End of Term Problem set
Problems added 2014
Econ 306

1. Lets reverse the directed search story and suppose that the firms approach the workers and make them offers. Firms have wages $w \in [0,1]$ that have been set long in the past, so they can't vary their wage offer when they meet a worker. The two firms don't know each other's wage. Each believe that the other firm has wage that is randomly drawn from the interval $[0,1]$ according to a continuously differentiable and monotonically increasing distribution function F. Each firm selects one and only one worker and offers to employ them at whatever their wage is. If both firms make a proposal to the same worker, then the worker selects the firm who offers the highest wage, or randomly chooses one of the firms if their wage is the same.
   A firm who hires a worker makes profit K if it hires a worker. The profit K is independent of the wage it offers, so all it wants to do is to hire a worker. Describe the Bayesian Nash equilibrium of this game.

2. Change the problem above and assume that firms can offer whatever wages they like. Each firm has a profit k which it knows, and which is unknown to the other firm. A firm who has profit k, offers a wage w and hires a worker gets payoff $k - w$. A firm who fails to hire a worker has payoff 0. Suppose that each firm believes the other firm's profit is drawn from a distribution F which is continuously differentiable and monotonically increasing. As in the previous problem, each firm makes an offer to one and only one of the two workers. When the worker has multple offers, she accepts the highest offer. In this case, the firm who offered the lower wage will not match and will earn payoff 0. Describe the Bayesian Nash equilibrium of this problem.

3. Use the deferred acceptance algorithm to match students to schools in the following problem:

$$UBC \rightarrow 1 \succ 3 \succ 2 \succ \emptyset \succ 4 \succ 5$$

$$SFU \rightarrow 2 \succ 3 \succ 1 \succ \emptyset \succ 4 \succ 5$$

$$UT \rightarrow 1 \succ 3 \succ 2 \succ \emptyset \succ 4 \succ 5$$

   and

$$1 \rightarrow UT \succ SFU \succ UBC \succ OUT$$

$$2 \rightarrow UT \succ UBC \succ SFU \succ OUT$$

$$3 \rightarrow UT \succ SFU \succ UBC \succ OUT$$

$$4 \rightarrow UT \succ SFU \succ UBC \succ OUT$$

$$5 \rightarrow UT \succ SFU \succ UBC \succ OUT$$

In this description, the $\emptyset$ means that the school would rather leave their spot unfilled than admit a student who is worse than $\emptyset$. So the schools preferences are interpreted to mean that students 4 and 5 do not meet their entry requirements. Do both the school proposing and school proposing versions of the algorithm. Are they the same? Explain how the schools could 'game' the algorithm by manipulating their entry standards.

4. In the problem described above, change the preferences so that the schools are willing to admit all 5 students (remove the $\emptyset$ option). Suppose, however, that each of the schools actually has two spots to fill instead of just one. So some of the schools should end up with two students. Use the student proposing version of the deferred acceptance algorithm to match students to schools.

5. Eight professional dancers - four male and four female - are to perform a synchronized dance in a pop music video. The director wants them to form pairs consisting of one dancer of each gender. The director doesn't care which pairs form, but he wants the pairs to stay together during the long period of rehearsals leading up to the performance. The performers all have strict preferences over partners. They look like this: for the women

	Kelby	Justin	Linden	Lysander
Britney	3	2	4	1
Taylor	4	2	1	3
Catness	1	3	2	4
Miley	1	4	3	2

;

and for the men

	Kelby	Justin	Linden	Lysander
Britney	2	3	2	4
Taylor	1	2	4	2
Catness	4	1	3	1
Miley	3	4	1	3

. In these tables, the numbers represent the rank of the corresponding person, so, for example, Britney's favorite partner is Lysander. You immediately think of stable matchings. Can you find them? (Hint: there are three of them).

6. Find the Bayesian Nash equilibrium of the first and second price auction and provide a formula for the case where there are n bidders whose values are uniformly distributed on $[0, 1]$. Give a mathematical argument to show that the equilibrium bid in the first price auction is smaller than it is in the second price auction. Write down a mathematical formula for the sellers expected revenue in each case and show that they are the same.

7. Do problem 7 again but assume that each bidder's value is distributed on $[0, 1]$ as $F(v) = v^2$.

8. In the second price auction version of our three bidder one seller problem, suppose that the seller sets a low reserve and the bidders are almost

convinced that the preferences are as we described them $v_1 > v_2 > v_3$. However suppose that bidders 1 and 2 are not quite sure of bidder 3's motivation. In particular, suppose that both of them assign a small probability (say 1/100) to the possibility that 3 is a 'religious' bidder who simply wants to submit the highest bid he can win or lose in order to show his devotion to the product. The second complication is that they just aren't sure how much money a religious bidder might have. Suppose that both of them think that a religious bidder has some amount of money $\tilde{m}$ which is uniformly distributed on the interval $[0, v_4]$ where $v_4 > v_1$. They don't think it is very likely that 3 is religious, but if he is, they expect him to bid whatever he has. Write down the expected payoffs of bidders 1 and 2 when they expect religious 3 to bid uniformly on the interval $[0, v_4]$ and the sensible 3 to bid a best reply to what they are doing. Are there any inefficient equilibrium outcomes in which 1 doesn't get the good even when 3 is sensible instead of religious. Can you see any advantage to 3 of cultivating an image as a religious bidder?