Assignment 2 - Econ306 - 2021

1. Use the Hungarian algorithm to find the optimal matching and potential for each of the following problems: (if you are determined not to work through it, you can use James' algorithm with sagemath).

	(0)	(0)	(0)	(0)
(0)	20	28	8	4
(0)	21	15	6	3
(0)	4	10	14	2
(0)	1	5	2	7

	(0)	(0)	(0)	(0)
(0)	28	20	8	4
(0)	21	15	6	3
(0)	14	10	4	2
(0)	7	5	2	1

To show that the row weights are part of a Nash equilibrium, you treat them as wage offers from workers followed by offers of employment made by the different employers to workers. Explain why a 'deviation' by a worker to a higher wage offer will not be profitable for a worker, especially how employers offers will change. When there is a deviation by a worker, do all employers have to change the offers in response, or only one of them? Does it matter whether you are interested in Nash equilibrium instead of subgame perfect equilibrium when you answer this question?

(1) Use the deferred acceptance algorithm to match students to schools in the following problem:

$$UBC \to 1 \succ 3 \succ 2 \succ \emptyset \succ 4 \succ 5$$

$$SFU \to 2 \succ 3 \succ 1 \succ \emptyset \succ 4 \succ 5$$

$$UT \to 1 \succ 3 \succ 2 \succ \emptyset \succ 4 \succ 5$$

and

$$1 \to UT \succ SFU \succ UBC \succ OUT$$

$$2 \to UT \succ UBC \succ SFU \succ OUT$$

$$3 \to UT \succ SFU \succ UBC \succ OUT$$

$$4 \to UT \succ SFU \succ UBC \succ OUT$$

$$5 \to UT \succ SFU \succ UBC \succ OUT$$

In this description, the $\emptyset$ means that the school would rather leave their spot unfilled than admit a student who is worse than $\emptyset$. So the schools preferences are interpreted to mean that students 4 and 5 do not meet their entry requirements. Do both the school proposing and school proposing versions of the algorithm. Are they the same? Explain how the schools could 'game' the algorithm by manipulating their entry standards.

(2) In the problem described above, change the preferences so that the schools are willing to admit all 5 students (remove the $\emptyset$ option). Suppose, however, that each of the schools actually has two spots to fill instead of just one. So some of the schools should end up with two students. Use the student proposing version of the deferred acceptance algorithm to match students to schools.

(3) Eight professional dancers - four male and four female - are to perform a synchronized dance in a pop music video. The director wants them to form pairs consisting of one dancer of each gender. The director doesn't care which pairs form, but he wants the pairs to stay together during the long period of rehearsals leading up to the performance. The performers all have strict preferences over partners. They look like this: for the women

	Kelby	Justin	Linden	Lysander
Britney	3	2	4	1
Taylor	4	2	1	3
Catness	1	3	2	4
Miley	1	4	3	2

;

and for the men

	Kelby	Justin	Linden	Lysander
Britney	2	3	2	4
Taylor	1	2	4	2
Catness	4	1	3	1
Miley	3	4	1	3

. In these tables, the numbers represent the rank of the corresponding person, so, for example, Britney's favorite partner is Lysander. You immediately think of stable matchings. Can you find them? (Hint: there are three of them).

(4) Find the Bayesian Nash equilibrium of the first and second price auction and provide a formula for the case where there are n bidders whose values are uniformly distributed on $[0,1]$. Give a mathematical argument to show that the equilibrium bid in the first price auction is smaller than it is in the second price auction. Write down a mathematical formula for the sellers expected revenue in each case and show that they are the same.

(5) Do problem 4 again but assume that each bidder's value is distributed on $[0,1]$ as $F(v) = v^2$.

(6) In the second price auction version of our three bidder one seller problem, suppose that the seller sets a low reserve and the bidders are almost convinced that the preferences are as we described them $v_1 > v_2 > v_3$. However suppose that bidders 1 and 2 are not quite sure of bidder 3's motivation. In particular, suppose that both of them assign a small probability (say 1/100) to the possibility that 3 is a 'religious' bidder who simply wants to submit the highest bid he can win or lose in order to show his devotion to the product. The second complication is that they just aren't sure how much money a religious bidder might have. Suppose that both of them think that a religious bidder has some amount of money $\tilde{m}$ which is uniformly distributed on the interval $[0, v_4]$ where $v_4 > v_1$. They don't think it is very likely that 3 is religious, but if he is, they expect him to bid whatever he has. Write down the expected payoffs of bidders 1 and 2 when they expect religious 3 to bid uniformly on the interval $[0, v_4]$ and the sensible 3 to bid a best reply to what they are doing. Are there any inefficient equilibrium outcomes in which 1 doesn't get the good even when 3 is sensible instead of religious. Can you see any advantage to 3 of cultivating an image as a religious bidder?

(7) Find the equilibrium bidding rule in a first price auction with reserve price $r > 0$ with n bidders whose values are independently and uniformly distributed on $[0, 1]$ according to a distribution F which is monotone increasing and differentiable.

(8) Suppose a seller is trying to sell to a single buyer whose valuation he/she doesn't know. Values are distributed as F on $[0, 1]$. What price should the seller offer in order to maximize expected revenue? How does it compare to the optimal reserve price in an auction with n bidders whose values are all independently and identically distributed F on $[0, 1]$. Given your answer, how does the optimal reserve price in an auction depend on the number of bidders?