

Econ304
Answers for the Final

1. Let u be the individual's utility for wealth function. Let p be the probability with which he is caught if he engages in crime. The individual solves

$$\max \left[u(w), \max_{0 \leq \alpha \leq 1} \{ pu(w\alpha - d) + (1-p)u(\alpha w + (1-\alpha)R) \} \right]$$

Notice that this has to be solved in two parts. The Lagrangian for the second part is

$$pu(w\alpha - d) + (1-p)u(\alpha w + (1-\alpha)R) - \lambda_1 \alpha + \lambda_2 (\alpha - 1)$$

where the two multipliers are associated with the two constraints $-\alpha \leq 0$ and $\alpha - 1 \leq 0$. The first order conditions are

$$pu'(\alpha w - d)w + (1-p)u'(\alpha w + (1-\alpha)R)(w - R) - \lambda_1 + \lambda_2 = 0$$

$$-\alpha \leq 0; \lambda_1 \leq 0$$

$$\alpha - 1 \leq 0; \lambda_2 \leq 0$$

where the latter two conditions hold with complementary slackness. The first order condition when $0 < \alpha < 1$ is

$$pu'(\alpha w - d)w + (1-p)u'(\alpha w + (1-\alpha)R)(w - R) = 0$$

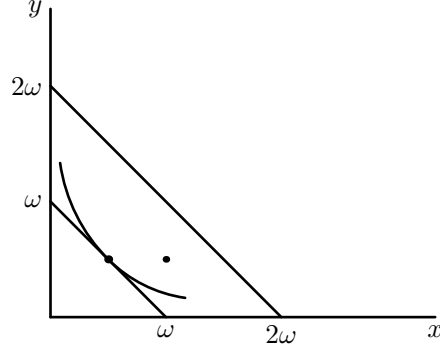
since the two multipliers have to be zero by complementary slackness. Then letting $\alpha[p]$ be the function that gives the α that satisfies this condition for each value of p . Differentiating implicitly with respect to p gives

$$\begin{aligned} & pu''(\alpha w - d)w^2 \frac{d\alpha}{dp} + u'(\alpha w - d)w + \\ & (1-p)u''(\alpha w + (1-\alpha)R)(w - R)^2 \frac{d\alpha}{dp} - \\ & u'(\alpha w + (1-\alpha)R)(w - R) = 0 \end{aligned}$$

Solving this for $\frac{d\alpha}{dp}$ gives

$$\frac{d\alpha}{dp} = \frac{u'(\alpha w - d)w - u'(\alpha w + (1-\alpha)R)(w - R)}{pu''(\alpha w - d)w^2 + (1-p)u''(\alpha w + (1-\alpha)R)(w - R)^2}$$

The denominator of this expression is negative because $u'' \leq 0$. To show the numerator is positive note that $\alpha w - d < \alpha w + (1-\alpha)R$ and $w > w - R$. Then since u is concave $u'(\alpha w - d) > u'(\alpha w + (1-\alpha)R)$ so $u'(\alpha w - d)w > u'(\alpha w + (1-\alpha)R)(w - R)$. So $\frac{d\alpha}{dp} \leq 0$.



2. The firm chooses (x, y) to maximize profits, i.e.,

$$\max px + y$$

subject to $y = 1 - x^2$. The solution is $-2x = -p$ or $x = \frac{p}{2}$. This gives $y = 1 - \frac{p^2}{4}$ and makes profits equal to $\frac{p^2}{2} + 1 - \frac{p^2}{4} = 1 + \frac{p^2}{4}$. From the Cobb Douglas property of preferences, demand for good x by each consumer is

$$\frac{\alpha}{\alpha + \beta} \frac{1 + \frac{p^2}{4}}{2p}$$

So setting the sum of the demands of both consumers to the total supply gives

$$\frac{\alpha}{\alpha + \beta} \frac{1 + \frac{p^2}{4}}{p} = \frac{p}{2}$$

which implies

$$\frac{\alpha}{\alpha + \beta} + \frac{\alpha}{\alpha + \beta} \frac{p^2}{4} = \frac{2p^2}{4}$$

Then solving for p gives $\sqrt{\frac{4\alpha}{\alpha + 2\beta}} = p$. Total production of good x is then $\frac{1}{2} \sqrt{\frac{4\alpha}{\alpha + 2\beta}}$, good y is $1 - \frac{1}{4} \frac{4\alpha}{\alpha + 2\beta} = \frac{2\beta}{\alpha + 2\beta}$. Consumption of good x for each consumer is $\frac{1}{4} \sqrt{\frac{4\alpha}{\alpha + 2\beta}}$ consumption of good y is $\frac{\beta}{\alpha + 2\beta}$.

3. Consumer 1 chooses x to maximize $x^{\frac{1}{2}} (\omega - x)^{\frac{1}{2}}$ since he knows consumer 2 can't produce any of the public good. The solution is $x = \omega/2$ (use the first order condition) so that $y = \omega/2$. Consumer 2 has consumption $(\omega, \omega/2)$ since he can't produce the public good. This solution is shown in Figure 1 For the second part, the firm maximizes $(p_1 + p_2)y + x$ subject to the constrain that $y = 2\omega - x$. If the sum of the prices of the public goods is not equal to 1, the firm will produce at one of the extremes - either only public good, or only private good. Since consumers have

Cobb-Douglas preferences, their demands for both goods will be strictly positive at all prices. So there must be excess demand for one of the two goods unless the sum $p_1 + p_2 = 1$. Total profits are then 2ω and each consumer has income ω to spend. Again using properties of Cobb Douglas, they will demand different levels of the public good unless they face the same prices, so $p_1 = p_2 = \frac{1}{2}$. This gives equilibrium demand for the private good as $\omega/2$ for both consumers. Using either the Cobb-Douglas formula, or the production possibilities frontier, the consumption of the public good equal to ω . In this solution, consumer 1 goes from the consumption bundle $(\frac{\omega}{2}, \frac{\omega}{2})$ to $(\frac{\omega}{2}, \omega)$ so is better off. Consumer 2 goes from $(\omega, \frac{\omega}{2})$ to $(\frac{\omega}{2}, \omega)$ which means that she is in the same position in either case.