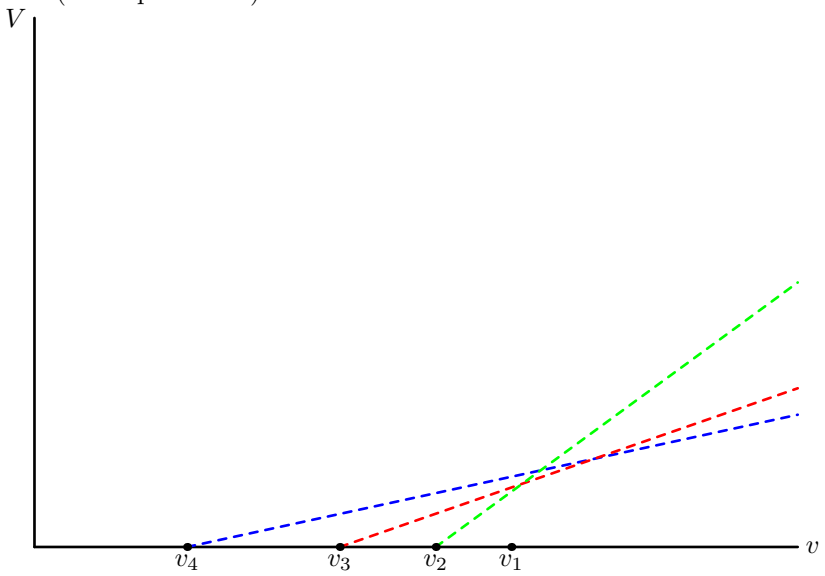
1. In a position auction, an equilibrium in which bidders bid their values may not exist at all. In the following diagram, the colored dash lines are interpreted as the lecture and bids are equal to values for all players. Write down the profitable deviations that are available to each player in this (non-equilibrium) outcome.



2. Can you give an algebraic condition (i.e., and inequality) that will ensure that there is a Nash equilibrium for this game in which each bidder bids their value?

3. Find the two Nash equilibrium in which the sellers expected revenue is highest and lowest

4. Using the diagrammatic method, show how to compute prices for the case where the bidder with the fourth highest value bids v_3 instead of v_4.

5. Write down a constrained maximization problem that shows how google would maximize its ad revenues from this action by setting slot prices so that each bidder prefers their assigned slot at the chosen slot prices. You don't have to solve this problem, just specify the objective and all the constraints.

6. Here is data from a position auction. There are three positions. The click through rates of the three positions are 7 for the first position, 3 for second position and 1 for the third position (in words, for every 100 page views, 7 users click on the first displayed ad, etc). There are 4 bidders. The money they expect to make from each viewer who clicks on an ad is 5,9,10 and 15. In other words, the bidder with value 15 expects to sell 15$ worth of stuff to anyone who clicks on their ad. The positions are sold using a second price auction, in other words the highest bidder is awarded the top

spot at a price equal to the bid of the second highest bidder, etc, without any reserve price. The values are all commonly know to the players

(a) Is there a Nash equilibrium for this bidding game in which each bidder bids his or her value? Explain fully why this is or isn't true.

(b) Find the equilibrium prices as we did in class that support a Nash equilibrium where the second and first highest bidders are indifferent between two positions?

(c) Pick a price between 5 and 9. Suppose the bidder whose value is 4 bids the price you choose, instead of 5. Find the bids of the other players that would support an equilibrium like the one we describe in class in which the higher valued bidders are indifferent between two positions.

(d) If Google wants to set a reserve price for its ads (in other words, you only get a position if you are one of the top three bidders and also submit a bid above the reserve price, what reserve price would it set.

7. Now following the second part of the reading on position auctions assume sellers' values are uniformly distributed on the interval $[0, 1]$.

   (a) Assuming firms use a monotonically increasing bidding rule, what is the expected quality of the firm who wins the top slot?

   (b) What is the expected quality of the firm who wins the second top spot.

   (c) What is the expected quality of the firm that fails to win a spot? How are these three related?

   (d) Now find the quality of the firm in the second top slot conditional on failing to trade with the firm in the top slot.
   Answer: Applying the formulas from the reading

8. Show that for the position auction where firms' qualities are distributed F on the interval $[0, 1]$, show that if the website is auctioning only a single slot instead of two slots, that the equilibrium bidding rule for firms in to bid their actual quality.