CAN MECHANISM DESIGNERS EXPLOIT BUYERS’ MARKET INFORMATION

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ABSTRACT. It is known that mechanism designers can extract agents’ information about competitors’ mechanisms in a competing mechanism game. This makes it possible for sellers to punish each other for deviations even when they do not directly observe other sellers’ mechanisms. This allows for very collusive equilibrium. This paper is concerned with whether these collusive equilibrium can be sustained in large markets when buyers either don’t understand sellers mechanisms, or find it costly to convey their market information. We provide conditions under which collusive equilibrium will persist in large markets and conditions under which all equilibrium of the competing mechanism game will become competitive.

1. INTRODUCTION

Digital markets represent a kind of classic example of competing mechanism games. Sellers sell through websites that output price offers that depend on messages received from clients. The sellers have perfect commitment power because the websites are just computer programs that respond to instructions created beforehand. The messages processed by these computer programs aren’t type or value reports. They are bits of the potential buyer’s browsing history that are passed to the program running the websites using cookies that are stored on the buyer’s computer. Alternatively, the message may be nothing more than the time at which a buyer requests a quote.¹

This is somewhat troublesome, because games in which sellers compete against each other using mechanisms can in general support a lot of equilibrium outcomes. The reason is twofold. First, sellers’ themselves need to send messages to explain their commitments. If other sellers’ mechanisms detect these messages, they can build in responses. For example, airline aggregator sites like Travelocity.com or Cheapair.com

¹For example, business class travelers on airlines are supposed to ask for quotes near the time of the flight, while leisure travelers ask for quotes much earlier.
publish price offers from many different airlines. It is a straightforward task for each airline to monitor these sites, then trigger changes in their own prices in response to what they learn. This basic point has been known since Salop described 'meet the competition pricing' ((Salop 1986)), and showed that guaranteeing to match competitors price offers is very anti-competitive.

Even if firms don’t directly monitor their competitors prices, they can do this indirectly by interpreting messages exchanged with their buyers. Buyer search behavior is no longer private, since firms can place cookies on browsers in order to track search behavior. If a buyer who might normally be expected to purchase from the firm delays that purchase, it might reflect the fact that that buyer has seen discounts offered by other firms. The firm can then tell its website to adjust prices if it observes this kind of behavior.

The implications are far more general that those suggested in Salop’s paper. The argument extends to more general games with many players and incomplete information.² If sellers can choose from a rich set of contracts that condition their price offers on messages sent by buyers, then any outcome function that is incentive compatible and individually rational in the sense of Myerson’s textbook (Myerson 1997) can be supported as a perfect Bayesian equilibrium outcome in a single competing mechanism game. It is worth noting that this is more than a simple restatement of the revelation principle. Like Salop’s game, all these outcomes can be supported as equilibrium in the same competing mechanism game.

The theorems in the competing mechanism literature that describe these very collusive equilibrium rely on three basic assumptions. First, they assume that buyers can convey very complex messages, and that sellers’ mechanisms are sophisticated enough to make use of these.³ Second, they assume that communication is costless. Third, they assume that market participants can both see and understand the commitments that are being made by others in the market.

The point of this paper is to relax these three assumptions to see whether mechanisms that depend on market information might be a practical issue. We focus on a market setting in which sellers make commitments that are based on very simple and realistic messages. Indeed, all sellers can do in our model is to monitor visits to a website. Our buyers have somewhat limited access to information about sellers’

²(Peters 2010)
³Conceptually, when a buyer tries to convey his market information he has to describe an infinite hierarchy of dependencies which will often require a complex message. See (Epstein and Peters 1999).
commitments, indeed most buyers won’t know anything about them at all. Finally, we’ll assume that buyers who do have important market information may bind it costly to communicate this information.

The main contribution here, apart from an illustration of how collusive equilibrium can arise when messages are very simple, is a theorem that illustrates the conditions under which collusive equilibrium breaks down. We show that as the market grows large, if either of two limiting conditions holds, then the set of perfect Bayesian equilibrium will collapse so that only the competitive equilibrium remains. What is perhaps surprising about these conditions is that the very things that make mechanisms work so well—communication and commitment—make markets work very badly. Markets work well when buyers don’t understand sellers’ commitments, and can’t communicate the information they do have.

It is worthwhile noting that the conceptual apparatus used in this paper is elementary, in contrast all of the existing literature in competing mechanisms.

1.1. Environment. The essence of a competing mechanism game is a set of commitments designed to influence buyers’ choice of trading partners. Since we aren’t interested here in the matching process per se, we ‘black box’ it, by focusing on a game built on top of a double auction. The auction carries out the matching of apparently willing buyers and sellers without the need to model complex search or communication strategies, while at the same time preserving the essential feature that sellers want to use price to compete for buyers. Furthermore, by using the highest losing bid pricing rule, we ensure that we can always construct equilibrium in which buyers bid their true values. This eliminates the need to worry about building incentive compatibility for buyers into mechanisms embedded in a market setting.

Similarly, the point of this paper is not to study the properties of double auctions in large markets. These properties are well known—double auctions support efficiency when markets are large.\(^4\) So we restrict attention to a very simple physical environment where it is very easy to solve for the equilibrium in the double auction. This makes it possible to focus on the main issues of buyer understanding and communication costs.

In this very simple environment, there are \(n\) buyers and \(n+1\) sellers. Each seller has a single unit of a homogeneous good to sell. Each buyer wants to acquire exactly one unit. The sellers’ goods are all perfect substitutes. The value of a buyer or seller with value \(v_i\) who

\(^4\)(Cripps and Swinkels 2006) or (Satterthwaite and Williams 1989).
trades at price \( p \) is \((v_l - p)\) for the buyer and \((p - v_l)\) for the seller. Buyers values are either high \( v_h \) or low \( v_l \) with \( v_h > v_l > 0 \). Each buyer’s valuation is independently drawn, which \( v_h \) being drawn with probability \( \pi \). We’ll assume that all sellers have cost 0, and that this is common knowledge.

In the terminology of (Peters 2010), the double auction is the default game. The competing mechanism game built around this is one in which sellers make commitments about how they will play this game. In the double auction, a seller submits an ask price, while buyers submit bids.\(^5\) The rules of the double auction stipulate that the \( n + 1 \) goods end up in the hands of the \( n + 1 \) traders who submitted the highest bids or asks (transferring the good to buyers in the case of ties), while all trades occur at a price that is equal to the \( n + 2^{nd} \) highest bid or ask.

To describe the competing mechanism game, we need to describe message spaces and mechanisms. We’ll assume that each buyer visits each seller’s website at least once, and at most twice. Since the first visit by buyers is uninformative, assume that each seller’s mechanism counts the number of buyers who visit his website a second time, then submits an ask price that depends on this total. The message space for each buyer could then be modeled as the pair \( \{0, 1\} \). The message 1 is conveyed when a buyer visits a seller’s website for the second time. The message 0 is conveyed when the buyer visits just once. Let \( a_j : \mathbb{N} \rightarrow [0, v_h] \) be the mechanism used by seller \( j \). Sellers write their programs before the market opens.

The logic we have in mind is that a buyer’s initial visit to a seller’s website is not informative to the seller, since all buyers behave the same way, and no buyer has any information when they first visit. What the buyer learns on the first visit is random. Some buyers learn nothing at all. With some probability \( \gamma(n) \) the buyer will learn something about the seller’s mechanism. All the buyer really needs to see is a single price commitment representing the ask price the seller commits to if no buyers visit him for a second time. Think of this as the price the seller’s website says the seller will submit as an ask price in the double auction. Whether buyers understand anything more about a seller’s mechanism isn’t important in what follows. Sellers understand that only some buyers will understand or see this commitment. Sellers do not know whether any particular buyer has understood their commitment or not.

\(^5\)In (Peters and Severinov 2006) it is shown that there are equilibrium in a competing auction game, where sellers host auctions on their own website which mimic the equilibrium in double auctions.
Note that buyers here are assumed to be fully rational even when they don’t learn anything about the seller’s commitment. They are assumed to fully understand what sellers are doing in the equilibrium that follow.

Once all buyers have made their first visit to each seller’s website and learned whatever they happen to learn, they choose whether or not to make a second visit. We’ll assume that the second visit is costly - in particular, a buyer who chooses to make a second visit pays a cost $\epsilon(n)$, where $\epsilon$ is a non-decreasing function of $n$. We assume that a buyer who chooses to pay this cost can visit all sellers’ websites without incurring any further cost.

After all buyers who want to make their second visits, buyers then submit bid prices in the double auction.

One obvious equilibrium of this competing mechanism game occurs when sellers ignore buyer messages entirely and all players simply play the double auction non-cooperatively. In the environment considered here there is only one equilibrium like this - all sellers ask 0, which is the trading price independent of the profile of buyer values. This equilibrium is efficient.

There is also a very bad equilibrium in which all buyers bid 0 and all sellers ask $v_h$.

The outcome we are interested in here is the one that maximizes the expected revenue of a representative seller. In this simple environment that outcome is either the one which all sellers submit the ask price $v_h$ and each seller sells at that price with probability $\frac{n}{n+1}$, or the one in which all sellers ask $v_l$ each each seller trades with probability $\frac{n}{n+1}$. The latter outcome is one in which efficient trade occurs, but sellers extract all the surplus. The former outcome has the property that sellers inefficiently hold back their output to raise prices, so we’ll assume that $v_h \pi > v_l$ and focus on the former outcome. The arguments made below can be applied equally in the other case.

The folk theorem for competing mechanisms would say that there is a perfect Bayesian equilibrium in which sellers expected revenue is $v_h \pi \frac{n}{n+1}$, and that this supports buyers purchasing if and only if their value is $v_h$. We’ll illustrate the logic behind that theorem shortly. For

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6The reason is simply that when there are sellers whose ask prices are positive, then the seller with the highest ask price will have at least the $n+1^{st}$ highest bid or ask price no matter what the buyers values are, and he won’t trade. If the $n+2^{nd}$ highest bid or ask is also positive for some profiles of buyer values, this high price seller will strictly gain by undercutting the lowest possible value of the $n+2^{nd}$ highest bid. If the $n+2^{nd}$ highest bid is always zero, then one of the existing sellers will have an incentive to raise his bid.
the moment, notice that existing folk theorems assume that all buyers
know the mechanisms that are being used by the sellers, and they can
communicate those mechanisms to other sellers. In this paper, we are
interested in what happens when this is not the case.

1.2. How the collusive outcome is supported as a perfect Bayesian
equilibrium. To describe the collusive equilibrium, recall there are \( n \)
buyers. Each sellers’ mechanism specifies an ask price as a function
of the number of buyers who visited their website. Call the following
program a reciprocal pricing program:

\[
a(j) =
\begin{cases} 
  v_h & j < 2 \\
  0 & \text{otherwise}.
\end{cases}
\]  

(1.1)

The idea behind the reciprocal pricing program is that the seller is
going to set a high price unless 2 or more buyers visit his website. The
interpretation is that when buyers see one of the sellers doing something
else, then they will pay the cost to visit sellers’ websites.

Proposition 1. If

\[
n\gamma(n)(1 - \gamma(n))^{n-1} + (1 - \gamma(n))^n < \frac{n}{n+1}\pi
\]

(1.2)

and

\[
n\gamma(n)(1 - \gamma(n))^{n-1} \geq \epsilon(n)/v_l
\]

(1.3)

there is a perfect Bayesian equilibrium for the competing mechanism
game in which each seller uses a reciprocal pricing mechanism. Buyers
trade if and only if their values are \( v_h \).

Proof. The rule we want buyers to use is for each buyer to visit each
seller’s website before bidding in the double auction if and only if one
or more of the mechanisms that the buyer observes is something other
than a reciprocal pricing mechanism. We’ll assume that once buyers
have visited websites they will bid their values in the double auction.

Given the buyers’ strategy rule, a seller who uses a reciprocal pricing
mechanism and expects all the others to use such a mechanism will earn
\( v_h \) when he trades. He trades with probability

\[
\sum_{l=0}^{n} \frac{n!}{(n-1)!!} \pi^l (1 - \pi)^{n-l} \frac{l}{n+1} = \frac{n}{n+1}\pi
\]

giving expected profit \( \pi v_h \frac{n}{n+1} \).
If the seller deviates, he should expect to trigger significant price cuts by the other sellers if two or more buyers observe his deviation. This event occurs with probability
\[ 1 - n\gamma(1 - \gamma)^{n-1} - (1 - \gamma)^n, \]
which is high if there are lots of buyers, but less than one. This notation suppresses the dependence of \( \gamma \) on \( n \). Since \( n \) is fixed in the rest of this argument, the balance of the proof suppresses this dependence for both \( \gamma \) and \( \epsilon \).

In this event, all of the other \( n-1 \) sellers will cut their ask prices to zero. Since all \( n \) buyers will have values above zero if they bid truthfully, the \( n+1 \)st highest bid or ask will be the ask price of a seller, who won’t trade as a result. The trading price will be 0 since the deviator’s ask price must be non-negative. So the deviating seller can’t earn a profit if his deviation is detected by two or more buyers.

The deviating seller can use any price function that he wants. To understand what happens when there is a deviation, the most important fact is that a buyer who decides to visit a website, always visits all of them. So all sellers are going to have the same number of messages after a deviation. If the number of messages is 0 or 1, all the other sellers will ask price \( v_h \) in the double auction. The most profitable deviation is one in which the deviator slightly undercuts this price in those two events. Otherwise the deviator can’t make a profit.

This deviation means that the deviator’s ask price will be the (unique) \( n+1 \)st highest bid or ask price if the buyers all have low values, the \( n+2 \)nd highest bid or ask price if exactly one bidder has a high value, and the \( n+3 \)rd or higher bid or ask price in every other case. In the case where there are no high value buyers, the deviator won’t trade. In all other cases, the deviator will trade for sure at a price that his arbitrarily close to \( v_h \).

There is a discontinuity in the payoff to the deviator at price \( v_h \) because the deviator will be rationed. Yet because he can price arbitrarily close to \( v_h \), a necessary and sufficient condition for the deviation to be unprofitable is that it be unprofitable when the deviator trades for sure at price \( v_h \) whenever there is at least one high value buyer.\(^7\)

A deviator who undercuts will then make a profit in two events, the first is the event in which no buyers visit websites, but at least one of them has a high value. The second event in which the deviator makes a

\(^7\)The deviating seller could also ensure trade when there are one or fewer website visits by setting price \( v_l \). As we assumed \( v_h \pi \frac{d}{n+1} > v_l \), this won’t be as profitable as a deviation to a price slightly less than \( v_h \).
profit is when 1 or more buyers visit websites. Formally, the deviator’s profit is
\[
\left( (1 - \gamma)^n + n\gamma (1 - \gamma)^{n-1} \right) (1 - (1 - \pi)^n) v_h.
\]
So the deviation is unprofitable provided this is less than or equal to
\[
v_h \pi \frac{n}{n + 1}.
\]
This is the condition in the theorem.

We now want to show that buyers want to inform sellers when they see a deviation, provided their cost of doing so isn’t too large. One issue is that when buyers see a deviation by one seller, their continuation behavior is based on their beliefs about the mechanism used by the sellers they didn’t observe. We’ll assume they believe that those sellers continue to use reciprocal pricing mechanisms. If two or more of the other buyers have seen the deviation, they will report it and price will fall anyway. If no other buyers have seen the deviation, then informing sellers will have no effect, since they ignore unilateral messages. So all the gains to reporting occur in the event that a single one of the other buyers observes the deviation. This occurs with probability \(n\gamma (1 - \gamma)^n\). In that case, the trading price if the buyer reports will trigger punishment by the others. The trading price will be 0 once the punishment has been triggered because \(n\) buyers will bid at least \(v_l\), the deviator will ask some price \(v' > 0\), so that the \(n + 2^{nd}\) highest bid or ask will be zero.

So the expected gain to reporting the deviation is \(n\gamma (1 - \gamma)^{n-1} v_i\), where \(v_i\) is either \(v_l\) or \(v_h\). Then there is a perfect Bayesian equilibrium in which all sellers use reciprocal pricing mechanisms provided \(n\gamma (1 - \gamma)^{n-1} v_i \geq \epsilon\), the condition in the statement of the theorem. □

To interpret the two conditions in the statement of the theorem, the left hand side of the first condition is the probability with which 1 or fewer buyers will see the deviation. If all buyers visit websites whenever they see a deviation, then this is the probability that a seller will get away with a deviation. This probability should be lower than the probability with which the seller is rationed if he goes along with the equilibrium.

The left hand side of the second condition is the probability that only one other buyer will see the deviation when it occurs. This is the only case in which it actually pays the buyer to report the deviation. So the condition becomes a simple statement that it pays the buyer to visit websites even though it is costly.

Of course, both these conditions depend on the probability with which buyers observe the mechanisms of individual sellers and the costs
of reporting. From the last line in the proof, it might seem that when there are many buyers, buyers will stop reporting deviations because it is so unlikely that their reports will make any difference. This is correct as far as it goes, except that the calculation assumes that buyers believe that all the other buyers will report. If the inequality in the proof fails, then buyers will report with probability less than one and we will have to modify the calculation. This is dealt with below.

This example also illustrates that the messages that buyers send do not have to be complex to support outcomes with inefficiently high prices. Compare the simple messages here with the recommendation mechanism in (Yamashita 2010), the universal type reports in (Epstein and Peters 1999) or the mechanisms that depend explicitly on other mechanisms in (Peters and Szentes 2012).

Notice that there is some similarity in this story to the dynamic pricing that airlines use to price tickets - if the airline isn’t filling as many seats as it expected at some point in time, it will discount. If the slow down is caused by a competitor’s sale, then buyers may anticipate the price response and delay their purchases, confident that the other buyers will confirm their behavior and ensure that the price response follows. As in the meet the competition argument, the competitor realizes that sales are pointless and doesn’t cut price.

1.3. Equilibrium in Large Markets. Large markets present two complications. When there are many buyers, reporting deviations by visiting websites no longer pays because each buyer wants to rely on the others to report. If we want to maintain reporting at all, then buyers will have to adopt mixed reporting strategies. Then, as the market grows, buyers may be less likely to see any particular deviation and will face much higher costs of reporting it.

Secondly, when the simple kind of equilibrium described above breaks down, there could in principle be many equilibrium outcomes. To keep from having to deal with very abstract asymmetric equilibrium outcomes, suppose we consider only equilibrium in which sellers use symmetric, anonymous and non-random mechanisms. Symmetric seems reasonable in the large markets we are interested in since asymmetric equilibrium would require buyers in particular to understand strategies of a lot of other players. In symmetric equilibrium they only need to have rational expectations about aggregates. Non-random mechanisms are simply to keep the analysis tractable. Anonymous mechanisms are reasonable in an environment where sellers only observe the number of buyers who visit their websites.
In a symmetric equilibrium, all sellers offer the same mechanism. Let $r_h$ or $r_l$ be the probabilities that high and low types, respectively, visit websites. Sellers then receive some random number of visits. Start with the equilibrium path outcome. In a symmetric equilibrium, all sellers receive the same number of messages $j$. Since all sellers are using the same mechanism, they all set the same ask price, say $a(j)$. Since there are $n + 1$ sellers asking $a(j)$, the trading price will be $a(j)$ for every profile of buyer values except possibly for the one in which all buyers have low value. In that case, if $a(j) > v_l$, no trades will occur and the $n + 2^{nd}$ highest bid or ask would be $v_l$.

There can be many price functions $a$ that are supported as equilibrium. What the next theorem shows is that this multiplicity is only possible if the most collusive equilibrium in which prices are equal to $v_h$ can be supported.

**Theorem 2.** Suppose there is a symmetric equilibrium in which all sellers use price function $a(\cdot)$ and $a(j) > 0$ with strictly positive probability on the equilibrium path. Then then there is also an equilibrium in which all sellers set ask price $v_h$.

**Proof.** Costly website visits pay off to the buyer when they succeed in lowering the ask prices of all the buyers. On the equilibrium path, all sellers set the same ask price $a(j)$, where $j$ is the realized number of visits. From the perspective of any buyer, the probability that any other buyer will report is $\pi r^* = \pi r^* = r^*$, where $r^*$, $r^*_h$ and $r^*_l$ are to be thought of as equilibrium visiting probabilities.

In what follows, we will suppress the dependence of $r$ on $r_h$ and $r_l$ to make the notation a bit simpler. Also, we’ll assume in what follows that $a(j) \leq v_h$. This entails no loss. If $a(j)$ doesn’t satisfy this condition, then we can replace each price $a(j) > v_h$ with $v_h$ without changing any buyer or seller’s payoff.

Since all sellers see the same number of messages $j$, and all use the same rule, the profile of ask prices is $a(j)$ when there are $j$ website visits. The $n + 2^{nd}$ highest bid or ask will then be $a(j)$ except for the case where $a(j) > v_l$ and there are no high value buyers. In that case no seller will trade.

Define $Q(j)$ to be the probability with which a seller will trade conditional on $j$ website visits. Let $p^* > 0$ be the expected payoff of each seller when all sellers stick to the equilibrium pricing rule. Obviously $p^* \leq v_h \pi \frac{n}{n + 1}$.

Now consider a deviation by some seller to an ask price mechanism that sets the ask price for each $j$ to $a(j) - \delta$, for some arbitrarily small $\delta$ and let $r(\delta)$ be the equilibrium visiting probability. Let $r$ be any limit
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point of \( r(\delta) \) as \( \delta \) goes to zero. The limit point \( r \) might be different from \( r^* \).

From the perspective of a high value buyer, there will always be at least one high value bidder in the double auction, so that the \( n + 2^{nd} \) highest value (which determines the trading price) at the limit of this sequence will always be \( a(m) \). The payoff associated with a website visit for the high value buyer must be

\[
\sum_{j=0}^{n-1} \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j} (a(j) - a(j+1)) .
\]

The argument is that if \( j \) of the other buyers choose to visit websites, then the high value buyer will trade and earn surplus \( v_h - a(j) \) if he doesn’t visit, and \( v_h - a(j+1) \) if he does.

For this to have a positive value to to the high value buyer, \( a(j) > a(j+1) \) for some \( j \).

From the perspective of a low value buyer, the only difference is that when all buyers have low values and \( a(j) > v_l \), the \( n + 2^{nd} \) highest bid or ask will be \( v_l \). However, since the low value bidder won’t trade in this event anyway, the low value bidder’s payoff will be the same as it would if the price had been \( a(j) \). So the payoff to a low value buyer who visits can be written

\[
\sum_{j=0}^{n-1} \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j} \{\max [0, v_l - a(j+1)] - \max [0, v_l - a(j)]\}
= \sum_{j=0}^{n-1} \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j} \left(\min [a(j), v_l] - \min [a(j+1), v_l]\right).
\]

The payoff received by the deviating seller is close to

\[
(1 - (1 - \pi)^n) a(0) + \sum_{j=1}^{n} \frac{n!}{j! (n-j)!} \gamma r^j (1 - \gamma r)^{n-j} a(j)
\]

if \( a(0) > v_l \), since the deviating seller sells for sure at price \( a(j) \) in the limit, provided there is at least one high value buyer. This is because the deviator always has a slightly lower price than the non-deviators.

To support the equilibrium, at least one of the following two inequalities must hold

\[
\sum_{j=0}^{n-1} \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j} (a(j) - a(j+1)) \geq \epsilon,
\]

\[
(1 - (1 - \pi)^n) a(0) + \sum_{j=1}^{n} \frac{n!}{j! (n-j)!} \gamma r^j (1 - \gamma r)^{n-j} a(j) \geq \epsilon.
\]
and
\[ \sum_{j=0}^{n-1} \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j} \left( \min [v_l, a(j)] - \min [v_l, a(j+1)] \right) \geq \varepsilon. \]

Whenever \( r_h \) or \( r_l \) respectively, are strictly positive but less than 1, these inequalities become equalities.

The deviation we are considering is unprofitable given the pricing function \( a \) and the limit point \( r \) of the continuation equilibrium. What we want to do now is to show that this information allows us to construct a punishment that supports the equilibrium where firms all ask \( v_h \).

In the continuation equilibrium following the deviation, buyers must visit websites with positive probability, otherwise the deviation would be profitable. Suppose first that in this continuation equilibrium, the high value buyer visits websites with strictly positive probability. The argument for the case where only low value buyers visit websites is an extension of this argument.

Consider the following minimization problem:
\[
\min_{a'(0), \ldots, a'(n)} (1 - (1 - \pi)^n) a'(0) + \sum_{j=1}^{n} b(j) a'(j)
\]
subject to
\[
\sum_{j=0}^{n-1} q(j) (a'(j) - a'(j+1)) \geq \varepsilon; \\
(1.7) \quad a'(0) = a(0); \\
(1.8) \quad v_h \geq a'(j) \geq 0.
\]

In this expression
\[ b(j) = \frac{n!}{j! (n-j)!} (\gamma r)^j (1 - \gamma r)^{n-j} \]
and
\[ q(j) = \frac{(n-1)!}{j! (n-1-j)!} (\gamma r)^j (1 - \gamma r)^{n-1-j}. \]

Observe from this that
\[ b(j) = \frac{n}{n-j} (1 - \gamma r) q(j). \]

In this program, we have fixed the limit reporting strategy at \( r \) and ignored the incentive constraint for the low value buyer. The original pricing function \( a \) satisfies all the constraints (since we assumed high
value buyers visit websites with strictly positive probability) and yields the deviator a lower payoff than he would have achieved in equilibrium. So the value provided by the solution to this program is also lower than the deviator’s equilibrium payoff.

Let \( \lambda_0 \) be the Lagrange multiplier associated with constraint (1.6), and \( \lambda_1 \) and \( \lambda_2 \) the multipliers associated with the two constraints in (1.8). The first order condition for \( a'(j) \) when \( j \geq 1 \) is

\[
\frac{n}{n-j} (1 - \gamma r) q(j) + \lambda_0 (q(j) - q(j-1)) + \lambda_1 + \lambda_2 = 0.
\]

Because the payoff function is linear, each \( a'(j) \) in the optimal solution will be either \( v_h \) or 0, except for possibly for one value of \( j \) where it will be interior.

The continuation equilibrium in which only low value buyers report is handled in exactly this way, except that in the constraint \( v_h \geq a'(j) \) is replaced with \( v_l \geq a'(j) \), and the value minimizing price function has prices equal to \( v_l \) for \( j \) less than some critical value, and 0 thereafter.

The first two terms in the first order condition can be written as

\[
q(j) \left[ (1 - \gamma r) + \lambda_0 \left( 1 - \frac{j(1-\gamma r)}{(n-j)\gamma r} \right) \right].
\]

The term \( 1 - \frac{j(1-\gamma r)}{(n-j)\gamma r} \) is monotonically decreasing in \( j \). So it changes sign from positive to negative at most once. For this reason, the entire expression changes sign at most once at some threshold value of \( j \).

The solution can’t have \( a'(j) = v_h \) for all \( j \), because buyers would not visit websites, and (1.6) would be violated. Similarly, the solution could not have \( a'(1) = 0 \), then \( a'(j) = v_h \) for larger values of \( j \), since visiting a website would then be more likely to raise prices by \( v_h \) than to lower them by a \( (0) \) - buyers would not visit websites and again (1.6) would be violated.

So the price function that minimizes the deviator’s payoff either falls immediately to zero (in which case (1.6) requires \( (1 - \gamma r)^{n-1} a(0) = \epsilon \), or prices rise immediately to \( v_h \), then falls to an intermediate value \( a' \) at some critical value of \( j \), then collapse to 0 for higher values of \( j \). The intermediate price at some value of \( j \) ensures that (1.6) holds with equality.

Now observe that we can make the expected payoff to the deviator even lower by relaxing the constraint (1.7). This shows that when buyers visit websites with probability \( r \), there is some \( k \geq 1 \) such that the pricing rule \( a'(j) = v_h \forall j < k \), \( a'(k) = a' \); and \( a'(j) = 0 \) otherwise, yields the deviating seller a payoff less than \( p^* \pi \frac{n}{n+1} \). Of course this implies the deviator’s payoff is less than \( v_h \pi \frac{n}{n+1} \).
To handle the case where the initial equilibrium has only low value buyers reporting, we begin by truncating the pricing rule so that 
\[ a'(j) \leq v_l. \] This obviously lowers the deviator’s profit without affecting the reporting incentives of the low value buyer. In the minimization problem, we replace the constraint \( v_h \geq a'(j) \) by \( v_l \geq a'(j) \) and use the same reasoning to show that there is a pricing function for which \( a'(j) = v_l \) for all \( j \) less than some critical value, and \( a'(j) = 0 \) otherwise, which provides a lower payoff to the deviator than what he gets against the continuation equilibrium.

This pricing function, however, provides exactly the same reporting incentives to the high value buyer as it does to the low value buyer. At that point we can revert to the argument given above for the case where only high value buyers report, and continue with the rest of the proof.

Since the pricing rule satisfies (1.6), we have
\[ b(k - 1) (v_h - a') + b(k) a' = \epsilon, \]
which implies
\[ b(k - 1) v_h + (b(k) - b(k - 1)) a' = \epsilon \]
or
\[ b(k - 1) = \frac{\epsilon}{\alpha(k - 1) v_h + (1 - \alpha(k - 1)) a'} \]
where \( \alpha(k - 1) = \frac{b(k-1)}{b(k)} \).

The deviator’s profit can then be written
\[ (1 - (1 - \pi)^n) v_k + \sum_{j=1}^{k-2} b(j) v_h + \frac{\epsilon}{\alpha(k - 1) v_h + (1 - \alpha(k - 1)) a'} v_h + b(k) a'. \]
Now allow \( r \) to vary so that (1.6) is satisfied, and modify the pricing rule until \( a(j) = v_h \) for \( j \leq 1 \) and \( a(j) = 0 \) otherwise. Then using (1.6), the deviator’s profit under this new pricing rule is
\[ (1 - (1 - \pi)^n) v_k + \frac{\epsilon}{v_k v_h} \]
which is less than (1.9).

So the deviator’s expected profit is under this new pricing rule is less than \( v_h \pi \frac{n}{n+1} \). The implication is that there is an equilibrium in which the pricing rule is \( a(j) = v_h, j \leq 1, a(j) = 0 \) otherwise. This rule has a continuation equilibrium in which buyers do not visit websites when sellers use this rule, and visit websites with some positive probability if someone doesn’t. This supports the payoff \( v_h \pi \frac{n}{n+1} \) as an equilibrium.  
\[ \square \]
We want to make the argument that if the market is very large, then the only equilibrium outcome will be competitive under appropriate conditions. In order to do this, we are going to make use of Theorem 2 by providing conditions under which the collusive outcome can no longer be supported as an equilibrium. By Theorem 2, it then follows that there is no equilibrium in which sellers make positive profits.

Theorem 2 showed that collusive equilibrium with a price equal to $v_h$ could be supported by having the price fall to zero after 2 or more website visits.

**Theorem 3.** Define $x \equiv \lim_{n \to \infty} \frac{1}{\gamma(n)} \frac{1}{\pi_n}$. For large $n$, the outcome where all sellers ask $v_h$ can be supported as a perfect Bayesian equilibrium if and only if

\[
\begin{align*}
\lim_{n \to \infty} \frac{\epsilon(n)}{v_h} &\leq \frac{1}{x} e^{-x} & x > 1 \\
\lim_{n \to \infty} \frac{\epsilon(n)}{v_h} &\leq e^{-1} & \text{otherwise}
\end{align*}
\]

and

\[
2 \lim_{n \to \infty} \frac{\epsilon(n)}{v_h} \leq \pi.
\]

To interpret this theorem, note first note how the two issue, observability and communication costs interact. The limit $x$ is the inverse of the average number of high value buyers who actually see the mechanism being used by a seller. So a high value of $x$ indicates that very few buyers actually know what any particular seller is doing.

The limit of $\epsilon/v_h$ is the proportion of the surplus that is taken up by communication in a large market. No matter how low communication costs are, it will make little difference if buyers don’t understand sellers’ mechanisms in the first place. This is what the first condition in (1.10) indicates. If $x > 1$, then communication costs have to be very low before collusive outcomes can be sustained.

What goes wrong when either of the conditions in (1.10) fails is that it becomes impossible to make buyers visit websites. The chance that they can make a difference in price is just too small to compensate for the cost of communication.

The second condition (1.11) determines whether reports will actually deter deviations even when they are possible. When the probability that buyers have high values is high, the collusive outcome is very attractive, and sellers will not risk it by undercutting if they know that some buyers are going to see and report their deviations.

We remark further, that if either of the conditions above fails, then by Theorem 2, the only remaining equilibrium are those in which sellers ask prices are all zero - in other words, all equilibrium are competitive.
Proof. From Theorem 2, the most collusive outcome can be supported if it can be supported with a pricing rule that sets price to zero after 2 or more website visits. Again, assuming that only high value buyers visit websites, the most collusive outcome can be supported only if the two conditions hold. First,

\[(1.12) \quad (1 - \gamma(n) \pi r_h)^n + \frac{n(1 - \gamma(n) \pi r_h)\epsilon(n)}{(n - 1)v_h}\]

ensures that a seller doesn’t want to defect. Second, the equation

\[(1.13) \quad (n - 1) \gamma(n) \pi r_h (1 - \gamma(n) \pi r_h)^{n-2} = \frac{\epsilon(n)}{v_h}\]

has to have a solution with \(r_h \in (0, 1)\).

The next step is to show that as long as \(x < 1\), \(\lim_{n \to \infty} \epsilon(n) < v_h/e\) guarantees that (1.13) always has a positive solution. The left hand side of this expression is continuous, and has value 0 when \(r_h = 0\). The maximum value of the left hand side can be found using the first order condition

\[(n - 1) \gamma(n) \pi r_h (n - 2) \gamma(n) \pi (1 - \gamma(n) \pi r_h)^{n-3} = (1 - \gamma(n) \pi r_h)^{n-2}(n - 1) \gamma(n) \pi.\]

Canceling the common terms gives

\[r_h \gamma(n) \pi (n - 2) = (1 - \gamma(n) \pi r_h)\]

\[r_h (\gamma(n) \pi (n - 2) + \gamma(n) \pi) = 1\]

\[(1.14) \quad r_h = \frac{1}{\gamma(n) \pi (n - 1)}.\]

Substituting this into the left hand side of (1.13) gives

\[\left(1 - \frac{1}{n - 1}\right)^{n-2}.\]

The limit of this expression as \(n \to \infty\) is \(e^{-1}\), which is larger than \(\frac{\epsilon(n)}{v_h}\) under the hypothesis of the theorem.

Finally, the second term in (1.12) has the same limit as \(\frac{\epsilon(n)}{v_h}\). We want to focus on the first term. This term is

\[(1 - \gamma(n) \pi r_h)^n = (1 - \gamma(n) \pi r_h)^2 (1 - \gamma(n) \pi r_h)^{n-2} =\]

\[\frac{(n - 1) \gamma(n) \pi r_h}{(n - 1) \gamma(n) \pi r_h} (1 - \gamma(n) \pi r_h)^2 (1 - \gamma(n) \pi r_h)^{n-2} =\]

\[\frac{(1 - \gamma(n) \pi r_h)^2 \epsilon(n)}{(n - 1) \gamma(n) \pi r_h v_h}.\]
By the properties of the binomial probability distribution, 
\[
\frac{(1 - \gamma(n) \pi r_h)^2}{(n-1) \gamma(n) \pi r_h} \leq 1
\]
(since the probability assigned to exactly \(k\) successes in \(n\) trials is non-decreasing in \(k\) when \(k\) is small (i.e., less than the mean number of successes).

By the argument that showed that a solution to (1.13) exists, we have \(r_h < \frac{1}{\gamma(n) \pi (n-1)}\). When \(r = \frac{1}{\gamma(n) \pi (n-1)}\),
\[
\frac{(1 - \gamma(n) \pi r_h)^2}{(n-1) \gamma(n) \pi r_h} = \left(1 - \frac{1}{n-1}\right)^2.
\]
Since the left hand side of this expression is decreasing in \(r\) we have
\[
1 \geq \frac{(1 - \gamma(n) \pi r_h)^2}{(n-1) \gamma(n) \pi r_h} \geq (1 - \frac{1}{n-1})^2.
\]
Taking limits as \(n\) goes to infinity gives
\[
\lim_{n \to \infty} \frac{(1 - \gamma(n) \pi r_h)^2}{(n-1) \gamma(n) \pi r_h} = 1.
\]
Substituting this into (1.12) and taking limits shows that the probability with which a defection remains unreported in the limit is \(\lim_{n \to \infty} 2e(n)/v_h\).

When \(x > 1\), the maximum value of the left hand side of (1.13) is attained when \(r_h = 1\). Then a solution to that equation with \(r_h < 1\) exists only if
\[
\lim_{n \to \infty} (n-1) \gamma(n) \pi \left(1 - \frac{\gamma(n) \pi n}{n}\right)^{n-2} =
\lim_{n \to \infty} n \gamma(n) \pi \lim_{n \to \infty} \left(1 - \frac{\gamma(n) \pi n}{n}\right)^n =
\frac{1}{x} e^{-x}
\]
is larger than the limit of \(\epsilon(n)/v_h\), which gives the second case in the statement of the theorem.

Since \(\epsilon(n)\) is non-decreasing, \(r_h\) goes to zero with \(n\), and the fact that the binomial probability distribution has the well known property that the second term in (1.12) is at least as large as the first, the probability a defection remains unreported is then less than or equal to \(2\epsilon(n)/v_h\). Substituting this into the seller’s incentive constraint proves the result.

We summarize the results in a diagram.
The shaded area includes the region where \( x < 1 \) and the limit of \( \epsilon(n)/v_h < e^{-1} \). Low communication costs support collusive equilibrium. Pushing communication costs even lower can, collusive equilibrium can also be supported when observation of sellers’ mechanisms is very unlikely.

1.4. Conclusion. The paper shows conditions under which collusive equilibrium can and can’t be supported as equilibrium in large markets where buyers try to communicate market information. There are two parts of the story that differ from traditional mechanism design. The first is that some of the people who participate in a mechanism may not know exactly how the mechanism works. Traders who don’t know what a mechanism designer is doing can’t give other mechanisms designers any useful information.

Full knowledge of sellers’ commitments and costly communication of type are absolutely fundamental assumptions in standard mechanism design, but they probably aren’t plausible in competitive contexts. One of the implications of the results here is that it is perhaps interesting to examine standard mechanism design to study the case where some agents don’t understand the rules that are being used to determine allocations.

As an example, the logic of second price auctions rapidly falls apart if none of the bidders actually knows for sure that the second price is being used. The buyers will sensibly believe that the seller will use first price techniques. In the extreme case in which no buyer understand the rules, a fixed price mechanism is the only feasible mechanism. In the intermediate case in which some buyers do understand the pricing rules, the analysis would probably resemble the analysis in this paper.

What this paper probably doesn’t get right is that most buyers will understand some aspects of a mechanism. For example, a mechanism
that generates offers for airline tickets uses information on customer flows, time until the flight, and the buyers search history. Buyers may not understand exactly how this works, but when they see a price quote, they do understand it. We leave the study of this kind of observability to another time.

REFERENCES


