Common Agency and the Revelation Principal: A Correction

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In (Peters 2001) at the bottom of page 1357, it is asserted that the outcome in the example on that page in which all three players get payoff 3/2 cannot be supported as an equilibrium if principals are only allowed to offer contracts that associate a single action with each different effort taken by the agent. This assertion is incorrect. If each principal offers a contract that selects action a_2 if e_2 is chosen and a_3 if e_1 is chosen, the agent will choose effort e_2 and there is will be no profitable deviation for either principal.

The agent's effort choice can be understood by observing that he would rather induce actions (a_2, a_2) by taking effort e_2, than actions (a_3, a_3) by choosing effort e_1. To see that there are no profitable deviations observe that the only way for a deviating principal to raise his payoff is to induce the agent to take effort e_1. Since the non-deviating principal takes action a_3 when he observes effort e_3, every action for the deviator yeilds less profit than $\frac{3}{2}$.

I thank Alredo Di Tillio for pointing this out to me.

The example given below has the desired property.

	e_1			e_2			
	a_1	a_2	a_3		a_1	a_2	a_3
a_1	$0,0,2$	$2,0,2$	$-1,5,3$	a_1	$0,0,0$	$0,0,0$	$0,0,0$
a_2	$0,2,2$	$1,1,1$	$0,0,1$	a_2	$0,0,0$	$\frac{3}{2},\frac{3}{2},\frac{3}{2}$	$0,0,0$
a_3	$5,-1,3$	$0,0,1$	$2,2,\frac{1}{2}$	a_3	$0,0,0$	$0,0,0$	$0,0,0$

In this example, each of the principals can choose from 3 distinct actions, labelled simply a_1 through a_3. The numbers in each cell represent the payoffs to respectively, principal 1 who chooses the row, principal 2 chooses the column and the agent who chooses the matrix. The rows and columns represent the simple actions available to each principal while the matrices are determined by the agent's action.

The *take it or leave it game* has two stages, each principal chooses a incentive contract which is a function that maps the set of efforts $\{e_1, e_2\}$ into the set of simple actions $\{a_1, a_2, a_3\}$ that are controlled by that principal. The agent then selects effort. The *menu game* is the same except that in the first stage each principal offers the agent a collection of incentive contracs. In the second

stage the agent chooses an incentive contract from each collection then picks his effort.

The example has the property that the outcome (a_2, a_2, e_2) cannot be supported in equilibrium in the take it or leave it game, but it can be supported in the menu game.

Claim 1 *The outcome (a_2, a_2, e_2) cannot be supported by pure strategy equilibrium in the menu game..*

Proof. First observe that if the actions (a_2, a_2, e_2) are supported by some equilibrium, then neither principal can offer the action a_1 in response to effort e_1. The reason is that the agent would then take effort e_1 no matter what action the other principal associated with this effort. So suppose that one of the principals commits to using a_2 when the agent's effort is e_1. The other principal's (strict) best reply is to offer a contract that associates a_1 with effort e_1. This would induce the agent to select effort e_1 instead of e_2 raising the deviating principal's payoff to 2. As mentioned above, the action a_1 cannot be associated with effort e_1 in any equilibrium that supports outcome (a_2, a_2, e_2).

So suppose instead that one of the principals offers a_3 when e_1 is observed (and a_2 when e_2 is observed). The other principal's strict best reply is to offer a contract that commits to a_3 in response to both effort levels. This would force the agent to use effort e_1 and raise this second principal's payoff to 2. ■

When principals offer menus of incentive contracts, deviations by one principal can induce the agent to select a different incentive contract. This will generally change both the action take by the non-deviating principal and the agent's effort. These change might harm the deviator, and this might support new equilibrium outcome.s

Claim 2 *The outcome (a_2, a_2, e_2) can be supported as a pure strategy equilibrium in the menu game.*

Proof. Suppose that each principal offers the agent the choice between a contract that commits to a_2 for *both* levels of effort, or a_3 for both levels of effort. The agent's best choice when offered these menus is evidently to choose the contracts giving a_2 from both principals, then select effort e_2.

As before, if there is a profitable deviation for one of the principals it must induce the agent to take effort e_1. The deviating principal can do this by allowing the agent to choose any action to associate with e_1, or by restricting the action associated with effort e_1 to either a_3 or a_1. In the first and last of these cases, the agent will choose a_1 from the deviator and a_3 from the menu offered by the non-deviator to attain the payoff 3. The deviating principal will get payoff -1 in this case. If the deviator instead restricts the agent's choice to a_3, the agent will choose a_2 from the non-deviator's menu and the deviator's payoff again falls. ■

References

PETERS, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.