

A proposer and responder share a monetary payoff of size 1. The utility from the monetary payoff follows the form of loss aversion. More specifically, we assume the monetary payoff has a linear form and the reference structure satisfies constant loss aversion (Tversky and Kahnman 1991), i.e.

$$(0.1) \quad V(s, r, \lambda) = \begin{cases} s - r & \text{if } s \geq r \\ \lambda(s - r) & \text{if } s < r \end{cases}$$

where s is the share of the monetary payoff received, and r is the reference point. The term $\lambda \geq 1$ is the degree of loss aversion. The variable λ is assumed to be privately known to the agent, with distribution G on a compact support bounded below by 1. We'll assume throughout that G is continuously differential - in particular, that it has a continuous derivative.

We'll add two additional behavioral terms. First, we'll add a fairness (spitefulness) component to a player's payoff whenever his or her payoff is smaller than the payoff of the other party. To keep things simple, we'll just add this as a fixed cost. Alternatively when the player's payoff is higher than the payoff of the other player, we'll add a competitiveness term as follows

$$V(x, y, \lambda, r) = \begin{cases} x - r - \frac{1}{2} & 0 \leq x < y; x \geq r \\ \lambda(x - r) - \frac{1}{2} & 0 < x < y; x < r \\ x - r + \alpha(x - y) & x \geq y; x \geq r \\ \lambda(x - r) + \alpha(x - y) & x \geq y; x < r \\ -\lambda r & x = y = 0. \end{cases}$$

In this formula, r is the reference offer. The reference offer is endogenously determined from the equilibrium distribution of offers. However, instead of using the entire distribution of offers to characterize this reference offer, we are going to use the lowest offer in the support of the equilibrium offer distribution. In words, this will be the highest offer having the property that responders expect to receive something at least as good with probability 1.

Notice two things about this reference offer. First, a responder who receives any offer perceives a cost λr to rejecting that offer. This cost is modelled as a lost *offer*, not a lost payoff. A responder who receives the reference offer r will be quite unhappy with it because of spitefulness. We don't include this spitefulness in calculating the responder's ex ante perception of the value of the opportunity.

Second, by using the lowest offer in the support of the equilibrium offer distribution as the reference point, we are taking the approach that assumes that the value of the bargaining opportunity is defined ex ante by a payoff the responder expects to receive *for sure*. We will show below how our results continue to hold when we instead assume the reference offer is defined by something that the responder expects to receive with very high probability.¹

These reference offers depend on the equilibrium of the game. However, some properties of these offers are straightforward. We'll just embed them in our description of the model so that our formal definition of equilibrium can be simplified.

First notice that whatever the responder's reference offer happens to be, he will accept an offer $s < \frac{1}{2}$ as long as

$$s - r - \frac{1}{2} \geq -\lambda r$$

If the responder is not loss averse (i.e $\lambda = 1$), this reduces to $s \geq \frac{1}{2}$. In other words, a responder who is not loss averse will reject all offers smaller than $\frac{1}{2}$ (though she will accept an offer equal to $\frac{1}{2}$). This is independent of what the reference offer is. On the other hand, for any offer less than $\frac{1}{2}$, there will be a subset of responders with low enough loss aversion (sufficiently close to 1) who would reject that offer. As long as the support of G includes 1, it implies that the only offer that the proposer can be

¹To see why this might matter, suppose the responder expects to receive an offer of .1 with probability ϵ while all other offers are at least .3, where ϵ is some very small number. Then the responder is close to being sure that he will receive an offer of at least .3. In this case, it would seem more reasonable to define the reference offer as .3. We are going to show that this situation won't arise here.

confident will be accepted for sure is $\frac{1}{2}$. We'll proceed with that presumption in what follows and take the reference offer for proposers to be equal to $\frac{1}{2}$.

As with the proposer, the responder's degree of loss aversion λ is private information, drawn independently from the distribution G (the same G that applies to proposers). The parameter α is common knowledge.

Since all offers are interpreted as shares, we refer in the rest of the paper to s as the responder's share of the surplus gain. Since offers above $\frac{1}{2}$ are always accepted, we focus on offers in the range $[0, \frac{1}{2}]$. The payoff to a proposer who offers a share s ($\leq \frac{1}{2}$) to the responder, when this offer is accepted, is then given by $(1-s) - \frac{1}{2} + \alpha(1-2s)$. When an offer is rejected, on the other hand, the proposer suffers a loss of $-\frac{\lambda}{2}$.

Let $Q(s)$ be the probability with which a proposer expects a share s to be accepted. The proposer is then intent on maximizing

$$(0.2) \quad Q(s, r) \left((1-s) - \frac{1}{2} + \alpha(1-2s) \right) - (1-Q(s, r)) \left(\frac{\lambda}{2} \right).$$

After receiving an offer s , the responder accepts it if and only if

$$(0.3) \quad (s-r) - \frac{1}{2} \geq -\lambda r,$$

when $s \geq r$ and

$$\lambda(s-r) - \frac{1}{2} \geq -\lambda r$$

otherwise.

A strategy rule for the proposer is a function $s(\lambda)$ that describes the offer made by each his possible types. A strategy rule for the responder is a type dependent acceptance set. Let $\alpha(\lambda)$ be the lowest offer in this acceptance set for a responder of type λ . Since the acceptance set is defined by (0.3) above, we have

$$Q(s, r) = \Pr \{s \geq \alpha(\lambda)\} = \Pr \left\{ (s-r) - \frac{1}{2} \geq -\lambda r \right\} = \Pr \left\{ \lambda \geq \frac{\frac{1}{2} + r - s}{r} \right\}$$

if $s \geq r$ and

$$Q(s, r) = \Pr \left\{ \lambda(s-r) - \frac{1}{2} \geq -\lambda r \right\} = \Pr \left\{ \lambda \geq \frac{1}{2s} \right\}$$

otherwise.

Simplifying gives

$$(0.4) \quad Q(s, r) = \begin{cases} 1 - G\left(\frac{\frac{1}{2} + r - s}{r}\right) & s \geq r \\ 1 - G\left(\frac{1}{2s}\right) & \text{otherwise.} \end{cases}$$

From this expression we get the following straightforward properties of Q :

Lemma 1. *The probability $Q(s)$ with which an offer of s accepted is an increasing function of s . Furthermore, $Q(s)$ is increasing in the reference offer provided $s \geq r$.*

Straightforward calculations give the derivative of Q as

$$(0.5) \quad Q'(s, r) = G' \left(\frac{\frac{1}{2} + r - s}{r} \right) \frac{1}{r}$$

when $s \geq r$ and

$$(0.6) \quad Q'(s, r) = G' \left(\frac{1}{2s} \right) \frac{1}{s^2}$$

otherwise. Notice that since $\frac{1}{r^2} > \frac{1}{r}$, the left derivative of Q at r is strictly larger than the right derivative. The probability function $Q(s)$ is therefore continuous but kinked at $s = r$, at which point the function becomes flatter.

Lemma 2. *The derivative of the function $Q(s, r)$ is increasing (decreasing, constant) in s when G is concave (convex, both). If G is convex or linear, $G'(s)$ is decreasing in r provided $s \geq r$. The left derivative of $Q(s, r)$ at r is strictly larger than the right derivative.*

Since the acceptance rule for the responder is straightforward, we can suppress it and define a perfect Bayesian equilibrium for the game as a strategy rule $s^* : \overline{G} \rightarrow [0, 1]$ for proposers, where \overline{G} is the support of G , and a reference point r^* for the responder such that

$$r^* = \inf_{\lambda} \{s^*(\lambda)\}$$

and

$$Q(s^*(\lambda), r) \left(1 - s^*(\lambda)(1 + 2\alpha) + \alpha - \frac{1}{2}\right) + \{1 - Q(s^*(\lambda))\} \left(-\lambda \frac{1}{2}\right) \geq$$

$$Q(s', r) \left(1 - s'(1 + 2\alpha) + \alpha - \frac{1}{2}\right) + \{1 - Q(s')\} \left(-\lambda \frac{1}{2}\right).$$

for each λ and each alternative offer s' .

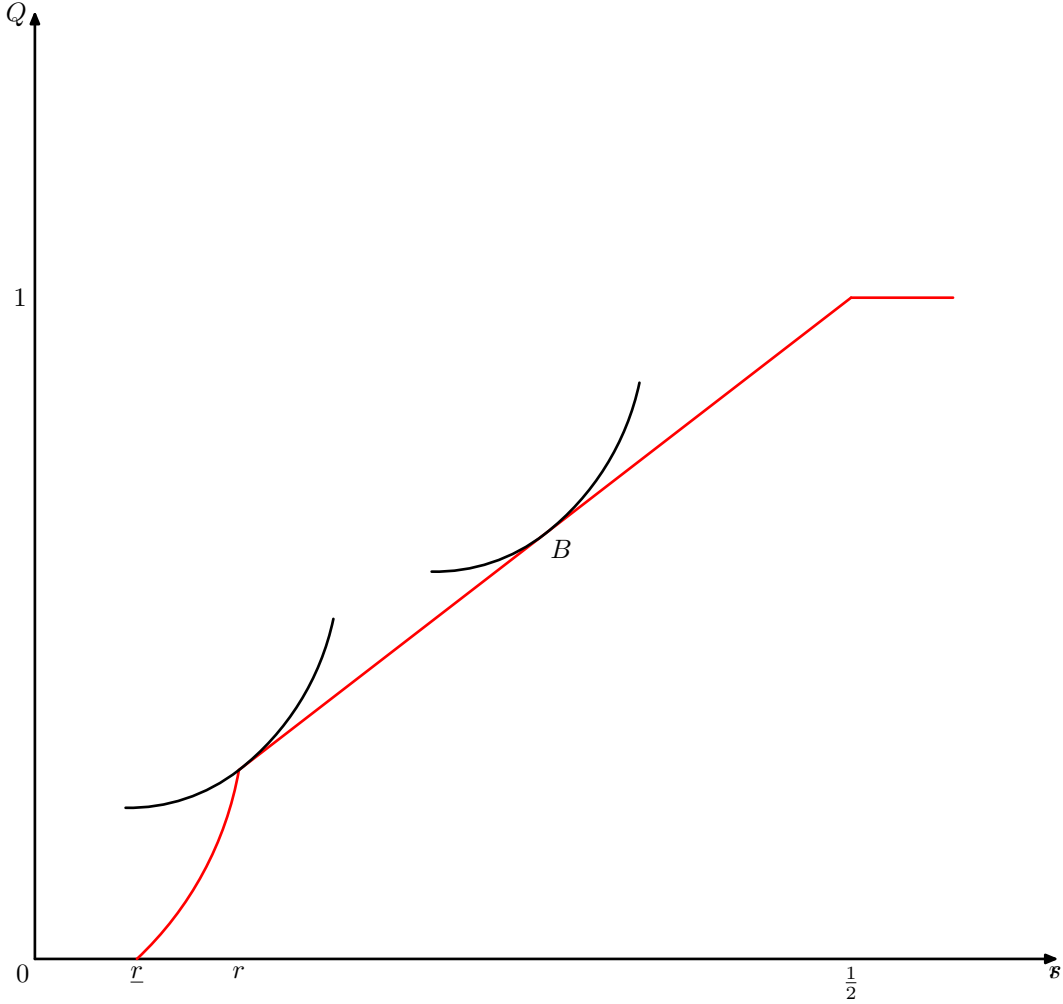
In this definition, Q is given by (0.4), and we assume that all offers in the range of s^* are less than or equal to $\frac{1}{2}$.

Equilibrium. For the most part, the equilibrium is straightforward. The function $Q(s, r)$ defines a kind of budget set that determines what (\tilde{Q}, \tilde{s}) pairs are attainable for the proposer. The proposer then chooses a (\tilde{Q}, \tilde{s}) pair that that maximizes

$$(0.7) \quad \tilde{Q} \left(1 - \tilde{s}(1 + 2\alpha) + \alpha - \frac{1}{2}\right) + (1 - \tilde{Q}) \left(-\lambda \frac{1}{2}\right)$$

subject to $Q(\tilde{s}, r) = \tilde{Q}$.

To visualize this equilibrium, it might help to look at the following Figure:



The red line represents the function $Q(s, r)$. There is some offer low enough that no one will accept it. This is \underline{r} in the figure. The function rises until it hits the reference offer at r , at which point it discontinuously flattens, then rises until $s = \frac{1}{2}$, at which point the probability of acceptance becomes 1 and can rise no further. Each proposer chooses an offer at which their iso-surplus curve is tangent to this 'budget line'. Since each proposer is maximizing a continuous function over a closed set, each proposer can find a best offer. Generally, these offers might be below the reference offer r or at $\frac{1}{2}$. Making an offer above $\frac{1}{2}$ is obviously a dominated strategy given the assumptions about proposer's preferences.

The solution to the proposer's problem is a non-decreasing function of the proposer's loss aversion. To see this, consider a proposer facing a trade-off between the share s that he offers the responder, and the probability as described above. For any λ , the solution to this problem occurs where an iso-surplus curve of the surplus function described in (0.7) is tangent to the function $Q(s, r)$ defined by (0.4).

The slope of an iso-surplus curve can be found by totally differentiating (0.7) with respect to \tilde{s} and \tilde{Q} , then setting the result to 0 and solving for $\frac{d\tilde{Q}}{d\tilde{s}}$. The result is

$$(0.8) \quad I'(\tilde{s}, \tilde{Q}, \lambda) = \frac{\tilde{Q}(1+2\alpha)}{\{(1-\tilde{s}(1+2\alpha)+\alpha) + \frac{1}{2}(\lambda-1)\}}$$

which is increasing in \tilde{s} and \tilde{Q} but falling in λ . In this formula

$$I(\tilde{s}, \tilde{Q}, \lambda : K) \equiv \left\{ \tilde{Q} : \tilde{Q} \cdot \left(1 - \tilde{s}(1+2\alpha) + \alpha - \frac{1}{2} \right) + (1-Q') \left(-\lambda \frac{1}{2} \right) = K \right\}.$$

From the formula given in (0.8) it is evident that an increase in λ (the proposer's loss aversion) makes this slope flatter. This gives the usual single crossing property that guarantees:

Lemma 3. *In every equilibrium, the proposer's strategy s^* is a non-decreasing function of the proposer's loss aversion λ .*

As we have argued, the best reply function exists for any reference offer $r \leq \frac{1}{2}$. All that remains is to verify that there is a fixed point for r having the property that when r is the reference offer, the lowest offer in the range of the best reply function is r . Since the best reply function is always non-decreasing by Lemma 3, all we need to do to accomplish this is to verify that the least loss averse proposer (the one for who $\lambda = 1$ will want to make the offer r .

Proposition 4. *If $G'(\bar{\lambda}) > 0$, where $\bar{\lambda}$ is the highest values in the support of G , and $G'(1)$ is bounded above by 1 then the set of equilibrium is non-empty, and every equilibrium has $0 < r < \frac{1}{2}$ and $s^*(\lambda)$ is a non-decreasing function with $s^*(1) = r$.*

Proof. As described above, each proposer chooses s to maximize (0.2) subject to (0.4). Since (0.4) is continuous on $[0, 1]$ and (0.2) is jointly continuous in (s, Q) , a best reply strategy rule $s(\lambda, r)$ exists for any $r \in [0, \frac{1}{2}]$. For (0.8), $s(\lambda, r)$ is non-decreasing.

Since r is the lowest point in the support of the equilibrium strategy, it must be offered by the least loss averse (loss-neutral) proposer. This means that any r at which the loss-neutral proposer cannot increase his expected surplus by offering a share other than r can be used to construct an equilibrium. The argument we use is to try to find conditions under which the highest iso-surplus curve for the proposer of type $\lambda = 1$ is tangent to the function $Q(s, r)$ at the point r .

From (0.5) and (0.6) this will be true if

$$G' \left(\frac{1}{2s} \right) \frac{1}{2s^2} \Big|_{s=r^-} = Q'(s, r)|_{s=r^-} \geq I'(s, Q(s, r), \lambda)|_{s=r, \lambda=1} \geq Q'(s, r)|_{s=r^+} = G' \left(\frac{\frac{1}{2} + r - s}{r} \right) \frac{1}{r} \Big|_{s=r^+}$$

or

$$(0.9) \quad G' \left(\frac{1}{2r} \right) \frac{1}{2r^2} \geq \frac{\{1 - G(\frac{1}{2r})\}(1 + 2\alpha)}{\{(1 - r(1 + 2\alpha) + \alpha)\}} \geq G' \left(\frac{1}{2r} \right) \frac{1}{r}.$$

Observe that from (0.3) and the assumption that the support of G is compact, there is some reference value \underline{r} low enough that the most loss averse responder in the support of G would be just indifferent between accepting and rejecting \underline{r} . Since G is assumed continuously differentiable, this offer is accepted with 0 probability. By the assumption that $G'(\bar{\lambda})$ is positive, $Q'(\underline{r}, \underline{r}) > 0$.

However, $I'(\underline{r}, 0, 1) = 0$, which can be seen by looking at the inner term in (0.9). Since the iso-surplus curve of the least loss averse proposer is flat at \underline{r} , while $Q'(\underline{r}, \underline{r}) > 0$, the least loss averse proposer has a profitable deviation if the reference offer is \underline{r} .

At the other extreme, if the reference offer $r = \frac{1}{2}$ we have

$$G'(1) \frac{1}{2} < \frac{1 + 2\alpha}{1 - \frac{1}{2}(1 + 2\alpha) + \alpha} = \frac{1 + 2\alpha}{\frac{1}{2}} > 3.$$

The implication is that $r = \frac{1}{2}$ can't be an equilibrium because $I'(\frac{1}{2}, 1, 1)$ is strictly larger than either the right or left derivatives of $Q(\frac{1}{2}, \frac{1}{2})$ (since both right and left derivatives are equal to $\frac{1}{2}$ at that point). This indicates that the least loss averse proposer has a profitable deviation to a lower offer.

In between, the three functions in (0.9) are continuous. Then by the Intermediate Value Theorem there is at least one point at which

$$G' \left(\frac{1}{2r} \right) \frac{1}{2r^2} = \frac{\{1 - G(\frac{1}{2r})\}(1 + 2\alpha)}{\{(1 - r(1 + 2\alpha) + \alpha)\}}$$

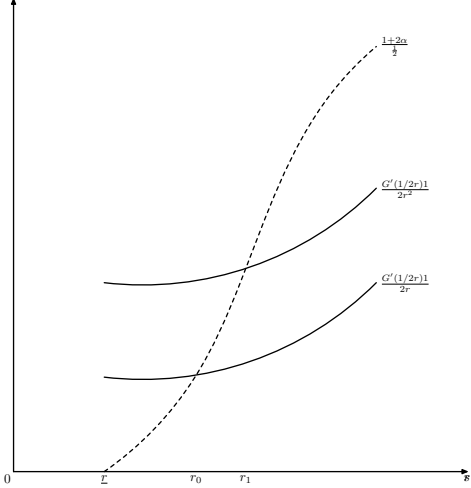
and a second point where

$$\frac{\{1 - G(\frac{1}{2r})\}(1 + 2\alpha)}{\{(1 - r(1 + 2\alpha) + \alpha)\}} = G' \left(\frac{1}{2r} \right) \frac{1}{r}.$$

These points are different because $G' \left(\frac{1}{2r} \right) \frac{1}{2r^2} > G' \left(\frac{1}{2r} \right) \frac{1}{r}$ whenever $r < \frac{1}{2}$.

The function $\frac{\{1-G(\frac{1}{2r})\}(1+2\alpha)}{\{(1-r(1+2\alpha))+\alpha\}}$ is strictly increasing, but the functions $G' \left(\frac{1}{2r} \right) \frac{1}{2r^2}$ and $G' \left(\frac{1}{2r} \right) \frac{1}{r}$ have ambiguous signs that depend on how the derivative G' varies. Let $r_0 < r_1$ be the largest pair of intersection points of these functions. By construction the inequalities (0.9) hold for any reference offer between r_0 and r_1 , which establishes the existence of a full equilibrium. \square

The determination of the reference point can be understood with the help of the following diagram. The dashed line describes the slope of the iso surplus curve of the least loss averse proposer at the reservation point, as the reservation point varies from \underline{r} to $\frac{1}{2}$. Since $G \left(\frac{1}{2r} \right) = 1$, the slope of this iso-surplus curve starts at 0 then rises to $\frac{1+2\alpha}{2}$.



The implication is that there is a continuum of equilibrium with reference points between r_0 and r_1 , the lowest and highest intersection points of the lower solid line and the dashed line, and the higher solid line and the dashed line. To see the logic, when $r > r_1$, the slope of the least loss averse proposer's iso-surplus curve is strictly larger than the left derivative of the function Q at the reservation point. So this proposer would strictly increase his payoff by cutting his offer. Conversely, at reservation values below r_0 , the least loss averse proposer will strictly gain by raising his offer.

There may be additional equilibria if $G' \left(\frac{1}{2r} \right) \frac{1}{2r^2}$ and/or $G' \left(\frac{1}{2r} \right) \frac{1}{r}$ intersect $\frac{\{1-G(\frac{1}{2r})\}(1+2\alpha)}{\{(1-r(1+2\alpha))+\alpha\}}$ at more than two points. The reference point at r_0 has the iso-surplus curve tangent to the Q function on the right. In other words, the least loss averse proposer has an iso-surplus curve at r that has the same slope as the function Q to the right of r . That means that all other proposers will make offers higher than r . For any of the other equilibrium reservations points that are larger than r_0 but less than or equal to r_1 , there will be a positive mass of proposer types who will make offers equal to the reserve price. If data from experiments can be explained by this feature it will exhibit pooling around the lowest observed offer.

One last point about this multiplicity is that it can be reduced by changing the model of the reference offer slightly. The reference offer is the highest offer such that responders expect that with probability 1 they will receive an offer at least as good. One might change the definition slightly so that the reference offer is the highest offer such that responders expect to get something better with probability 1. The only reference offers that satisfy this criteria are those for which the least risk averse proposer's iso-surplus curve is tangent to Q from the right (the intersection at r_0 in the picture). As described above, the probability that a proposer makes a better offer than the reference offer is equal to 1. At all the reference offers above r_0 there will be a strictly positive probability that the proposer will make an offer exactly equal to r . We return to this distinction in the next section.

Equilibrium with Uniformly Distributed types. To illustrate the main comparative static result, we'll revert to the assumption that proposer and responder types are uniformly distributed in this

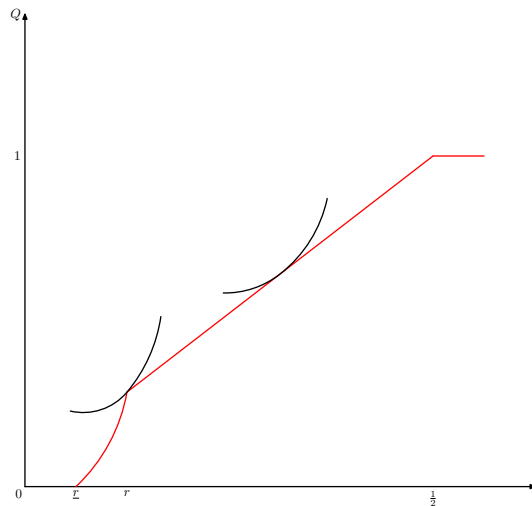
section. The results could also be shown if there is not too much variability in the derivative of the distribution function G or if it is concave. We'll focus on the uniform case just because it makes the analysis simple. We'll also focus on those equilibria where there is pooling both at $\frac{1}{2}$ and at the reference point r - primarily because this pooling was apparent in the data. Pooling at $\frac{1}{2}$ requires the upper bound of the distribution of loss aversion to be large (i.e., it is possible for proposers to be very loss averse).

In the uniform case, an offer equal to the reference offer r will be accepted only if

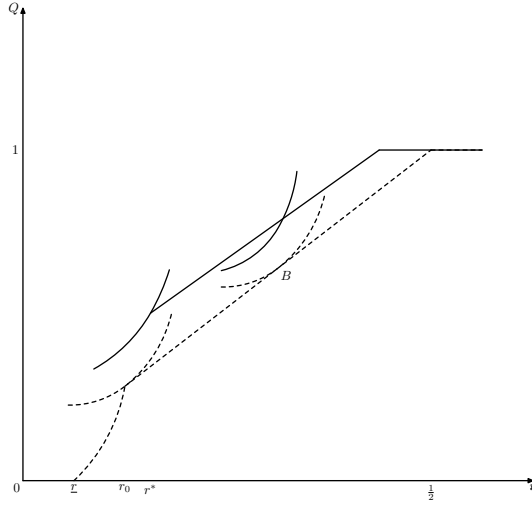
$$-\frac{1}{2} \geq -\lambda r.$$

This means that in every equilibrium, the reference offer has to exceed $\underline{r} = \frac{1}{2\lambda}$. The values for r_0 and r_1 can be calculated in a straightforward way by simplifying (0.9) using the uniform assumption.

There is an equilibrium with reference point is r_0 in which the least loss averse proposer has an iso-surplus curve that is just tangent to the function Q at r_0 . However, for reasons that will become clearer in the next section, we focus on equilibrium in which there is some pooling at the reference offer. The corresponding picture resembles Figure and appears here:



The comparative static exercise we are interested in is increasing the reference offer, r . By Lemma 1 and Lemma 2, the function $Q(s, r)$ shifts upwards and becomes flatter because of the fact that G is linear. From (0.8), the iso-surplus curve for any proposer at any offer s becomes steeper when the reference offer rises. The consequence of all this is that the iso-surplus curve for each proposer will be strictly steeper than the function $Q(s, r)$ at the offer the proposer made in the old equilibrium. The two empirical implications are that all offers will be accepted with higher probability and each proposer type will make a lower offer in the new equilibrium. The changes look approximately like those in the following diagram, where the dashed lines represent the equilibrium at the old reference offer, while the solid lines illustrate the way the solution to the proposer's problem changes after the alteration in the reference offer.



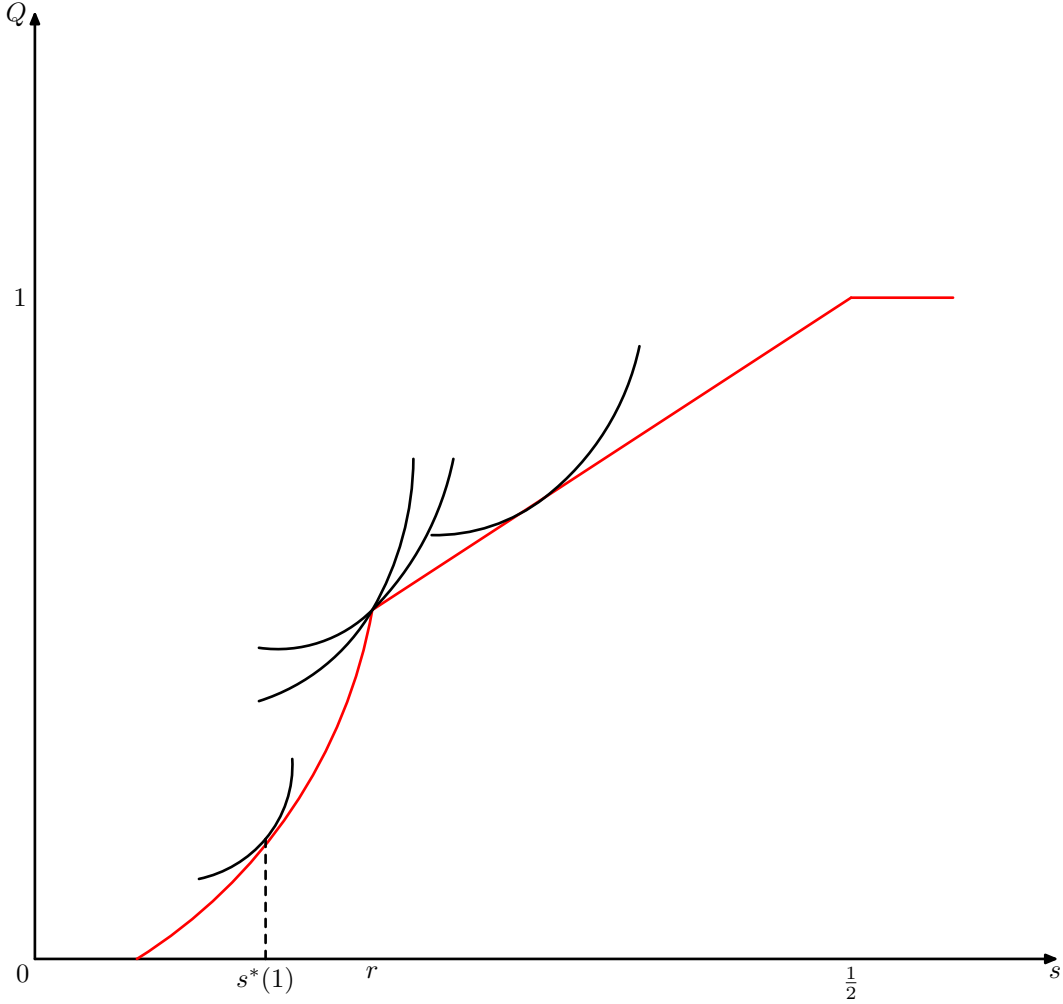
Robustness. Our analysis of the reference offer relies on the idea that a reference offer is something a responder (or proposer for that matter) expects to get for sure. This ignores most of the properties of the equilibrium distribution. For example, one might imagine modelling the reference offer as the offer responders *expect* to receive, rather than one they are sure to receive.²

In this section, we'll examine a richer model of the reference offer by allowing it to be determined by the τ^{th} quantile of the equilibrium distribution of offers. In particular, we'll assume the reference offer r_τ is the highest offer such that the probability of being offered something at least as good is $1 - \tau$.

In the first part of the paper, the reference offer was one in which the responder expected something at least as good with probability 1. This is a special case in which we would say that $\tau = 0$ - i.e., we use the 0^{th} quantile of the distribution of offers to parameterize the reference point. Alternatively, if $\tau = \frac{1}{2}$, then the responder expects to receive an offer at or above the median half of the time.

Lemma's 1 through 3 describe properties of the proposer's best reply conditional of a reference offer, so they continue to apply to the situation here. The primary difference that emerges when $\tau > 0$ is that offers below the reference offer may appear along the equilibrium path. As the probability of acceptance $Q(s, r)$ is kinked at r , there will generally be pooling at the reference offer as illustrated in the following figure:

²They receive the offer r for sure in the sense that they expect they will receive an offer r for sure, and they might get more as well.



Full equilibrium requires an additional fixed point condition which is that the proportion $1 - \tau$ of best replies of proposers must lie at or above the reference offer.

The left and right derivatives of Q are given by

$$(0.10) \quad Q'(s, r) = G' \left(\frac{\frac{1}{2} + r - s}{r} \right) \frac{1}{r}$$

when $s \geq r$ and

$$(0.11) \quad Q'(s, r) = G' \left(\frac{\frac{1}{2}}{s} \right) \frac{1}{s^2}$$

otherwise.

To simplify, we assume again that G is uniform, so that the left derivative of Q evaluated at r is $\frac{1}{2r^2}$ while the right derivative is $\frac{1}{r}$, both of which are decreasing functions of r .

The slope of the proposer's iso-surplus curve is

$$\frac{Q'(1 + 2\alpha)}{\{(1 - s'(1 + 2\alpha) + \alpha) + \frac{1}{2}(\lambda - 1)\}}$$

which must also be a decreasing function of r since Q' is decreasing.

Now, to verify the existence of an equilibrium when the reference offer is the offer at the τ^{th} quantile of the equilibrium offer distribution, we can follow the approach adopted in the proof of Proposition 4. By Lemma 3, s^* if it exists is non-decreasing. So the condition that the reference point has to satisfy is

$$Q'(s, r)|_{s=r^-} \geq I'(s, Q(s, r), \lambda)|_{s=r, \lambda=G^{-1}(\tau)} \geq Q'(s, r)|_{s=r^+}.$$

In words, the iso-surplus curve of the proposer at the τ^{th} quantile of the distribution G has to have an iso-surplus curve that is at least as steep as the right derivative of Q evaluated at r and no steeper than the left derivative of Q evaluated at r . Of course, the derivative of Q is independent of λ so this reduces to

$$G' \left(\frac{1}{2r} \right) \frac{1}{2r^2} \geq \frac{\{1 - G \left(\frac{1}{2r} \right)\} (1 + 2\alpha)}{\left(1 - r(1 + 2\alpha) + \alpha + \frac{1}{2} (G^{-1}(\tau) - 1)\right)} \geq G' \left(\frac{1}{2r} \right) \frac{1}{r}.$$

It should be apparent that the only thing that has changed in this expression relative to (0.9) is the appearance of the constant $\frac{1}{2} (G^{-1}(\tau) - 1)$ in the denominator of the expression for the slope of the iso-surplus curve. This will reduce the expression in the middle of the set of inequalities relative to what it was in (0.9). If τ is small, this will cause both 'intersection points' to increase relative to what they were in the expression. In this sense, the set of equilibrium reservation offers will increase.

It is also worth while to note that that if we start in the case where τ is equal to zero, and the equilibrium reference offer is such that there is pooling of offers at the reference offer, then raising τ slightly will leave the equilibrium reference point unaffected. To see why, suppose that in the initial equilibrium when $\tau = 0$, 5% of all responder types pool at the equilibrium reference offer r . As required by the reference offer, responders are sure to get an offer at least as good as r in equilibrium.

Of course, this also means they will get an offer at least as good as r with probability 95%, so r will also be an equilibrium for any $\tau < .05$. As described above, increasing τ above 0 will also create some new equilibrium reference points by shifting the intersection point at r_1 to the right.