

Negotiation and Take it or Leave it in Common Agency

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Abstract

This paper contrasts models of common agency in which principals compete in incentive contracts (that is, they make take it or leave it offers) with models where principals offer agents menus of incentive contracts from which the final contract is negotiated. It is shown that *pure strategy equilibria* in incentive contracts are robust to the possibility that principals might offer menus. In addition, a *no-externalities* condition is given such that *any* equilibrium allocation with menus can be supported with competition in incentive contracts. The no-externalities condition is restrictive, but it is shown that it applies in most well known common agency problems, including, for example, the Bertrand pricing problem.

Common agency provides a convenient way to model incentive problems in which there is competition. For example Bernheim and Whinston (1986a), Bernheim and Whinston (1986b) or Dixit, Grossman, and Helpman (1997) offer contracts to agents that make the transfer that the principal makes to the agent depend on the agent's effort. Delegation problems can be analyzed similarly (for example Prat and Rustichini (1998), Prat and Rustichini

(2000), Fershtman and Kalai (1997)). 'Supply function equilibria' Klemperer and Meyer (1989) can be interpreted as a common agency problem in which firms are the principals offering incentive contracts to the auctioneer who chooses quantities from each supplier to maximize apparent surplus. Multi-unit auction problems could be interpreted in much the same way¹ with the auctioneer as the agent. Non-linear pricing equilibria make the price depend on the quantity chosen Biais, Martimort, and Rochet (2000) or Martimort and Stole (1999a). Common agency need not involve incentive contracts, for example Parlour and Rajan (1997) model competition among lenders (the principals) for a single borrower (the agent) by having the lenders simply offer interest rates and loan amounts. Finally a simple Bertrand pricing problem could be interpreted as a common agency problem where the principals (sellers) simply compete in price for a single buyer (the agent).

All of the models adopt a common and very natural approach to modelling the competition among the principals - each of them offers a single incentive scheme to the agent. The ability of the agent to control his payoff is then exogenously determined by the conditions of the economic environment - for example in a Bertrand problem the buyer can choose whether or not to buy from each seller.

As natural as it might seem to do this, quite different outcomes can prevail when principals communicate with the agent before settling on a particular incentive contract. Whatever method the principal uses to negotiate an incentive contract effectively provides the agent with a menu of incentive contracts he or she could obtain by choosing different negotiation strategies. Since the agent's optimal choice from a menu can depend on what other principals have offered, the principal can use the menu to learn what other principals are doing, then improve his action or punish deviations. As all 'interesting' equilibrium allocations with general communications mechanisms and common agency can be supported as equilibrium allocations when principals offer menus², this paper compares outcomes when principals of-

¹A practical example occurs in a uniform price pay your bid auction with multiple units. Collusive equilibria can be supported if bidders submit bid prices that vary depending on how much output they are eventually awarded Klemperer (2000).

²Martimort and Stole (1999b) or Peters (2001a). More formally, one of two mutually exclusive things is possible - either an equilibrium allocation supported by some set of communication mechanisms is not robust with respect to the possibility that principals might use more complex mechanisms (so it is uninteresting), or the equilibrium allocation can also be supported as an equilibrium allocation when principals use menus. The formal

fer menus of incentive contracts with situations where they offer degenerate menus consisting of a single incentive contract. To make the terminology a little less awkward, I will adopt the natural terminology that a principal who offers a degenerate menu makes a *take it or leave it* offer. To emphasize the fact that restricting attention to menus is simply an analytical shortcut for analyzing equilibria with quite general communications mechanisms, I will sometimes say that a principal who offers a menu *negotiates* the incentive contract with his agent.

The possibility that principals might offer menus creates two difficulties for the more natural modelling approach in which principals are only allowed to make take it or leave it offers. First, since the richer set of mechanisms creates new deviations that aren't feasible with take it or leave it offers, equilibrium allocations supported by competition in take it or leave it offers may not be *robust* in the sense that they will no longer be supported as equilibria once menus are allowed. This possibility is somewhat startling - it raises the question whether the results and implications of much of the literature on common agency are simply artifacts of the modelling technique. On the other hand it also presents the possibility that negotiation may be a way to refine away some unwanted equilibria.

The first pair of theorems in this paper addresses this difficulty. Quite generally under common agency with symmetric information it turns out that *pure strategy* equilibria in take it or leave it offers are robust to the possibility that principals might offer menus. Since most of the literature has focussed on pure strategy equilibria, this result is especially useful.³ This result applies to most of the best known papers on common agency which focus on pure strategy equilibria and assume symmetric information. (For example, Bernheim and Whinston (1986a), Bernheim and Whinston (1986b) or Dixit, Grossman, and Helpman (1997). Symmetric information is a typical assumption in many interesting applications as well - for example Parlour and Rajan (1997)). If there are problems with the intuition that has emerged from this literature, they are problems of omission rather commission.

With asymmetric equilibria principals will always want to communicate with agents to learn about their tastes. Even when the principal can write incentive contracts, the actual contract the principal wants will typically

description of menus and communications mechanisms is given below.

³Mixed strategy equilibria are generally not robust. Examples are provided in Epstein and Peters (1999) that illustrate why. Furthermore, even pure strategy equilibria are not robust when there are multiple agents. For a counterexample see Peters (2001b).

depend on the agent's marginal disutility for effort, no matter what kinds of contracts the other principals offer. The problem that arises in this instance is that an agent's *type* in a common agency problem is not the same as it is in a single principal problem since the agent has market information that may be relevant to the principal. In general, it is difficult to describe type in such problems (see Epstein and Peters (1999)). The second theorem in this paper deals with this. Formulate the set of feasible mechanisms as the set of direct mechanisms that would be available to the principal in the same economic environment if he were the only principal. The second theorem says that any pure strategy equilibrium in common agency relative to this set of direct mechanisms is robust to the possibility that principals might offer more complex mechanisms (which offer the agent menus of direct mechanisms, or ask the agent to report his full type).

These two theorems by themselves provide strong support for the existing literature on common agency. It might not capture all relevant behavior, but the equilibria that it does characterize are accurate. Much of the recent literature on common agency has focussed on the fact that menus may be used to support equilibrium allocations that could not be understood using models where principals are forced to compete in take it or leave it offers alone. The question then arises whether there are interesting economic environments in which all interesting equilibrium behavior can nonetheless be captured by having principals compete in take it or leave it offers. The paper provides a condition, referred to as the 'no externalities' condition, which ensures that the payoffs associated with *pure strategy* equilibria relative to the set of menus can be supported as pure strategy equilibria with take it or leave it offers. The condition involves two restrictions. First, each principals' payoff should depend only on his own action and the effort of the agent but should not depend on the actions taken by other principals. Secondly, *conditional on effort*, the agent's ranking of the actions to be taken by any single principal should be independent of the actions taken by any other principal.

When the principal attempts to communicate with the agent about the market, he needs to provide the agent with incentives to report market information truthfully. When the agent is allowed to choose from a menu, the principal can 'learn' what is happening in the market if the agent's optimal choice from the menu depends on what is happening elsewhere in the market. New equilibrium allocations can be supported if, in addition, the payoffs of other principals depend on how the agent chooses from this menu. The no-externalities condition eliminates these two considerations, so when

competing relative to the set of menus, the principal does not bother trying to communicate with the agent because it is not incentive compatible for the agent to reveal his information. Even if it was, the agent's choice will not affect the payoffs of the other principals.

The remarkable aspect of this may be that so many standard agency problems that have been studied in the literature actually satisfy these conditions. For example, Bernheim and Whinston (1986b), Dixit, Grossman, and Helpman (1997), Martimort and Stole (1999b), as well as standard agency like models like the Bertrand model, or Klemperer and Meyer (1989) all satisfy the no-externalities assumption, as will almost any problem in which the principal offers only monetary incentives to affect the agent's effort. In all such problems, the principal's action can be interpreted as a transfer of money that depends on the agent's effort. These monetary transfers involve no externalities across firms, and the agent always prefers either the smallest, or the largest transfer (depending on whether the agent's effort is a demand or a supply). A more general example is provided below to illustrate the condition. On the other hand, the condition will not generally apply in Bernheim and Whinston (1986a) because the principal's action is a *distribution* of monetary transfers. The agent's ranking of distribution will typically depend on what other distributions they are mixed with. Some additional restrictions are provided so that the no externalities assumption does apply in this case.

1 Basic Properties

Common agency models have the following general structure: there are n principals dealing with a single agent. Each principal $j \in \{1, \dots, n\}$ controls a simple action in the set Y_j , while the agent takes some effort from a set E . It will be assumed that the sets Y_j and E are both compact metric spaces.

The principal can write *contracts* contingent on all or part of the effort level $e \in E$ taken by the agent. These contracts are referred to henceforth as *pay for effort contracts*. The principal may not be able to contract fully on agent effort - for example some effort is simply unobservable. It will be assumed that for each principal there is a collection of measurable equivalence classes \mathcal{E}_j whose union is E such that principal j is constrained to respond to each effort level in the same equivalence class the same way. Formally the set of feasible pay for effort contracts is a subset \mathcal{A}_j of the set

$\{a_j : E \rightarrow Y_j : a_j \text{ is } \mathcal{E}_j \text{ measurable}\}$ where \mathcal{E}_j measurable means measurable with respect to the σ -field generated by \mathcal{E}_j . The set of feasible pay for effort contracts will generally be a subset of the set of \mathcal{E}_j measurable functions because of physical constraints on the principal's actions. For example, a firm may not be able to charge a buyer who chooses not to buy from it. These physical constraints play no role in the discussion that follows.

The agent's preferences are private information and are parameterized by elements in some set Ω . We refer to elements of the set Ω as *valuations* instead of types though the valuations themselves may be more complex than simple willingness to pay. Principals commonly believe that the agent's valuation is distributed according to some distribution F on Ω .

Agents and principals have expected utility preferences. The payoff to principal $j \in \{1, \dots, n\}$ is represented by $v_j : \prod_{k=1}^n Y_k \times E \times \Omega \rightarrow [0, 1]$. For the agent, payoffs are represented by the function $u : \prod_{k=1}^n Y_k \times E \times \Omega \rightarrow [0, 1]$. It is assumed that the agent's payoff function is continuous in all its arguments.⁴

Principals compete with each other by offering the agent 'menus' of pay for effort contracts. A menu for principal j is a mapping $\gamma_j : \mathcal{A}_j \rightarrow \mathcal{A}_j$ such that either $\gamma_j(a) = a$ or $\gamma_j(a) = \bar{a}$ for some $\bar{a} \in \mathcal{A}_j$ and such that the set $\{y \in Y_j : y = \gamma_j(a)(e)\}$ is a compact subset of Y_j for all $e \in E$. The interpretation is that the agent names the pay for effort contract that he wants, and if it belongs to the 'menu' γ_j , he gets it. The graph of the correspondence from effort into action that is defined by the menu is required to be closed in order that the agent always have a favorite element in the menu. Let Γ_j^* be the set of menus. A *degenerate menu* for which $\gamma_j(a_j) = \bar{a}_j$ for all a_j is called a take it or leave it offer. Let $\Gamma_j^C \subset \Gamma_j^*$ denote the set of feasible take it or leave it offers for principal j .

Let $\Gamma = \prod_{j=1}^n \Gamma_j$ be the set of feasible mechanisms. We consider only situations where the set of feasible mechanisms is a subset of the set of menus Γ^* . The game form is as follows: (i) each principal simultaneously selects a mechanism from Γ ; (ii) after observing the mechanisms $\{\gamma_1, \dots, \gamma_n\}$ offered by each principal, the agent simultaneously sends a message to each principal (if that is possible) and selects his effort; (iii) all payoffs are realized.

The message that the agent sends to each principal, and the effort that

⁴For most of the paper, we ignore the possibility that principals might offer 'random' contracts. The reason is that Y_j is itself general enough to include random variables and so already builds in the possibility of random contracts.

the agent takes depend on the agent's valuation and on the mechanisms that he or she observes being offered by all the principals. A *continuation strategy* for the agent is a mapping $\tilde{c} : \Omega \times \Gamma \rightarrow \Delta(\mathcal{A} \times E)$ that describes the joint probability distribution over messages and effort that the agent will use as a function of the agent's valuation and the array of mechanisms that he is offered by the principals. The continuation strategy c^* constitutes a *continuation equilibrium* relative to Γ if for every array of mechanisms $\gamma \in \Gamma$ offered by the principals and for almost every (with respect to F) valuation $\omega \in \Omega$, (i) the randomization $c^*(\omega, \gamma)$ maximizes

$$\int_{\mathcal{A} \times E} u(\gamma_1(\tilde{a}_1, e), \dots, \gamma_n(\tilde{a}_n, e), e, \omega) d\tilde{c}$$

This notation is somewhat awkward since messages are functions which determine other functions. Heuristically, the randomization \tilde{c} assigns a probability to each effort and array of messages $\{\tilde{a}_1, \dots, \tilde{a}_n, e\}$. The messages $\{\tilde{a}_1, \dots, \tilde{a}_n\}$ determine the incentive contracts that each principal actually adopts. The incentive contracts then determine the actions that the principals take in response to the effort level the agent takes. So the elements $\gamma_j(\tilde{a}_j, e)$ in the formula give the action that principal j will take when he receives a message \tilde{a}_j then 'sees' effort e .

A continuation equilibrium c^* determines a normal form game among the principals in which the actions spaces are Γ for each principal and payoffs are given by

$$\bar{v}_j(\gamma_1, \dots, \gamma_n, c^*) = \int_{\Omega} \int_{\mathcal{A} \times E} v_j(\gamma_1(\tilde{a}_1, e), \dots, \gamma_n(\tilde{a}_n, e), e, \omega) dc^*(\gamma, \omega) dF(\omega)$$

An *equilibrium* relative to the set of feasible mechanisms Γ is an array of randomizations $\{\delta_1, \dots, \delta_n\}$ and a continuation equilibrium c^* such that $\{\delta_1, \dots, \delta_n\}$ is a Nash Equilibrium for the normal form game defined by the continuation equilibrium c^* .

The principals' simple actions could be more complex than a simple monetary payment. For example, industry lobbyists trying to bribe a regulator could promise to restrict their output, or otherwise curb their behavior in a way that the regulator likes. This formulation is also flexible with respect to the degree to which the principal's action can depend on the agent's effort.

When lobbyists try to bribe a regulator, for example, the payment can depend fully on the agent's action (for example Dixit, Grossman, and Helpman (1997)). When multiple sellers are selling to the same agent, the price that each principal charges the agent could depend on the quantity that the agent buys from him, but not on the quantity that the agent buys from other sellers (this is the non-linear pricing problem discussed by Martimort and Stole (1999a) or Biais, Martimort, and Rochet (2000)). It also incorporates the usual 'moral hazard' problem in which the principal only sees a signal that is correlated with the amount of work the agent did. In this case the agent's effort is the probability distribution over signals that the agent chooses, while the principal's simple action is a probability distribution over payments he responds with (as in for example Bernheim and Whinston (1986a)).

To illustrate, two very simple examples come to mind. First, in one version of the simple Bertrand pricing problem, there are two firms each of whom has a single unit of capacity that they would like to sell to a single consumer. The capacity is costless. The units are indivisible and the consumer wants exactly one unit, but is willing to pay no more than 1 for the unit. The firms both know this, so information is symmetric. Payoffs are linear in the monetary transfer. The consumer's effort is simply a choice of either firm 1 or firm 2, while the firm's action is a monetary transfer (paid by the consumer to the firm) bounded above by 1 (a negative transfer means that the firm pays the consumer). A pay for effort contract is a pair of transfers, one for the case when the consumer chooses to buy from the firm and another transfer (naturally constrained to be non-positive) when the consumer chooses not to buy from the firm.

Alternatively, if the firms have unlimited capacities but convex production costs and the good is divisible so that the consumer can buy from both firms, then once again the firm's action is a transfer. In this case the consumers 'effort' is the quantity that the consumer chooses to buy from each of the firms. A pay for effort contract is a mapping from quantities purchased to transfers (in other words, a non-linear pricing scheme). Typically a particular firm can only verify the quantity that the consumer chooses to purchase from it. In this sense, the quantity that the consumer chooses to purchase from firm j represents an equivalence class of efforts (since the quantity that the consumer buys from the other firm can vary) by the consumer for which principal j must implement the same transfer.

2 Pure Strategy Theorems

The first theorem applies directly to the problem studied by Bernheim and Whinston (1986a) or Dixit, Grossman, and Helpman (1997) in which principals and the agent are assumed to have *symmetric information* in the sense that the distribution F of valuations for the agent is degenerate.

Theorem 1 *Suppose the principals and the agent have symmetric information and let $\gamma^* = \{\gamma_1^*, \dots, \gamma_n^*\}$ be any pure strategy equilibrium relative to the set of take it or leave it offers Γ^C . Then γ^* is also an equilibrium relative to the set of menus Γ^* .*

Proof. Let c be the continuation equilibrium relative to Γ^C . Since each $\gamma_j^* \in \Gamma_j^C$ it follows that γ_j^* provides a single pay for effort contract $a_j^*(\cdot)$ for every message that the agent sends. The agent's continuation strategy is then simply a probability distribution over the set E of feasible efforts and the agent's type is known. So principal j 's payoff in the equilibrium relative to Γ^C can be written as

$$V_j^C(\gamma^*) = \int_E v_j(a_1^*(e), \dots, a_n^*(e), e) dc(\gamma^*)$$

where $c(\gamma^*)$ is simply the probability distribution over effort used by the agent who is offered γ^* . The problem at hand is to extend the continuation equilibrium strategy c from Γ^C to a continuation equilibrium strategy \tilde{c} defined on all of Γ^* in a way that preserves the equilibrium. For any array of offers $\{\gamma'_j\} \in \Gamma^*$ such that $\gamma'_j \in \Gamma_j^C$ for all j , define

$$\tilde{c}(\gamma'_1, \dots, \gamma'_n) = c(\gamma'_1, \dots, \gamma'_n).$$

In words, if every principal makes a take it or leave it offer even though menus are feasible, the agent responds exactly as he does to those offers in the original equilibrium. If two or more offers in the array $\{\gamma'_j\}$ are non-degenerate menus, the continuation strategy for the agent can be defined in any convenient way⁵.

⁵An optimal continuation strategy exists because expected utility implies that preferences are linear (hence continuous) in the distributions in $\Gamma^* = \Delta(\mathcal{A} \times \Delta(\mathcal{A}))$, while by definition, menus are compact (in the weak topology) subsets of Γ^* .

If a single principal j deviates from γ_j^* to a non-degenerate menu γ_j'' of contracts in Γ^* choose $a_j''(\cdot) \in \gamma_j''$ and e'' to maximize

$$u(a_1(e''), \dots, a_j''(e'') \dots a_n(e''), e'')$$

Let $\tilde{\gamma}_j$ be the take it or leave it offer for which the principal responds to every message with contract $a_j''(\cdot)$ and set

$$\tilde{c}(\gamma_1^*, \dots, \gamma_{j-1}^*, \gamma_j'' \dots \gamma_n^*) = c(\gamma_1^*, \dots, \gamma_{j-1}^*, \tilde{\gamma}_j, \dots, \gamma_n^*)$$

In words, when principal j offers the menu γ_j'' , the agent selects a_j'' from the menu and then randomizes over effort as if the principal had offered a degenerate menu $\tilde{\gamma}_j$ consisting only of the offer a_j'' . The strategy $\tilde{c}(\gamma_1^*, \dots, \gamma_{j-1}^*, \tilde{\gamma}_j, \dots, \gamma_n^*)$ must be at least as good as e'' , else $c(\gamma_1^*, \dots, \gamma_{j-1}^*, \tilde{\gamma}_j, \dots, \gamma_n^*)$ cannot be a continuation equilibrium.

In this continuation equilibrium the agent reacts to γ_j'' in the game relative to Γ^* exactly as he would have reacted to a_j'' in the game relative to Γ^C . So if $V_j(\gamma_1^*, \dots, \dots, \gamma_n^*)$ is the payoff of the deviator in the competition relative to Γ^* when the agent uses the continuation equilibrium strategy \tilde{c}

$$V_j^*(\gamma_1^*, \dots, \gamma_j^*, \dots, \gamma_n^*) = V_j^C(\gamma_1^*, \dots, \gamma_j^*, \dots, \gamma_n^*) \geq$$

$$V_j^C(\gamma_1^*, \dots, a_j'', \dots, \gamma_n^*) = V_j^*(\gamma_1^*, \dots, \gamma_j'', \dots, \gamma_n^*)$$

and the deviation is unprofitable. ■

The argument in the proof is straightforward, when one principal unilaterally deviates from a take it or leave it offer it is always optimal for the agent to choose an element from the menu, then select effort the way he did in the original game when he was simply offered his choice from the menu by itself. There could, of course, be other ways for the agent to choose from the menu and then select effort. Nonetheless, this shows that there always exists a continuation equilibrium in which the deviation is unprofitable. So the original equilibrium is robust.

It is worth noting that by Peters (2001a), equilibria relative to the set of menus are robust in the sense that the payoffs associated with these equilibria can still be supported when the set of mechanisms is made even larger. It follows that the payoffs associated with pure strategy equilibria relative to Γ^C can be supported as equilibria no matter what kind of communications mechanisms are feasible.

It is tempting to interpret this theorem by arguing that principals offer menus to learn market information, but in a pure strategy equilibrium market information is already known to all principals because they know each other's strategies in any Nash equilibrium. This interpretation is incorrect for two reasons. First, the theorem is not generally true with multiple agents in the sense that expansions of the set of feasible mechanisms to allow prior communication between principals and agents can destroy pure strategy equilibria (of course the notion of menu doesn't have much meaning with multiple principals).⁶ Secondly, it should be emphasized that principals are not so much interested in what other principals do on the equilibrium path, they want to know when some principal tries to *deviate* from the equilibrium path. So knowledge about equilibrium play has no implication for the value of market information.

When the principal does not know ω then he will want to offer a menu just to learn about the agent's preferences. The set of menus can still be a fairly difficult space to work with if Y is, for example, a Euclidean space representing probability distributions over some finite set of simple actions. So it is worthwhile to try to simplify the set of feasible mechanisms in this case. To get at this let Γ^{DM} be the set of 'direct mechanisms'. This consists of all menus γ for which the image of γ_j in \mathcal{A}_j can be embedded in Ω in the sense that there exists a one to one mapping from $\gamma_j(\mathcal{A}_j)$ into a subset of Ω . Heuristically a menu is a direct mechanism if it would have been possible to offer the same set of alternatives with the message space Ω . This is the most natural candidate for the set of mechanisms available to the principals.⁷

Equilibrium in direct mechanisms need not generally involve truthful reporting. With a single mechanism designer this is not important since each equilibrium in which the agent misrepresents his type can be represented as an equilibrium in which the agent reports his type truthfully. This is not true with competing mechanisms. The reason is that if the agent reports his type truthfully to each principal, then conditional on his type and the effort level he chooses, the principals' actions must be independent. When he lies about his type in equilibrium, he can randomize over his reports such that reports and therefore actions taken by principals conditional on effort are correlated. So in general, not all equilibrium allocations with direct mechanisms can be represented as equilibrium allocations with incentive compatible direct

⁶See Peters (2001b) for an example.

⁷In the case of competing mechanisms, this is the approach used in McAfee (1993).

mechanisms.

The point of the theorems in this section is to check the robustness of specially selected equilibrium. So it is appropriate to focus on those equilibria that can be represented as equilibrium with truthful reporting. For this reason we define an equilibrium in *incentive compatible direct mechanisms* to be one in which the continuation equilibrium is such that the agent uses a pure strategy when choosing among the incentive contracts offered in any principal's menu (though he might still like to randomize over effort).⁸

Theorem 2 *Let $\gamma^* = \{\gamma_1^*, \dots, \gamma_n^*\}$ be any pure strategy equilibrium in incentive compatible direct mechanisms in Γ^{DM} . Then γ^* is an equilibrium relative to the full set of menus Γ^* .*

The proof resembles the proof of Theorem 1 so it is included in the appendix. The essential idea is that when one principal deviates into the larger space of contracts, he essentially offers the agent a *menu* of direct mechanisms. The agent responds by choosing one of these direct mechanisms, so the deviating principal could have attained the same payoff by offering the direct mechanism that the agent picks by itself. Theorem 1 is actually a special case of Theorem 2 for the case when Ω is degenerate. In problems with asymmetric information, if it seems sensible to concentrate on pure strategy equilibria then equilibria in simple direct mechanisms of the usual sort are interesting in the sense that they are robust to the possibility that principals might invent more sophisticated ways to communicate. Once again, the possibility exists that there are alternative equilibria that cannot be understood as pure strategy equilibria in direct mechanisms.

3 Environments without Externalities

Most of the literature on common agency simply ignores menus. Theorem 1 shows the sense in which this can be a perfectly sensible thing to do. However, this theorem does not say anything about what this may leave out. The literature has identified two distinct sorts of equilibria with menus that are eliminated by restricting contracts. In one sort of equilibrium,

⁸Of course we could allow randomized choices provided that these choices were all independent of one another. Independence along with linearity of the payoff functions would then make these random strategies equivalent to pure strategies.

deviations by one principal are deterred by the possibility that the agent will change his selection from the other principal's menu, or change his effort in a way that hurts the deviating principal (this is the argument proposed in Martimort and Stole (1998)). Alternatively, (Peck (1995)) the agent can act as a correlating device for the principals (or the principals can act as correlating devices for the agent) in a way that generates payoffs that cannot be attained in the absence of such devices. In either case, the contracts used to support the new equilibria provide the agent an incentive compatible way of communicating his valuation to the principal, where his type includes a description of what he knows about the behavior of the other principals (see Epstein and Peters (1999)).

The various ways negotiation affects the outcome can be shown in a simple example. In the following table, the first two payoffs in each cell are the payoffs to the principals, who select among the actions $\{a, b\}$. The agent's payoff is the third entry. The agent takes no effort in this example. Thus a take it or leave it offer is simply a probability distribution over actions. There is a single pure strategy equilibrium in take or leave it offers in which the payoffs to the principals are 3 and the agent gets 0. The offer a is weakly dominated for each principal, so there are no mixed strategy equilibria

	a	b
a	0, 0, 1	2, 1, 1
b	1, 2, 1	3, 3, 0

Once menus are allowed, there are a continuum of new equilibrium outcomes, none of which work out very well for the principals. First suppose that both principals offer to negotiate the outcome by offering the agent a menu consisting of the actions $\{a, b\}$. The agent randomizes with probability q between the two off diagonal outcomes when he makes his choice from the menus. This supports payoffs to the principals anywhere between 1 and 2. If either principal tries to deviate and force the action b by making a take it or leave it offer, the agent responds with a choice that minimizes the deviator's payoff (that is, by choosing a from the non-deviator's menu).⁹

In this equilibrium the agent plays two roles. He correlates the choices of the principals by randomizing in his choice from the menu. He also enforces

⁹The same argument works if the deviator tries to hide his action by offering a random contract. In that case the agent strictly prefers to choose the action a from the non-deviator's menu.

the equilibrium by punishing deviations. Both these roles are supported by the fact that the payoff that one principal gets is sensitive to what the other principal does. When the agent correlates the outcomes he induces the principals to take actions that are only profitable when they are undertaken jointly. When the agent punishes a deviation he chooses an action by one principal that reduces the payoff of the deviating principal.

The environments studied by many of the best papers in common agency do not allow this sort of externality, and the deviator will not be hurt by a change in the non-deviator's action unless the agent also changes his effort. For example, in Bernheim and Whinston (1986a), once the agent's effort has been decided, no principal cares what monetary offers that other principals have made to the agent. In a Bertrand competition, once the buyer has chosen to buy from one seller, that seller does not care what price offers the other sellers have made. In problems like these, menus will only be useful in punishing deviations if the deviation induces the agent to switch his effort in a way that hurts the deviator. If the change in the agent's effort is something that the agent would have been willing to do without a change in the non-deviator's action, then it should be possible to implement the punishment without using menus. This is exactly the motivation for the assumptions that follow.

Say that the *no-externalities* assumption holds relative to the set of actions Y if conditional on every level of effort that principal j can contract on, the agent has a weak preference ordering over the elements of Y_j that is independent of his type, his actual effort and the actions in Y_{-j} being offered by the other principals; and if the principal's utility is determined only by his own action and the agent's effort. Formally for each j

- (i) there exists a function $V_j : Y_j \times E \times \Omega$ such that for all $(y_1, \dots, y_j, \dots, y_n) \in Y$; $e \in E$ and $\omega \in \Omega$;

$$v_j(y_1, \dots, y_j, \dots, y_n, e, \omega) = V_j(y_j, e, \omega);$$

and

- (ii) for each equivalence class $\hat{e} \in \mathcal{E}_j$ and any closed subset $B \subset Y_j$ there is a $y \in B$ such that,

$$u(y, y_{-j}, e, \omega') \geq u(y', y_{-j}, e, \omega')$$

for all $y' \in B$; $\omega' \in \Omega$ and $y_{-j} \in Y^{n-1}$ and $e \in \hat{e}$.

The no-externalities assumption is obviously a strong one, but there are a number of interesting environments that satisfy it. The simplest is probably the ordinary Bertrand pricing model in which several sellers compete in price to sell to a single buyer who wants exactly one unit of output. In this case Y_j is a set of prices, say the unit interval $[0, 1]$. The buyer only wants a single unit of output, so effort is just the choice the buyer makes about which seller to buy from, i.e., $E = \{0, 1\}$. In this case the principal can observe perfectly and contract completely on the agent's effort, so $\mathcal{E}_j = \{0, 1\}$. The seller's incentive contract will then specify the transfer the buyer makes contingent on the seller she chooses to buy from. So if seller 0 wants to sell at a price $p \in [0, 1]$, he is implicitly using a pay for effort contract that specifies a transfer p if the buyer takes effort 0 (i.e., chooses to buy from seller 0) and a transfer 0 if the buyer chooses to buy from seller 1. Obviously the buyer who is offered any closed set of prices will prefer the lowest price for each level of effort no matter what price is charged by the other seller.

The usefulness of the assumption stems from the fact that the agent's ranking is restricted conditional on observable effort. The assumption does not require that agent's rank pay for effort contracts in a way that is independent of the contracts offered by other principals. This makes it possible to apply the assumption and the Theorems that follow from it to non-linear pricing problems quite generally. For example, the models in Biais, Martimort, and Rochet (2000) or Martimort and Stole (1999a) both satisfy the no-externalities assumption provided sellers are restricted to non-random contracts (as they are in those papers).

This can be seen by considering a problem described in Martimort and Stole (1999a). There are two downstream sellers (the principals) who sell differentiated commodities. These commodities are both produced by a single supplier. The supplier's cost function is $C(q_1, q_2)$ where q_1 and q_2 are the quantities that the supplier produces for downstream seller 1 and 2 respectively. The downstream sellers make monetary transfers to the producer. Downstream seller i 's monetary transfer to the producer can only depend on q_i . If the downstream sellers decide on transfers t_1 and t_2 respectively, then

$$u(q_1, q_2, t_1, t_2) = t_1 + t_2 - C(q_1, q_2)$$

and

$$v_i(q_1, q_2, t_1, t_2) = D_i(q_1, q_2) q_i - t_i$$

where $D_i(q_1, q_2)$ is the market price of good i when outputs are q_1 and q_2 .

One way to model this problem is to allow the principals to compete in non-linear pricing schemes. To proceed with this, think of the agent's effort as being the pair of outputs (q_1, q_2) that he chooses to produce. Let the equivalence classes \mathcal{E}_i for principal i be defined as the collection of sets $\{(q'_1, q'_2) : q'_i = q_i\}$ indexed by the output of the principals own product q_i . Pay for effort contracts are simply maps from agent effort into transfers that are measurable with respect to the σ -algebra defined by \mathcal{E}_i , that is, just the set of non-linear pricing schemes of the form $t_i(q_i)$. Now it is clear that the principal's payoff depends only on his own action and the agent's effort, so the first part of the no-externalities assumption is satisfied. For the second part observe that for any closed set of transfers for principal i the agent will prefer the highest one for any effort level for which q_i is fixed (indeed, for any level of effort at all). So this non-linear pricing problem satisfies the no-externalities assumption.

No externalities applies to non-linear pricing problems in which sellers are required to offer a single price for each quantity. More generally, if random pricing is allowed, the no-externalities assumption applies provided buyer utility is quasi-linear. Then buyers can compare contracts based on expected payments.

The no-externalities assumption can be applied to the environments studied by Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997) where principals offer agents non-random monetary rewards that are contingent on the effort that the agent takes (for example bribes to a government agency that depend on the policy that the agency adopts). The payoffs to the principals in these models depend only on the agent's action and on the monetary payment that the principal makes to the agent. Once again, conditional on the agent's effort choice, the agent will always weakly prefer a higher payment to a lower one, and this gives the no-externalities assumption.

To begin we focus on the case of symmetric information, since this has been assumed in most of the previous literature. Furthermore, to attempt to make the proof more transparent, we also restrict attention to pure strategy continuation equilibrium and non-random contracts. The theorem is generalized below to allow for asymmetric information, random contracts and mixed continuation equilibrium. The essential logic is the same.

Theorem 3 *Suppose that the no-externalities assumption holds relative to the set of simple actions available to the principals and that the principals*

and the agent have symmetric information. Let $\{\gamma_1^*, \dots, \gamma_n^*\}$ be a pure strategy equilibrium relative to the set of menus Γ^* and suppose that in the continuation equilibrium associated with Γ^* the agent uses pure strategies. Then there exist an equilibrium $\{\tilde{\gamma}_1, \dots, \tilde{\gamma}_n\}$ relative to the set of take it or leave it offers Γ^C which preserves all payoffs.

Proof. For each $\hat{e} \in \mathcal{E}_j$ and for every $j = 1, \dots, n$ the menu γ_j^* generates a closed set $A_j^*(\hat{e})$ of simple actions for principal j that the agent could induce by selecting among the different pay for effort contracts in principal j 's menu and choosing some level of effort in \hat{e} . Since the no-externalities assumption holds relative to the set of simple actions, there is an element of $A_j^*(\hat{e})$ that is at least as good as any other element in $A_j^*(\hat{e})$ for the agent no matter what simple actions are taken by the other principals and no matter what effort in \hat{e} he decides to take. Let $\tilde{a}_{\gamma_j}(\hat{e})$ be the set of actions in $A_j^*(\hat{e})$ that satisfy this property.

Let e^* be the agent's equilibrium effort under γ^* . Suppose that $e^* \in \hat{e}_j^*$ and let a_j^* be the simple action that the agent chooses with each principal on the equilibrium path. Define the contract $\tilde{\gamma}_j \in \Gamma^c$ so that $\tilde{\gamma}_j$ assigns a pay for effort contract that associates the simple action a_j^* with observable effort \hat{e}_j^* and otherwise associates any one of the elements of $\tilde{a}_{\gamma_j}(\hat{e})$ with each of the other equivalence classes $\hat{e} \in \mathcal{E}_j$. If every principal offers the contract $\tilde{\gamma}_j$ and the agent takes effort e^* , the agent and every principal must get the same payoff as they do in the initial equilibrium because they are all doing the same thing as in the original equilibrium.

To show that it is an equilibrium for principals to offer $\tilde{\gamma}$ and for the agent to take effort e^* we need to construct a continuation equilibrium relative to Γ^C that makes deviations from these things unprofitable.

It is immediate that it is optimal for the agent to choose effort level e^* when all principals offer $\tilde{\gamma}_j$. This follows because for any other effort level e' in equivalence class \hat{e}' , $\tilde{\gamma}_j(\hat{e}') \in \tilde{a}_{\gamma_j}(\hat{e}') \subset A_j^*(\hat{e}')$, so that the actions $\{\tilde{\gamma}_j(\hat{e}')\}_{j=1 \dots n}$ are all actions that the agent could have induced in the original game by deviating to some effort level in \hat{e}' and selecting from the principals' equilibrium menus. Since e^* is an equilibrium level of effort for the original game, a deviation to e' cannot be profitable.

The continuation equilibrium when many principals deviate to contracts in Γ^C is irrelevant. The existence of continuation equilibria in this case is guaranteed by continuity of the agents payoff function and compactness of the various spaces, so any convenient continuation equilibrium can be assigned

when there are multiple deviations.

The important argument is to construct the continuation equilibrium for unilateral deviations. When feasible mechanisms are Γ^C , principals can only offer a single pay for effort contract to the agent. As a take it or leave it offer is a degenerate menu, each potential deviation $\gamma'_j \in \Gamma_j^C$ is also in Γ_j^* . So suppose that when one principal deviates to γ'_j and all other principals continue to use $\tilde{\gamma}_{-j}$, the agent responds with the effort e' that he would have chosen in the original game if one principal deviated to γ'_j while every other principal continued to use γ_{-j}^* . Let $e' \in \hat{e}'_k$ for principal k . It is possible that in the original game, the agent's choice from the menus of the other principals might not coincide with $\tilde{\gamma}_{-j}(\hat{e}')$, but by part (i) of the no-externalities condition, this would not affect the payoff to the deviating principal. Then provided it is optimal for the agent to respond to the unilateral deviation to $\tilde{\gamma}_j$ by taking effort e' , the principals payoff when he offers γ' must then be the same as his payoff in the original equilibrium when he deviated to γ' . This makes the deviation unprofitable because γ' is not an equilibrium contract in the original game.

So the last detail is to show that the effort level e' is optimal for the agent when there is a unilateral deviation to γ'_j . This effort is optimal in the original game when there is a deviation to γ' . This means that there are a set of simple actions $\{a'_{-j}\}$, such that by taking effort e' and selecting appropriately from the non-deviators' menus, the agent induces an outcome $(\gamma'_j(\hat{e}'_j), a'_{-j}, e')$ that is at least as good for the agent as any other feasible outcome. By construction and part (ii) of the no-externalities assumption, $\{\gamma'(\hat{e}'_j), \tilde{\gamma}_{-j}(\hat{e}'_{-j}), e'\}$ is at least as good as $(\gamma'(\hat{e}'_j), a'_{-j}, e')$, so the result follows. ■

Despite the pure strategy and symmetric information assumptions, the theorem can nonetheless be applied to the models of Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997). In the models described in these papers, the principals' actions are simply transfers that they make. The agent's effort is a policy (in Dixit, Grossman, and Helpman (1997)) or a quality and choice of partner (in Bernheim and Whinston (1986b)). In both cases this effort is fully contractible for the principal. Conditional on a policy or quality, the agent always prefers higher transfers and principals get utility from the policy or quality less the cost of the transfer. So the no-externalities assumption holds with respect to the set of non-random transfers. These papers assume symmetric information and focus on pure strategy equilibrium. Then by Theorem 3, no new equilibrium payoffs can be supported

by allowing the principals to offer menus of pay for effort contracts to the agents.

This begs the question about randomization. It also makes it impossible to apply the theorem to non-linear pricing problems like Biais, Martimort, and Rochet (2000) where all the interest comes from the fact that principals and agents are asymmetrically informed. To deal with this, a more general version of the theorem is presented that allows randomization in two dimensions, agents are allowed to use random strategies both when choosing from menus and in choosing effort, and agents are allowed to have private information about their valuations.

There is one complication associated with this generalization. When the agent uses a random selection strategy in choosing among elements in a menu, the principal's action conditional on observable effort becomes random. This cannot be reproduced when principals make take it or leave it offers that associate simple actions (but not random actions) with each observable effort. The agent is not going to care about this randomization since he will restrict to actions about which he is indifferent. If principals are risk averse, their payoffs might depend on this randomization.

To capture this the following theorem reproduces equilibrium with menus as equilibrium with take it or leave it offers in a new environment where the set of simple actions is the set of probability distributions over the set of simple actions in the original environment.

Theorem 4 *Let $\{\gamma_1^*, \dots, \gamma_n^*\}$ be a pure strategy equilibrium relative to Γ^* in an environment where the set of simple actions available to each principal is Y_j . Suppose that the no-externalities assumption holds relative to Y . Then there exists a pure strategy equilibrium in take it or leave it offers (i.e., relative to Γ^C) in an environment in which the set of simple actions available to each principal is $\Delta(Y_j)$ that preserves all payoffs.*

The proof of Theorem 4 mimics the proof of Theorem 3 so it is included in the appendix. Note that pure strategy equilibria relative to Γ^C explicitly allow the principals to offer random contracts which assign a probability distribution over simple actions to each effort level that the agent might undertake. If the original equilibrium involves each agent using pure selection strategies from menus conditional on effort and type then random contracts are no longer required in the theorem.

4 Examples

It is worthwhile at this point to discuss briefly two examples. In the first, the no-externalities assumption holds, in the second it doesn't. First consider a non-linear pricing problem. There are two firms who are the principals in this problem, each firm produces two commodities according to a cost function $C_i(x, y)$ where (x, y) denotes the quantities of the two outputs that the firm chooses to produce. There is a single consumer (the agent) who buys all four commodities from a fixed budget. The commodities produced by the different firms are substitutes but not perfect substitutes. The consumer's utility function is $u(x^1, y^1, x^2, y^2, w, \omega)$ where w is the amount of money left over after the consumer purchases the four commodities and ω is the consumer's valuation distributed in some fashion that is common knowledge to both principals and the agent.

There are a variety of ways to model this problem depending on what kinds of contracts the modeler thinks are feasible. One natural assumption is that the firms can use non-linear pricing schemes but cannot commit themselves to random prices. At this stage, one approach is to use 'direct mechanisms' in which each firm asks the agent his or her type, then provides a different non-linear pricing rule for each possible response. If a pure strategy equilibrium for this game can be found, then it is an equilibrium relative to the set of menus by Theorem 2.

Now check for no-externalities. The agent's effort is naturally interpreted as the vector of demands that he or she makes to each principal. Each principal's action is the price that he or she charges for each bundle. So

$$v_i(p_1^1, p_1^2, p_2^1, p_2^2, x^1, y^1, x^2, y^2, \omega) = p_i^1 + p_i^2 - C(x^i, y^i)$$

Evidently each principal's payoff depends only on the prices that he charges and the quantity that the agent chooses to buy.

The equivalence classes for principal i of observable effort for the agent are simply the vectors of demands indexed by the agent's demand from principal i . The principal's actions are no longer an ordered set, but conditional on the quantities that the agent wants to buy from seller i , the agent will always prefer the combination of transfers that gives the lowest sum (or cost for the bundle if the actions are per unit prices) independent of his type, the prices offered by the other seller and the quantities purchased from the other seller. So the no-externalities assumption holds in this environment.

Theorem 3 can then be applied in two ways. First, it isn't really necessary to think about principals asking the agent his type then offering a non-linear pricing scheme. Every pure strategy equilibria of that form is an equilibrium relative to the set of menus. Furthermore, if attention is restricted to incentive compatible direct mechanisms the agent will report his type then receive a non-random non-linear pricing scheme in return. In such an equilibrium the agent cannot randomize his choice of action conditional on his type and effort. Then by Theorem 3, all such equilibria can be reproduced with competition in take it or leave it offers, in other words, in simple non-linear pricing schemes.

Furthermore, there won't be any other equilibria in menus that cannot be understood with simple competition in non-linear pricing schemes, except possibly those that induce the agent to randomize his choice of prices. These can be understood by allowing the principals to compete in what are effectively non-linear lotteries over prices.

An alternative is to assume that firms' simple actions are linear prices. Non-linear schemes and randomization are not allowed. The firms' payoffs are easily seen to satisfy part (i) or the no-externalities condition. The buyer now has utility

$$u(x^1, y^1, x^2, y^2, w - p_x^1 x^1 - p_y^1 y^1 - p_x^2 x^2 - p_y^2 y^2, \omega)$$

The firm now offers the buyer a vector of prices. This example better illustrates the usefulness of theorems 3 and 4. Conditional on each vector of demands, the buyer always prefers the vector of prices that minimizes his expenditure independent of the prices offered by the other firm and independent of the buyer's valuation. So again the no-externalities assumption holds and no new payoffs can be generated by allowing sellers to compete by offering menus of linear price vectors to the agent. (Of course the sellers would like to deviate to non-linear pricing schemes but it is assumed they are not allowed to). It should be clear in both these examples why menus and negotiation play no role. The agent's preferences over simple actions offered by the principals don't depend on what the other principals are doing, so there is no incentive compatible way for the principal to extract market information. Since many incentive problems are going to involve monetary rewards for effort, the no-externalities assumption will hold and market information will play no role.

Menus may play a role when the principals' simple actions are more complex than monetary transfers. One example is the common agency model of

Bernheim and Whinston (1986a). The principals in that model make payments to an agent contingent on a signal whose distribution is chosen by the agent. The simple actions of the principals should then be interpreted as the distributions of payments rather simple transfers. The agent's effort is just the distribution of signals that he chooses. As in Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997), the principal's payoff depends only on the effort that the agent takes and the expected payment that is made to the agent, so the first part of the no-externalities assumption is satisfied. If the agent is also risk neutral, then the agent will always prefer a distribution of payment with a higher mean, and the no externalities assumption holds.

If the agent is risk averse, the second part of the assumption is more difficult to satisfy.¹⁰ The agent's payoff in Bernheim and Whinston (1986a) is a separable function of monetary payments and effort. To make things concrete suppose that the agent chooses some real number w that generates a joint distribution of signals $\{s_1, \dots, s_n\}$ for the principals. Suppose further that conditional on w , these signals are independent. Suppose that the principals simple actions are given by the signal contingent payments $t_j(s_j)$. The agent's expected utility is

$$\mathbf{E}_w u \left(\sum_{j=1}^n t_j(s_j) \right) - k(w, \omega)$$

where $k(w, \omega)$ is the cost to an agent of valuation ω to generate the joint distribution of signals w . When the agent is risk averse, the way the agent ranks two different distributions of transfers $t_j(\cdot)$ and $t'_j(\cdot)$ will typically depend on the transfers they are mixed with simply because the transfers are partly used to hedge risks created by the other principals' transfer schemes.

The no externalities assumption can be ensured by imposing additional restrictions on the environment. Suppose that $u(x) = -e^{-\beta x}$ so that the agent has constant absolute risk aversion. Then the agent's expected payoff is

$$\mathbf{E}_w \left(-e^{-\beta \sum_{l \neq j} t_l(s_l)} \right) \mathbf{E}_w \left(-e^{-\beta t_j(s_j)} \right) - k(w)$$

and contingent on effort w , the agent will prefer a distribution that generates a higher value for $\mathbf{E}_w \left(-e^{-\beta t_j(s_j)} \right)$ no matter what the other principals

¹⁰The following argument was suggested by a referee.

do, which resurrects the no externalities assumption. Applying Theorem 4 there are no new equilibrium payoffs generated by looking at pure strategy equilibrium in which the principal offers the agent menus of pay for effort contracts (even to learn about the agent’s type in this case).

5 Conclusion

Complex negotiations can result in interesting new behavior in common agency when there are many principals. Standard approaches to common agency developed by Bernheim and Whinston (1986a) or Dixit, Grossman, and Helpman (1997) ignore this. This paper shows that provided one is interested in pure strategy equilibria, these approaches lead to errors in omission rather than errors in commission. With symmetric information, pure strategy equilibria in take it or leave it contracts are robust against the possibility that principals might offer far more complicated communication schemes. The role of communication is then to support new kinds of equilibrium outcomes. With asymmetric information, the same can be said of equilibrium in direct mechanisms in which the agent is required only to report his preference information.

The second part of the paper studies environments considered in some of the best known applications of the common agency model. It shows that these environments have a special property which is referred to as ”no-externalities”. When this property holds, allocations that can be supported with general communications mechanisms can also be supported by having principals make take it or leave it offers. So in the best known applications of common agency, there is nothing new to learn by looking at menus or more complicated kinds of communication.

6 Appendix

Proof. of Theorem 2 Let c be the continuation equilibrium relative to Γ^{DM} . Since each γ_j^* is incentive compatible the agent uses a pure strategy in choosing among contracts. It follows that for each type ω , the agent will choose a single incentive contract $a_j^*[\omega]$ from Γ^{DM} . Principal j ’s payoff can be written as

$$\int_{\Omega} \int_E v_j(a_1^*[\omega](e), \dots, a_n^*[\omega](e), e, \omega) dc[\omega](e) dF(\omega)$$

where, in a somewhat awkward notation, $a_i^*[\omega]$ is the incentive contract chosen by the agent when his type is ω and $a_i^*[\omega](e)$ is the action in Y_j that the contract specifies when the agent's effort is e . $c[\omega](e)$ is the randomization over effort used by an agent *conditional* on type ω .

As in the Proof of Theorem 1, the problem is to extend the continuation equilibrium to all of Γ^* in a way that preserves the initial equilibrium. Let \tilde{c} be the continuation equilibrium on the extended set of mechanisms. As before, the agent should respond to all offers in Γ^* that are also in Γ^{DM} in exactly the same way as he does in the initial equilibrium - so set

$$\tilde{c}(\gamma, \omega) = c(\gamma, \omega).$$

for each $\gamma \in \Gamma^{DM}$, each $\omega \in \Omega$.

Since the equilibrium offers γ^* from Γ^{DM} are also in Γ^* , this extension preserves all payoffs provided no principal changes his offer. Furthermore, since no new deviations are available to the agent in this case than there were in the original equilibrium, the behavior constitutes a best reply for the agent to all such arrays of contract. As before, when two or more of the offers in γ lie outside Γ^{DM} , choose the continuation equilibrium in any convenient fashion.

Finally, focus on the case where a single principal, say principal j , deviates from Γ^{DM} to some $\gamma'_j \notin \Gamma_j^{DM}$. As before, we wish to construct the continuation equilibrium following this deviation so that it has the same consequences as a deviation to a contract in Γ_j^{DM} .

For each type $\omega \in \Omega$ choose an array of incentive contracts $\{a'_1[\omega], \dots, a'_n[\omega]\} \in \{\gamma_1^*, \dots, \gamma'_j, \dots, \gamma_n^*\}$ and a level of effort $e[\omega] \in E$ such that

$$u(a'_1[\omega](e[\omega]), \dots, a'_n[\omega](e[\omega]), e[\omega], \omega) \geq$$

$$u(a''_1(e'), \dots, a''_n(e'), e', \omega)$$

for all $e' \in E$; $a''_k \in \gamma_k^*$; $k \neq j$; $a''_j \in \gamma'_j$. For the deviating principal j , formulate the direct mechanism γ_j^d by collecting the contracts $\{a'_j[\cdot]\}$ for all possible values of ω . Since the choice set available to the agent when principals offer $\{\gamma_1^*, \dots, \gamma_j^d, \dots, \gamma_n^*\}$ is contained in the choice set available when principals offer $\{\gamma_1^*, \dots, \gamma'_j, \dots, \gamma_n^*\}$, the choices $\{a'_k[\omega]\}_{k=1, n}$ and $e[\omega]$ must still be optimal within the smaller choice set. Now set the continuation equilibrium so that

$$\tilde{c}(\gamma_1^*, \dots, \gamma'_j, \dots, \gamma_n^*, e) = c(\gamma_1^*, \dots, \gamma_j^d, \dots, \gamma_n^*, e)$$

Since the continuation equilibrium in Γ^{DM} relative to $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\}$ must have the agent choose an incentive contract and level of effort optimally for each value of his type, the continuation equilibrium randomization $c(\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*, e)$ is at least as good for the agent as the selection $\{a'_1[\omega], \dots, a'_n[\omega]\}$ and $e[\omega]$. So it must be at least as good for every type of agent as any array of options open to him when principals offer $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\}$. Hence $\tilde{c}(\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*, e)$ must be a continuation equilibrium.

Finally, since the distribution of actions and efforts chosen by the agent is the same relative to $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\} \in \Gamma^*$ as it is relative to $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\} \in \Gamma^{DM}$, and since the distribution chosen by the agent relative to $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\}$ is at least as good for the principal as the distribution relative to $\{\gamma_1^*, \dots, \gamma_j^d \dots \gamma_n^*\}$, this construction ensures that it does not pay the principal to deviate. ■

Proof. of Theorem 4: For each equivalence class $\hat{e}_j \in \mathcal{E}_j$ define the set $A_j^*(\hat{e}_j) \equiv \{y_j = a_j(\hat{e}_j); a_j \in \gamma_j^*\}$. This is the set of simple actions by principal j that the agent could induce by selecting an effort $e \in \hat{e}_j$. By the no-externalities assumption there is a subset $\tilde{a}_j^*(\hat{e}_j)$ such that $y_j \in \tilde{a}_j^*(\hat{e}_j)$ implies that,

$$u(y_j, y_{-j}, e, \omega) \geq u(y'_j, y_{-j}, e, \omega)$$

for each $y' \in A_j^*(\hat{e}_j)$, each $\omega \in \Omega$; each $y_{-j} \in Y_{-j}$ and each $e \in \hat{e}_j$.

Let c^* denote the joint distribution on $Y \times E \times \Omega$ induced by the agent's equilibrium continuation strategy given offers $\{\gamma_1^*, \dots, \gamma_n^*\}$. Let $c^*(\omega)$ be the distribution on $Y \times E$ associated with c^* conditional on the agent's type ω . Since the agent's continuation strategy must be optimal given the contracts offered by the principals, it must be that

$$(y, e) \in \text{supp } c^*; e \in \hat{e}_j \Rightarrow y_j \in \tilde{a}_j^*(\hat{e}_j) \quad (1)$$

where $\text{supp } c^*$ means the support of c^* . Let $g_j^*(\hat{e}_j)$ be the distribution of actions used by principal j in equilibrium that is induced by c^* conditional on observing effort \hat{e}_j . By (1) $\text{supp } g_j^*(\hat{e}_j) \subset \tilde{a}_j^*(\hat{e}_j)$. Observe that by expected utility and the definition of $\tilde{a}_j^*(\hat{e}_j)$, the agent must be indifferent over all lotteries over $\tilde{a}_j^*(\hat{e}_j)$ whenever the agent takes an effort $e \in \hat{e}_j$. Let $\pi^*(\omega)$ be the marginal distribution on effort for an agent of type ω that is induced by c^* .

To construct the take it or leave it offer for each principal define the pay for effort contract \bar{a}_j^* such that $\bar{a}_j^*(\hat{e}_j) = g_j^*(\hat{e}_j)$ for each \hat{e}_j such that there is

some $e \in \hat{e}_j$ such that e is in the support of c^* . Otherwise define $\bar{a}_j^*(\hat{e}_j)$ to be any convenient randomization over actions in $A_j^*(\hat{e}_j)$.

Suppose that each agent responds to the array of offers $\bar{a}^* \in \Gamma^C$ by adopting the strategy $\pi^*(\omega)$. Then observe that \hat{e}_j can be thought of as a measurable function of e so that

$$\begin{aligned} \int_{\mathcal{A} \times E \times \Omega} v_j(y_j, y_{-j}, e, \omega) dc^* &= \int_{\mathcal{A}_j \times E \times \Omega} V_j(y_j, e, \omega) dc^* \\ &= \int_{\Omega} \int_E \int_{Y_j} V_j(y_j, e, \omega) dg^*(\hat{e}_j) d\pi^*(\omega) dF(\omega) \end{aligned}$$

The first equality follows from the no-externalities assumption for principals. Since the first expression is the principal's expected payoff in the equilibrium relative to Γ^* while the last is the payoff of the principal when his contract is $\bar{a}_j^*(\hat{e}_j)$ and the agent chooses strategy according to $\pi^*(\omega)$, each principals payoff in this situation is preserved.

For the agent's payoff observe that

$$\begin{aligned} &\int_{Y \times E} u(y_1, \dots, y_n, e, \omega) dc^*(\omega) \\ &\equiv \int_{Y_{-j} \times E} \int_{Y_j} u(y_j, y_{-j}, e, \omega) dc^*(\omega, y_{-j}, e) dc_{e, y_{-j}}^*(\omega) \end{aligned}$$

where $c^*(\omega, y_{-j}, e)$ is the distribution on y_j conditional on ω and y_{-j} and e in the original equilibrium relative to Γ^* while $c_{e, y_{-j}}^*$ is the corresponding marginal distribution. By no-externalities for the agent conditional on \hat{e}_j , the optimal set of actions available in principal j 's menu $\tilde{a}_j^*(\hat{e}_j)$ is independent of the agent's type, the agent's effort within \hat{e}_j and the actions of the other principals. By (1) $c^*(\omega, y_{-j}, e)$ has its support contained in $\tilde{a}_j^*(\hat{e}_j)$. By expected utility preferences the agent must be indifferent to all randomizations over $\tilde{a}_j^*(\hat{e}_j)$ so this last expression must be equal to

$$\int_{Y_{-j} \times E} \int_{Y_j} u(y_j, y_{-j}, e, \omega) dg_1^*(\hat{e}_j) dc_{e, y_{-j}}^*(\omega)$$

Repeating this argument for the $j - 1$ other firms yields the expression

$$\int_E \int_{Y_j} \int_{Y_{-j}} u(y_j, y_{-j}, e, \omega) dg_1^*(\hat{e}_j) dg_{-j}^*(\hat{e}_{-j}) d\pi^*(\omega)$$

where in an obvious notation $dg_{-j}^*(\hat{e}_{-j})$ means $dg_1^*(\hat{e}_1) \cdots dg_{j-1}^*(\hat{e}_{j-1}) \cdot dg_{j+1}^*(\hat{e}_{j+1}) \cdots dg_n^*(\hat{e}_n)$. So the agent's payoff is also preserved in this new situation.

The next step is to try to show that the continuation equilibrium relative to Γ^C can be constructed so that no one has an incentive to deviate from this behavior. The first step in this regard is to show that the randomization $\pi^*(\omega)$ over effort is a best reply for each valuation for the agent. This is immediate since for every \hat{e}_j and each j , $g_j^*(\hat{e}_j)$ has support contained in $A_j^*(\hat{e}_j)$. So if there is a profitable deviation in effort when principals offer $\bar{a}^*(\hat{e})$, then this same distribution of effort contingent actions could have been achieved in the original equilibrium by choosing appropriately from the principals menus. Since the deviation was unprofitable in the original equilibrium it must also be unprofitable for the agent in the new continuation.

The next step is to show that the continuation equilibrium can be constructed in such a way that no firm has an incentive to unilaterally deviate in Γ^C from this construction (multilateral deviations can be treated in any convenient fashion). Suppose that some principal j deviates to γ'_j which offers the randomized incentive contract $g'_j(\hat{e}_j)$. Let c' be the joint distribution on $Y \times E \times \Omega$ in the continuation equilibrium relative to $\{\gamma_1^*, \dots, \gamma'_j, \dots, \gamma_n^*\}$. Let $c'(\omega)$ be the same joint distribution conditional on ω and let $\pi'(\omega)$ be the marginal distribution over effort associated with this conditional distribution. This is well defined since a deviation in Γ^C is also a deviation in Γ^* . Reasoning as above

$$\begin{aligned} \int_{Y \times E \times \Omega} v_j(y_j, y_{-j}, e, \omega) dc' &= \int_{Y_j \times E \times \Omega} V_j(y_j, e, \omega) dc' \\ &= \int_{\Omega} \int_E \int_{Y_j} V_j(y_j, e, \omega) dg'_j(\hat{e}_j) d\pi'(\omega) dF(\omega) \end{aligned}$$

which indicates that the deviating principal's payoff in Γ^C is the same as the payoff that he gets by making the same deviation in Γ^* . Since the deviation is unprofitable in the original equilibrium, it must also be unprofitable here.

Lastly, since the support of $g_k^*(\hat{e}_k)$ is contained in $A_k^*(\hat{e}_k)$ for each principal other than the deviator, if there were a profitable deviation in effort relative to Γ^C then this deviation would have been profitable in the original continuation as well, a contradiction. ■

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