$

  for consumer 1 and

$$u_2\left(x_2, y\right)$$

  for consumer 2
- x_1 and x_2 are the time that consumer 1 and 2 respectively get to themselves, while y is the total number of events organized by the club - both consumers enjoy all the events at the club equally

- events are produced when the two consumers spend time organizing them
- the relationship between the number of events and the time each consumer has to themselves is

  $$y = f(\omega_1 + \omega_2 - x_1 - x_2)$$

  where ω_1 and ω_2 are the total amount of time that each consumer has to allocate between the two activities
- there are no rules about volunteering time, each consumer spends whatever time they like organizing

- this is called the *voluntary contribution game*
- the Nash equilibrium is given by a pair of private consumptions x_1^* and x_2^* such that

$$u_1\left(x_1^*, f\left(\omega_1+\omega_2-x_1^*-x_2^*\right)\right) \geq u_1\left(x', f\left(\omega_1+\omega_2-x'-x_2^*\right)\right) \tag{1}$$

for any alternative contribution $x' \in [0, \omega_1]$ and

$$u_2\left(x_2^*, f\left(\omega_1+\omega_2-x_1^*-x_2^*\right)\right) \geq u_2\left(x', f\left(\omega_1+\omega_2-x_1^*-x'\right)\right) \tag{2}$$

for any alternative contribution $x' \in [0, \omega_2]$.

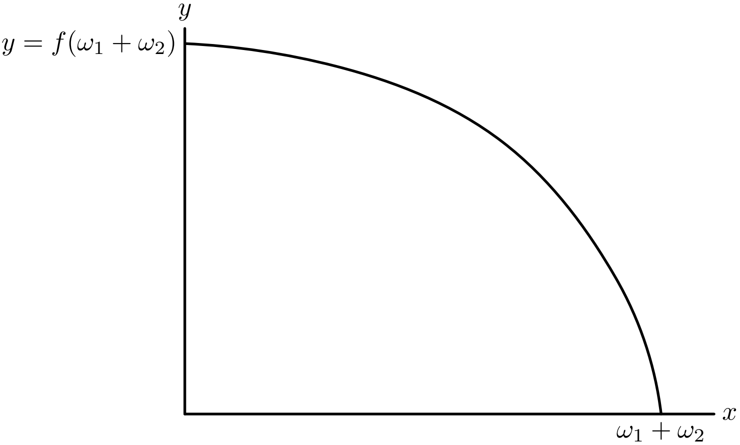
- in a Nash equilibrium, each consumer believes that he knows how much time the other consumer is going to volunteer
- if consumer 1 believes that consumer 2 is going to take x_2 hours for himself, then his or her problem is to maximize

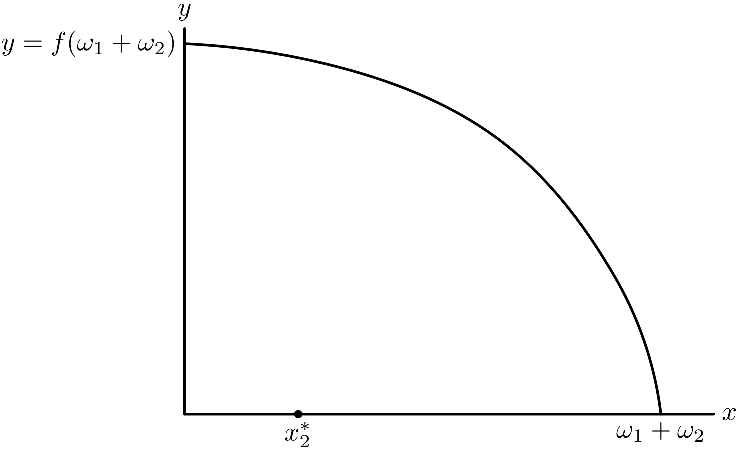
  $$u_1(x_1, y)$$

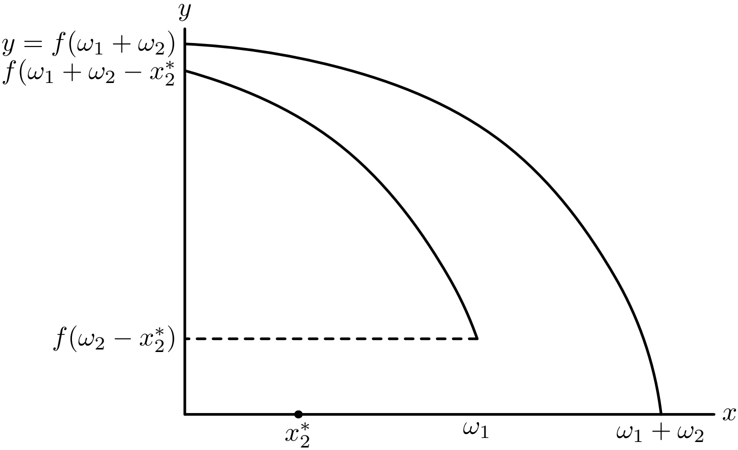
  subject to

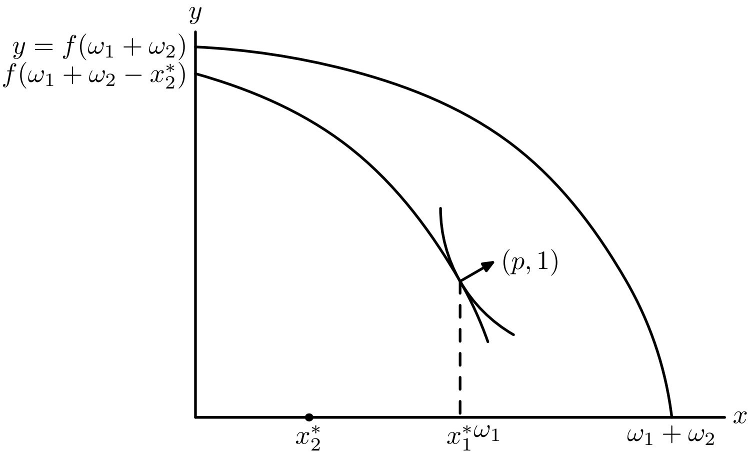
  $$y = f(\omega_1 + \omega_2 - x_1 - x_2).$$

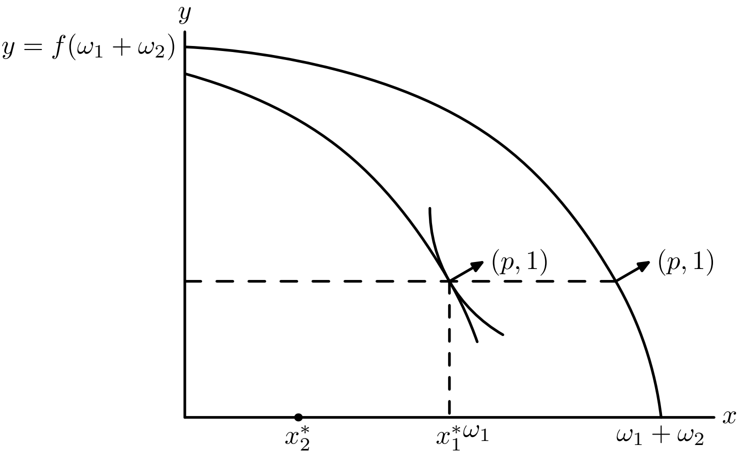
- this is a problem you have seen many many times before - so we can draw a picture

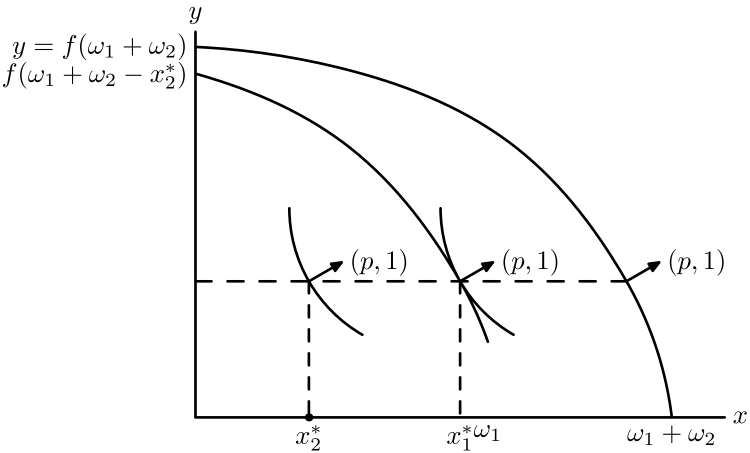

y
$y = f(\omega_1 + \omega_2)$
x
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
x
x_2^*
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
$f(\omega_1 + \omega_2 - x_2^*$
$f(\omega_2 - x_2^*)$
x
x_2^*
ω_1
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
$f(\omega_1 + \omega_2 - x_2^*)$
$(p, 1)$
x
x_2^*
$x_1^* \omega_1$
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
$(p, 1)$
$(p, 1)$
x
x_2^*
$x_1^*\omega_1$
$\omega_1 + \omega_2$



y
$y = f(\omega_1 + \omega_2)$
$f(\omega_1 + \omega_2 - x_2^*)$
$(p, 1)$
$(p, 1)$
$(p, 1)$
x
x_2^*
$x_1^* \omega_1$
$\omega_1 + \omega_2$

Find Equilibrium

- in algebra, let $u_1(x, y) = \alpha \ln(x) + (1-\alpha)\ln(y)$ for both players, and suppose that the production function is just $y = (\omega_1 + \omega_2 - x_1 - x_2)$.
- given his expectation that consumer 2 will contribute $\omega_2 - x_2$ to producing the public good, consumer 1 should solve

$$\max_{x_1} \alpha \ln(x_1) + (1-\alpha)\ln(\omega_1 + \omega_2 - x_1 - x_2)$$

- the first order condition is

$$\frac{\alpha}{x_1} = \frac{(1-\alpha)}{\omega_1 + \omega_2 - x_1 - x_2}$$

- this gives the simple solution

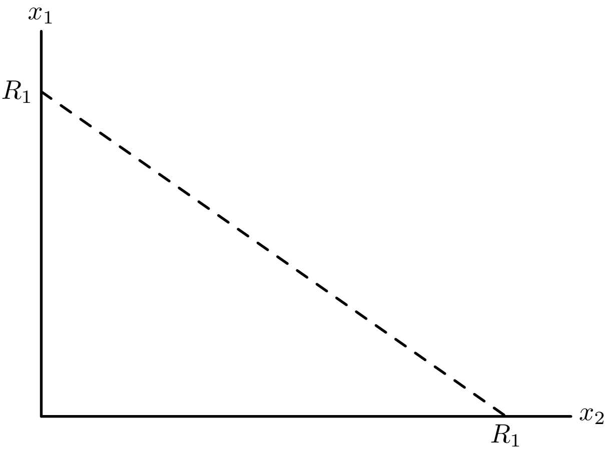
$$x_1 = \alpha(\omega_1 + \omega_2 - x_2).$$

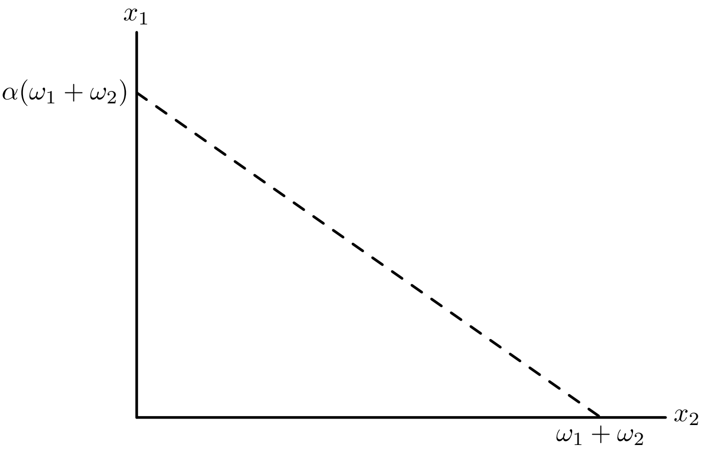
- if you write down the same equation for consumer 2 and solve both equations simultaneously for x_1 and x_2, you will find they are both the same and equal to $\frac{\alpha}{1+\alpha}(\omega_1 + \omega_2)$
- the question we want to ask is - if we (as dictators) could choose x_1 and x_2 to be anything at all, would we be happy with the players choices in a Nash equilibrium? or would we want to try to force them to do something else.
- the algebra doesn't address this question, so lets go back to the diagrams

- the equation

$$x_1 = \alpha \left(\omega_1 + \omega_2 - x_2\right).$$

  is player 1's best reply function
- we could draw this on a graph

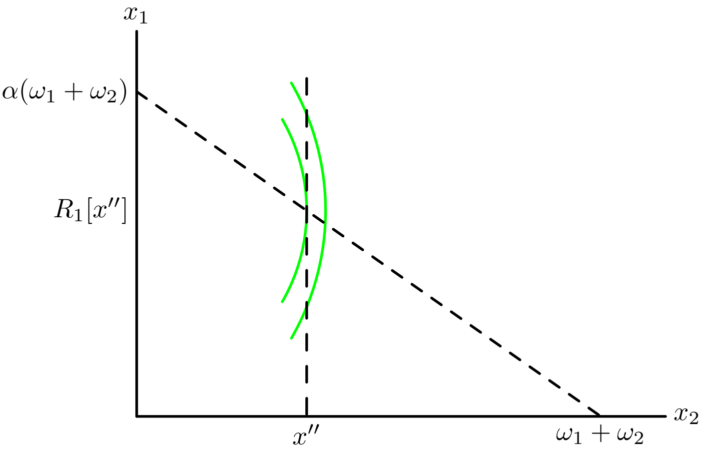

x_1
R_1
x_2
R_1

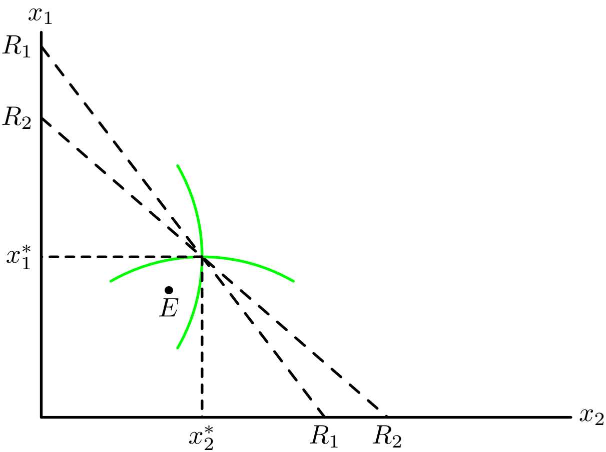

x_1
$\alpha(\omega_1 + \omega_2)$
x_2
$\omega_1 + \omega_2$

- back to the "what if we could pick anything" approach, we could ask what 1 would do if he could pick both x_1 and x_2
- the he would solve the problem

$$\max_{x_1, x_2} u_1\left(x_1, f\left(\omega_1 + \omega_2 - x_1 - x_2\right)\right)$$

- of course that would make him a lot better off
- one way to think of the Nash equilibrium is that he does exactly this, but he is constrained to choose x_2 so that it is equal to what he expects 2 to choose

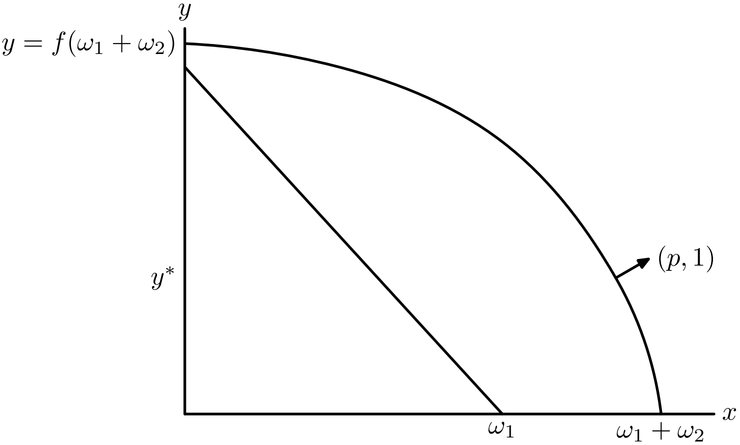

x_1
$\alpha(\omega_1 + \omega_2)$
$R_1[x'']$
x_2
x''
$\omega_1 + \omega_2$

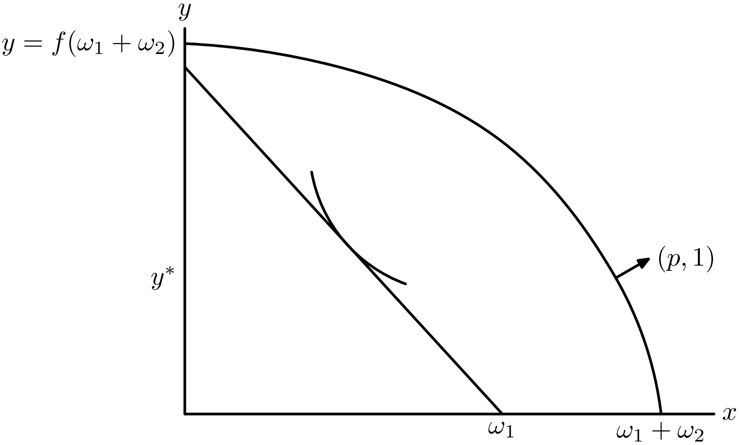


x_1
R_1
R_2
x_1^*
E
x_2^*
R_1
R_2
x_2

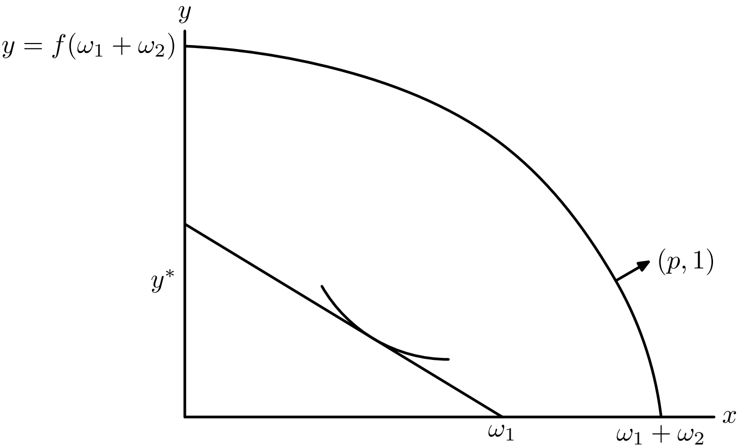
Patents

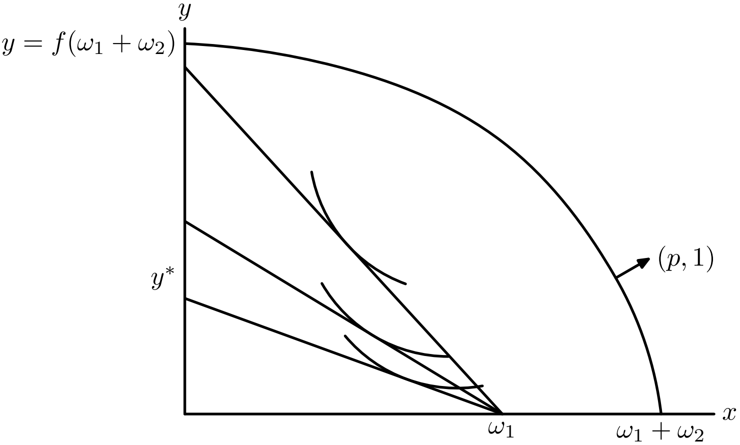
- give player 2 a patent
- she controls the public good and charges player 1 for all the public good that is produced
- no competition is allowed
- if p is the (relative) price of x, the $\frac{1}{p}$ is the relative price of the public good


y
$y = f(\omega_1 + \omega_2)$
y^*
$(p, 1)$
x
ω_1
$\omega_1 + \omega_2$

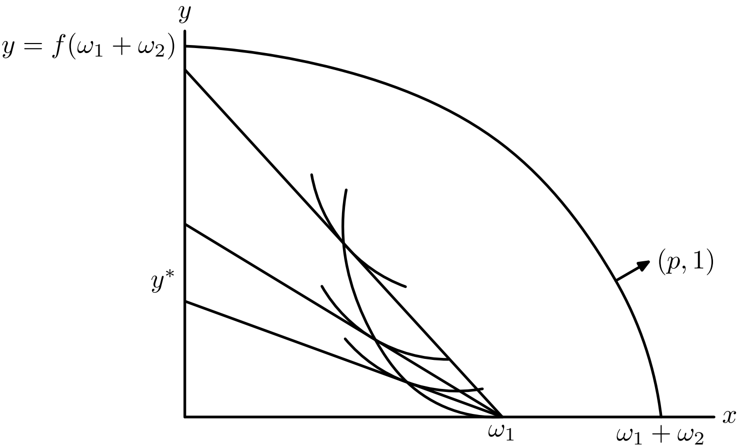


y
$y = f(\omega_1 + \omega_2)$
y^*
$(p, 1)$
x
ω_1
$\omega_1 + \omega_2$

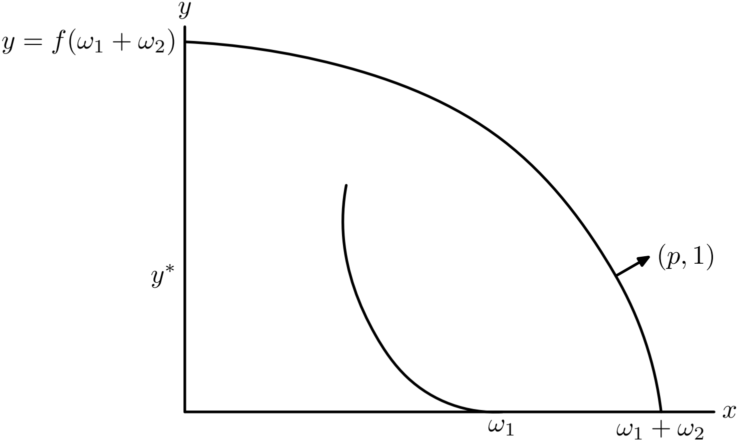

y
$y = f(\omega_1 + \omega_2)$
$(p, 1)$
y^*
ω_1
$\omega_1 + \omega_2$
x

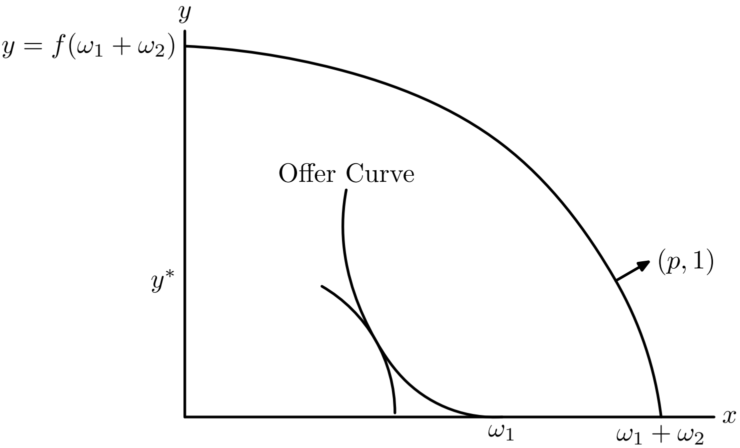


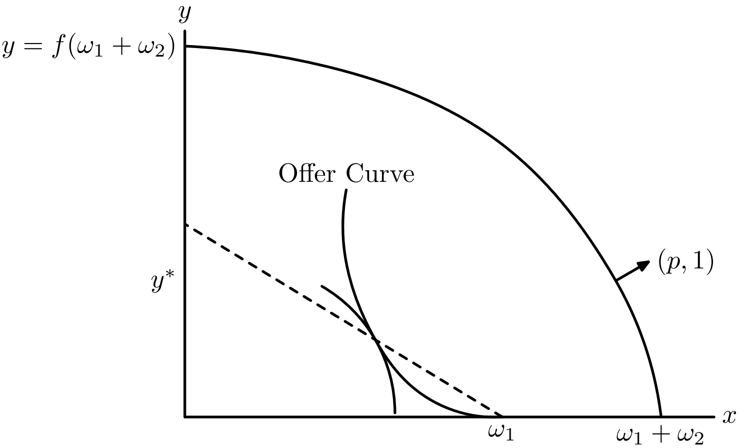
y
$y = f(\omega_1 + \omega_2)$
$(p, 1)$
y^*
x
ω_1
$\omega_1 + \omega_2$

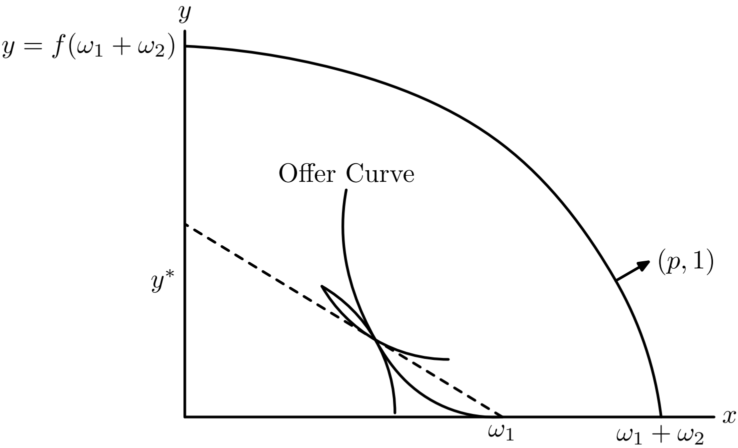


y
$y = f(\omega_1 + \omega_2)$
y^*
$(p, 1)$
x
ω_1
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
y^*
$(p, 1)$
x
ω_1
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
Offer Curve
$(p, 1)$
y^*
x
ω_1
$\omega_1 + \omega_2$


y
$y = f(\omega_1 + \omega_2)$
Offer Curve
$(p, 1)$
y^*
x
ω_1
$\omega_1 + \omega_2$



y
$y = f(\omega_1 + \omega_2)$
Offer Curve
$(p, 1)$
y^*
x
ω_1
$\omega_1 + \omega_2$