

Mid Term Answers

1.

$$\max_q [\bar{p}q - C[q]] = \max_q \mathbb{E}_p \{pq - C[q]\} \leq \mathbb{E}_p \left[\max_q (pq - C[q]) \right]$$

so he will enter the market with the variable price.

No. Consider two degenerate lotteries which give respectively p and p' for sure. The compound lottery which gives p with probability λ and p' with probability $(1 - \lambda)$ should be indifferent to the lottery p' for sure by the independence axiom $(\lambda p' + (1 - \lambda)p')$. Just use the reasoning from part *a* above to show this won't be true

2. trader i has a portfolio z_i such that $Rz_i = y_i \in \mathbb{R}^S$. By the market clearing condition in the portfolio market

$$R \sum_{i=1}^n z_i = \sum_{i=1}^n y_i = 0$$

Let R_S be the submatrix consisting of the first S columns of R . Since these are linearly independent there is a unique portfolio w_i for each i such that $R_S w_i = y_i$ and since $\sum_{i=1}^n y_i = 0$ then $\sum_{i=1}^n R_S w_i = R_S \sum w_i = 0$ which means that $\sum w_i = 0$ because R_S has full rank. Finally there is no arbitrage in a Radner equilibrium, so by the state price theorem, there is a vector of strictly positive state prices $p \in \mathbb{R}^S$ such that for any security $t \in \mathbb{R}^S$ that lies in the linear subspace spanned by R the Radner equilibrium price of the security must be $p't$. Then $p'y_i = p'Rz_i = q'z_i = 0$ and so $p'y = p'Rw_i = \sum_{s=1}^S q_s w_i^s = 0$. So there is a portfolio for each player consisting only of the first S Radner securities which is feasible and in the budget set for each trader that supports the same pattern of state contingent returns. Since each trader has the same income in each state, markets will clear at the existing spot price.

3. The production possibilities frontier is $y = 60 - \frac{3}{2}x$ so profit maximization by firms forces the relative price of good x to be $\frac{3}{2}$. If the price of y is normalized to 1, then firm 1 can find a solution to its profit maximization condition only if $\frac{3}{2}x - w\frac{x}{2} = 0$ which gives $w = 3$. Then aggregate demand for good x must be equal to supply i.e.,

$$\frac{\frac{2}{5}10 \cdot 3}{\frac{3}{5}} + \frac{\frac{1}{2}10 \cdot 3}{\frac{3}{5}} = x$$

so $x = 18$ and $y = 33$.

4. There are four states for this problem $(y, y), (y, y - d), (y - d, y), (y - d, y - d)$. The consumers agree on the probability of each state, for example $(y, y - d)$

occurs with probability $\pi(1 - \pi)$. Since the consumers are Cobb-Douglas, the market clearing conditions are

$$\begin{aligned} & \frac{\pi^2 (q_{(y,y)}y + q_{(y,y-d)}y + q_{(y-d,y)}(y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y,y)}} \\ & + \frac{\pi^2 (q_{(y,y)}y + q_{(y,y-d)}(y-d) + q_{(y-d,y)}y + q_{(y-d,y-d)}(y-d))}{q_{(y,y)}} \\ & = 2y \end{aligned}$$

$$\begin{aligned} & \frac{\pi(1 - \pi) (q_{(y,y)}y + q_{(y,y-d)}y + q_{(y-d,y)}(y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y,y-d)}} \\ & + \frac{\pi(1 - \pi) (q_{(y,y)}y + q_{(y,y-d)}(y-d) + q_{(y-d,y)}y + q_{(y-d,y-d)}(y-d))}{q_{(y,y-d)}} \\ & = 2y - d \\ & \frac{\pi(1 - \pi) (q_{(y,y)}y + q_{(y,y-d)}y + q_{(y-d,y)}(y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y-d,y)}} \\ & + \frac{\pi(1 - \pi) (q_{(y,y)}y + q_{(y,y-d)}(y-d) + q_{(y-d,y)}y + q_{(y-d,y-d)}(y-d))}{q_{(y-d,y)}} \\ & = 2y - d \end{aligned}$$

and

$$\begin{aligned} & \frac{(1 - \pi)^2 (q_{(y,y)}y + q_{(y,y-d)}y + q_{(y-d,y)}(y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y-d,y-d)}} \\ & + \frac{(1 - \pi)^2 (q_{(y,y)}y + q_{(y,y-d)}(y-d) + q_{(y-d,y)}y + q_{(y-d,y-d)}(y-d))}{q_{(y-d,y-d)}} \\ & = 2y - 2d \end{aligned}$$

The middle pair gives $q_{(y,y-d)} = q_{(y-d,y)}$ reducing the conditions to

$$\begin{aligned} & \frac{2\pi^2 (q_{(y,y)}y + q_{(y,y-d)}(2y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y,y)}} \\ & = 2y \end{aligned}$$

and

$$\begin{aligned} & \frac{2\pi(1 - \pi) (q_{(y,y)}y + q_{(y,y-d)}(2y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y,y-d)}} \\ & = 2y - d \end{aligned}$$

and

$$\frac{2(1-\pi)^2 (q_{(y,y)}y + q_{(y,y-d)}(2y-d) + q_{(y-d,y-d)}(y-d))}{q_{(y-d,y-d)}} = 2y - 2d$$

which gives

$$\frac{\pi}{1-\pi} \frac{q_{(y,y-d)}}{q_{(y,y)}} = \frac{2y}{2y-d}$$

$$\frac{\pi^2}{(1-\pi)^2} \frac{q_{(y-d,y-d)}}{q_{(y,y)}} = \frac{2y}{2y-2d}$$

and

$$\frac{\pi}{(1-\pi)} \frac{q_{(y-d,y-d)}}{q_{(y,y-d)}} = \frac{2y-d}{2y-2d}$$

which gives the four arrow debru prices. The Radner securities are arrow securities in this case.

5. Each consumer takes the consumption of the private good chosen by the other to be fixed. With Cobb Douglas preferences, 0 consumption of either good is never optimal. However, eating all the endowment may be optimal if consumer 2 is contributing enough to the public good. If consumer 1 chooses consumption less than his endowment the choice must be a solution to the first order condition

$$\alpha x^{\alpha-1} (1-x_1-x_2)^{1-\alpha} - (1-\alpha) x^\alpha (1-x_1-x_2)^{-\alpha} = 0$$

which solves to

$$x_1 = (1-\alpha) x_2 \tag{1}$$

so the overall solution is

$$x_1 = \min [\omega_1, (1-\alpha) x_2]$$

To keep things simple suppose that the endowments are such that both choose to contribute to the public good. Since their first order conditions are the same $x_1 = x_2$ in this case, so substituting this into (1) gives

$$x_1 = x_2 = \frac{1}{1+\alpha}$$

0.0.1 Lindahl Prices

Let x and y be the output choices of the firm. Profits are then $(p_1 + p_2)y + x$ so consumer 1 has budget

$$\omega_1 ((p_1 + p_2)y + x)$$

while consumer 2 has budget

$$\omega_2 ((p_1 + p_2) y + x)$$

Since consumer 1 has Cobb Douglas preferences, his demands are

$$x_1 = \alpha \omega_1 ((p_1 + p_2) y + x)$$

for the private good and

$$y_1 = (1 - \alpha) \omega_1 \frac{((p_1 + p_2) y + x)}{p_1}$$

The sum of the demand for the private goods from 1 and 2 must equal the total output of the private good by the firm, so

$$\alpha \omega_1 ((p_1 + p_2) y + x) + \alpha (1 - \omega_1) ((p_1 + p_2) y + x) = x$$

or just

$$\alpha ((p_1 + p_2) y + x) = x \quad (2)$$

Each of the two consumers has to demand exactly the amount of the public good that the firm produces so

$$(1 - \alpha) \omega_1 \frac{((p_1 + p_2) y + x)}{p_1} = y \quad (3)$$

and

$$(1 - \alpha) (1 - \omega_1) \frac{((p_1 + p_2) y + x)}{p_2} = y \quad (4)$$

Finally, the production technology says that $y = 1 - x$. That means as well that the slope of the PPF must be 1 for profit maximization to occur, so $p_1 + p_2 = 1$. Finally, notice from (3) and (4) that if we set

$$p_1 = p_2 \frac{\omega_1}{1 - \omega_1}$$

then whenever the second condition holds, the final one will as well. Now substitute all this information into (2) to get

$$\alpha = x$$

From (3)

$$(1 - \alpha) \omega_1 \frac{1}{p_1} = (1 - \alpha)$$

or $p_1 = \frac{1}{\omega_1}$ and the rest of the question follows. Plot this information in Figure 4 in the reading for practise.