Econ 600
Assignment 1

1. A consumer is trying to decide what color of paint to use to redecorate their apartment. He has decided on the color pink, and asks the paint store to give him the brightest shade of pink paint that they have. The sales person has no idea what that means so, she shows him samples of all the pink paints (there are many) that the store offers and asks him to say which sample he finds 'brighter'. He quickly finds that there are some shades he can't distinguish - he just can't tell which shade is brighter. If x and y are shades of pink, then we could define a binary relation $\succ$ with the property that $x \succ y$ if and only if shade x is brighter than shade y. Is this binary relation complete? Is it transitive? Suppose instead we define the binary relation $(! \succ)$ which is defined by $x\,(! \succ)\,y$ if and only if x is not brighter than y. Is this binary relation complete? Is it transitive? Assuming that there are a finite set of shades of pink, does either of these binary relations support a choice correspondence as we defined it in class (use the formal definition, not your intuition)? Just as a matter of intution, does it seem likely that consumer's favourite shade would be one that he could not distinguish from some other shades? If you think the answer is no (there is no formally right answer to this question), do you think there is any relation between this and the Ellsberg paradox?

2. Again using a finite set of alternatives, suppose the choices among alternatives are made by a family consisting of three people. They each have their own strict preferences over these alternatives (no family member is indifferent between any pair of alternatives), they vote. If some alternative from the choice set receives the majority of votes from the family members they choose it - otherwise they are deadlocked (like Democrats and Republicans) and can't make any choice. So use 'being deadlocked' as a choice. Suppose you can run all the experiments you like, and construct a choice correspondence from the resulting outcomes. Will it satisfy the weak axiom? (Either give a proof or provide a counterexample). Focussing only on the apparent preference relation you get by looking at experiments with only two options, will it be complete? transitive?

3. In the proof of existence of the utility function that we did in class, suppose that instead of using the 45^0 line to construct the utility function, we use a plane in which the first coordinate of the consumption bundle is equal to one (a flat line through the point $(1, 1)$. Given the assumptions we used in class, does the proof we used (utility is defined by the point where an indifference curve cuts the flat line through 1) there still work? Why or why not? If not, how would you strengthen the assumption to get the result.

4. Prove that if a consumer has transitive and complete preferences over a finite set $\mathcal{X}$, then the set $\mathcal{X}$ must have a best and worst element.

5. Let C be a choice correspondence that satisfies the weak axiom over a collection of experiments that includes all two and 3 elements sets, so that the choice correspondence supports a preference relation $\succeq_c$. Then use the preference relation to define a choice correspondence $C_{\succeq_C}$. Show that C is equivalent to $C_{\succeq_C}$. (This is all about writing a proof).

6. Let $\mathcal{P}$ be a collection of Anscombe Aumann lotteries (lists of state contingent lotteries over a common set of outcomes $\mathcal{X}$). Use all the regularity assumptions we used in class (except you don't need state uniformity) along with the idependence axiom to show that for any $p, p' \in \mathcal{P}$ $p \succeq p'$ if and only if there is a set of $S \times J$ constants $\{u_{sj}\}$ such that

$$\sum_{s=1}^{S}\sum_{j=1}^{J} p_{js}u_{sj} \geq \sum_{s=1}^{S}\sum_{j=1}^{J} p'_{js}u_{sj}.$$

7. In the Ellsberg problem suppose there are two urns. In the first urn, it is known that the probability of drawing a white ball is $x > \frac{1}{2}$ and the probability of drawing a black ball is $1 - x$. The second urn has 100 white and black balls in unknown proportions. As always the choice to be presented to the decision maker is to select an urn conditional on betting on one of the two colors. Our decision maker views the urn with an unknown number of white balls as one in which the number of white balls is either 25, 50 or 75. The way he evaluates this subjective uncertainty is to first calculate the expected utility associated with his bet (white or black) conditional on the number of white balls in the urn, then take a weighted average of these expected utilities. So if he chooses the ambiguous urn and is betting on white, his value is

$$\frac{1}{3}\{\text{Expected utlity of betting on white when there are 25 white balls}\}+$$

$$\frac{1}{3}\{\text{Expected utlity of betting on white when there are 50 white balls}\}+$$

$$\frac{1}{3}\{\text{Expected utlity of betting on white when there are 75 white balls}\}.$$

Suppose expected utility is computed using a strictly concave utility function, u, so that, for example, $\mathbb{E}\,(\text{betting on white}|\text{ there are 25 white balls}) = \frac{25}{100}u\,(10) + \frac{75}{100}u\,(0)$. If x is very close to $\frac{1}{2}$ which urn will this decision maker choose when betting on white? Which urn will he choose when he is betting on black and why?