PROBLEM SETS

Here are problems taken from various tests.

1. Comp

(1) A cost minimizing firm has a production technology, $Q = F(K, L)$ where output Q is produced using capital, K and labour, L. Inputs are purchased at prices w and r for labour and capital respectively. The firm bargains with a union that proposes a requirement that $L = 2Q$. Explain how the firm would calculate the costs of this requirement.

(2) A consumer has a utility function $U(X_1, \ldots, X_n)$ which is homogeneous of degree one. Show that the consumer's demand functions have a constant income elasticity equal to one.

(3) A consumer buys only apples and oranges. x denotes the quantity of apples and y the quantity of oranges. The consumer's preferences are such that for any bundle (x'', y''); any pair of bundles (x, y) and (x', y') ; and any $\lambda \in [0, 1]$, $u(x, y) \geq u(x', y')$ implies that

$$u(\lambda x + (1-\lambda)x'', \lambda y + (1-\lambda)y'') \geq$$
$$u(\lambda x' + (1-\lambda)x'', \lambda y' + (1-\lambda)y'')$$

Show that for this consumer, apples and oranges are perfect substitutes.

(4) Persons 1 and 2 each choose actions. Each person cares about both her own action and the other player's action. Is person 1 necessarily better off if she chooses her action before person 2 than if she chooses her action after person 2? Necessarily worse off?

(5) In the past, the Olympic games were awarded by having each city who wanted the games offer secret bribes to the Olympic delegates. The delegates would then award the games to the city whose total bribes were largest. To prevent corruption, the committee has decided to make it illegal for delegates to accept bribes, and now requires that cities build their bribes directly into their proposals (which are still kept secret) so that only the winning city would be required to pay the delegates. The city that offers the largest bribe is still awarded the games. What

effect will this have on the expected revenue of the Olympic delegates and the expected costs to the bidding cities?

(6) True, False or Uncertain: Discuss: In a private value second price auction, the unique perfect Bayesian equilibrium outcome occurs when every bidder bids his or her true valuation.

(7) There are two firms in an economy and two workers. If a firm hires a worker it produces output of value 1 and pays the worker some wage w for net profit $1 - w$. A worker who is hired has utility equal to the wage he or she receives. Model the competition between firms as follows - each firm simultaneously posts a wage, then each worker simultaneously applies to one and only one firm. If two workers apply to one firm, the firm chooses one at random, the other remains unemployed. If a firm gets no applications it cannot produce any output.

(a): Suppose that the firms post wages w_1 and w_2. Describe the continuation equilibrium for this case in which both workers use identical *mixed* strategies to choose among firms.

(b): Solve the two stage game by assuming that firms anticipate the *mixed strategy* continuation equilibrium that will follow each pair of wage offers, and choose best replies to each others' wages.

(c): Solve the game by assuming that the market sets a payoff $\overline{u}$ that firms must offer to workers. Each firm then chooses a wage *and* common choice probability for workers to maximize its profits subject to the constraint that if workers used the choice probability that the firm offered, they would receive the market payoff $\overline{u}$. An equilibrium is a value for $\overline{u}$ such that the choice probabilities that firms choose are actually used by workers.

(d): How do the solutions in (b) and (c) differ?

(8) A consumer has an inter temporal utility function of the form: $U = \log C_0 + \log C_1$, where C_0 and C_1 are current and future consumption. The consumer owns a firm with an inter temporal production frontier

$$3 = Y_0^2 + 2Y_1^2,$$

where Y_0 and Y_1 are current and future output. The consumer has a current income of \$1 but no future income. The consumption price are $p_0 = \$1$ and $p_1 = \$3$ and the rate of interest is 50%.

Determine the optimal consumption plan. Interpret your results with respect to borrowing and lending.

(9) A consumer has an income of \$3 in each of two years and faces prices, $p_0 = \$2$ and $p_1 = \$3$. Her utility function is

$$U = C_0 C_1$$

where C_0 and C_1 are current and future consumption. She can borrow at an interest rate of 100% and lend at an interest rate of 50%. What is the consumer's utility maximizing consumption bundle?

(10) (c) An incumbent monopolist (M) plays the game in Figure 1 against a succession of N potential entrants (E_1 in the first period, E_2 in the second period, and so on), each of whom stays in the market for at most one period. (The first payoff is to the entrant, the second to the monopolist.) M's payoff in the whole game is the discounted sum of its payoffs in all periods, with discount factor $0 < \delta \leq 1$. There is perfect information. (When making a choice, every player knows the actions taken in all previous periods.)

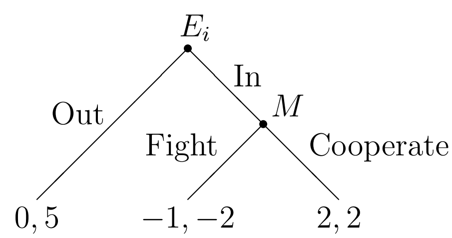

FIGURE 1. One stage of the game in Problem 10.

(a) Find all the subgame perfect equilibria when N is an arbitrary finite number and $\delta = 1$. (State both the equilibrium *strategies* and the equilibrium *outcome*.)

(b) Find the set of values of δ in $(0, 1)$ for which the following pair of strategies is a subgame perfect equilibrium when N is infinite. (You may use without proof the result that for $\delta < 1$ a strategy profile is a subgame perfect equilibrium of this game if and only if it satisfies the one deviation property.)

- Strategy for each entrant E_i, $i = 1, 2, \ldots$: enter if and only if there was entry in some previous period and M cooperated. (E_1 does not enter.)

- Strategy for M: fight entry after any history in which either there was no entry or all previous entrants were fought; cooperate after any history in which M cooperated in some period in which there was entry.

(11) (c) Consider a "hidden information" principal-agent model with two types, $i = 1, 2$. Let π denote the principal's subjective probability that the agent is of type 1. For each i, let $c_i : R_+ \to R_+$ denote the cost function for an agent of type i. That is, for each output level $x \geq 0$, $c_i(x)$ is the cost to an agent of type i of producing x. Assume that the reservation utility for each type of agent is zero.

(a) Define a concept of a **menu of contracts** in this model.
(b) State the participation or individual rationality constraints.
(c) State the incentive-compatibility constraints.
(d) Define formally the **optimal menu of contracts**.
(e) Suppose that $\pi = 1/3$ and that c_1 and c_2 are defined by

$$c_1(x) = 0.2x^2$$
$$c_2(x) = 0.8x^2$$

Find the optimal menu of contracts.

2. Final

(1) There are two consumers and one competitive price taking firm. Each of the consumers owns a single unit of oil which the firm uses as an input into production. One of the consumers owns the entire firm. The firm produces grain with the oil, but the firm's output of grain is uncertain. In a good state, the firm produces $y^H = a + x$ and in the bad state the firm produces $y^L = x$ where x is the total input of oil and $a > 0$ is a constant. Consumer 1 believes that the good state occurs with probability 1/2. Consumer 2 is the owner of the firm and believes that the good state occurs with probability 3/4. Suppose first that consumers only want grain (they don't consume oil directly) and that they are expected utility maximizers with utility for wealth function $u(w) = \ln(w)$. Compute the Arrow Debreu equilibrium prices for oil and state contingent grain. Describe the equilibrium allocations. Also describe how the solution would change if consumers utility for wealth functions were linear, or if they agreed on the probability with which the different states occur.

(2) Give proofs of Walras Law (the value of excess demand is non-positive) and the first welfare theorem (every Walrasian equilibrium allocation is pareto optimal). Start by explaining in words how your proof will work, then write out the proof carefully. You can use definitions from class, but state all assumptions clearly. You will be marked on the clarity of your argument. Explain which part of your proof of the first welfare theorem fails when there is a consumption externality.

(3) There are two securities, one pays one unit of output when the weather is nice, but not otherwise. The second security pays one unit of output when the weather is bad, but not otherwise. There are two consumers, one consumer is endowed with a single unit of output no matter what the weather. This consumer has prior belief that good and bad weather are equally likely. The second consumer observes a signal that can be either good or bad. If the signal is good, then good weather occurs with probability q and the informed consumers endowment is ω when the weather is nice and 1 unit when the weather is bad. If the signal is bad, then the weather is equally likely to be good or bad and the informed consumers endowment is 1 unit no matter what the weather. Suppose the consumers are expected utility maximizers with utility for wealth functions given by $u(w) = \ln(w)$. For what values of q and ω does a fully revealing rational expectations equilibrium exist. For what values of q and ω does a non-revealing rational expectations equilibrium exist. Are there values of q and ω for which no rational expectations equilibrium exists?

(4) An excess demand function $z_i : \mathbb{R}^k \to \mathbb{R}^k$ satisfies the *gross substitutes property* if for any pair of price vectors p, p'such that $p_k = p'_k$ for all $k \neq l$, and $p_l > p'_l$ then $z_i^k(p) > z_i^k(p')$ for each $k \neq l$. Prove that if the excess demand correspondence of every trader in an exchange economy satisfies the gross substitutes assumption then the Walrasian equilibrium price vector is unique. (Hint - if there are two equilibrium price vectors p and p', use homogeneity of degree zero of aggregate excess demand to argue that one can take $p > p'$ and at least one component the same in each vector without loss of generality. Then use gross substitutes.) Illustrate your argument in the case with 2 goods and 2 consumers.

(5) Use the *first welfare theorem* to show that in a public goods economy with one private and one public good, but $n > 2$ consumers, pareto optimality can be attained by (in part) choosing

an allocation so that the sum of the consumers marginal willingness to pay for the public good expressed in units of the private good is equal to the marginal cost of the public good expressed in terms of the private good.

(6) Suppose there are four commodities x, y, l, and t, two firms A and B, and one consumer. The firms use l and t to produce x and y. Firm A produces x according to the production function $x = \min[l, t]$, while firm B produces $y = (lt)^{\frac{1}{2}}$. The single consumer values only x and y according to $u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$ and has exactly one unit of l and one unit of t to sell to firms to finance his consumption. Give the Walrasian equilibrium prices and allocations. If the production function for firm A changes to $x = \{\min[l, t]\}^2$ will the original Walrasian equilibrium allocation still be an equilibrium? pareto optimal?

(7) A market consists of one buyer and one seller. The seller has exactly one indivisible unit of output to sell, the buyer wants to buy either exactly one unit or no units. The values of the unit to the buyer and seller are both determined by the state ε which is observed by the seller but not by the buyer. The prior distribution of ε is uniform on $[0, 2]$. The buyer's value if $6 - \varepsilon$ and if he pays p in this case his utility is $6 - \varepsilon - p$. The seller has value $3 + 2\varepsilon$ and would get $p - 3 - 2\varepsilon$ by trading in this case. If there are two market clearing prices the auctioneer always picks the lower one (so with perfect information, the market clearing price would be the seller's value whenever there is trade). What is the rational expectations equilibrium for this problem (remember you are looking for a belief function) and the allocation it supports - is it revealing? is it ex post efficient?

(8) Consider a Radner equilibrium with production where each firms' production set is defined on $\mathbb{R}^{KS}$ where K is the number of physical goods, and S is the number of states. Suppose that the matrix R of Radner securities has rank S. Explain why there must be an equilibrium in which firms shares are not traded.

(9) Let $\mathcal{X}$ be a *finite* set of consumption bundles. Suppose that preferences over the set $\mathcal{X}$ are transitive and complete. Show there must exist a consumption bundle x^* such that $x^* \succeq x \forall x \in \mathcal{X}$. Use this argument to construct a utility function that represents preferences over bundles in $\mathcal{X}$.

(10) Two traders have utility for wealth given by $u(w) = \ln(w)$. The following table provides information about the random events

that can occur in the market.

	w	1	2
$\frac{1}{4}$	G	ω	1
$\frac{1}{4}$	B	1	γ
$\frac{1}{4}$	G	1	γ
$\frac{1}{4}$	B	ω	1

The probabilities of the four events are given in the first column. The second column gives the weather, either good (G) or bad (B) (I got this idea from a skit). The third column gives consumer 1 's wealth, while the fourth gives consumer 2's wealth. There are two securities traded in this market before this event is realized. The first security pays one dollar if and only if the weather is good, the other pays one dollar if and only if the weather is bad. Give the matrix R of asset returns that applies in the Radner equilibrium for the market. Are markets complete or incomplete? Let q be the price of the first security and 1 the price of the second. Write the maximization problem that consumer 1 solves when she chooses her portfolio of securities. Can you find the price q that clears the asset market? (You will get more marks if you do this without writing down any first order conditions). What are the 'state' prices in this case?

(11) Suppose two consumers have identical preferences $x^{\alpha}y^{(1-\alpha)}$ over private goods x and the public good y. The aggregate endowment of the private good is 1. Half of this endowment is owned by each consumer. The production technology is such that each unit of the private good yields exactly one unit of the public good. Compute the equilibrium of the voluntary contribution game and the Lindahl allocations and prices.

(12) Suppose there are two commodities and two consumers. The preferences of consumer 1 depend on the state s and are given by $s\ln(x) + (1-s)\ln(y)$. Consumer 1 doesn't observe the state, but believes that it is uniformly distributed on $[0,1]$. Consumer 2 has preferences $\frac{1}{2}\ln(x) + \frac{1}{2}\ln(y)$. Consumer 1 is endowed with one unit of each good in each state. Consumer 2 has an endowment of one unit of good 1 in every state, but has state dependent endowment $2-s$ of good 2. Consumer 2 observes the state. Describe the rational expectations equilibrium for this problem. Show by contradiction that every rational expectations equilibrium for this problem must be fully revealing.

(13) Choose *one* of the following assertions. Give a formal statement of the assertion and the key assumption(s) involved and prove the assertion.

(a) Every Walrasian equilibrium is pareto optimal provided all traders utility functions are 'monotonic'.

(b) Suppose that there is a safe asset and a risky asset. The proportion of his wealth that an investor devotes to the risky asset will be independent of his wealth as long as the investor has constant relative risk aversion.

(c) (No trade theorem) Show that if the endowment in an exchange economy is pareto optimal and a competitive equilibrium exists, then there will be a competitive equilibrium in which no one trades provided preferences are convex.

(d) If the set of securities spans the space of state contingent return vectors, then the budget set in a 'Radner equilibrium' is equal to the budget set in an 'Arrow Debreu equilibrium'.

(14) 2. Suppose that there are two assets. One is a safe asset paying a return R while the other pays r_1 in state 1 and r_2 in state 2. The states occur with probability π and $1-\pi$ respectively. Total wealth W must be allocated between the two assets. Suppose the investor is an expected utility maximizer with a strictly concave utility for wealth function.

(a): Show that $\pi r_1 + (1-\pi)\, r_2 > R$ is a necessary and sufficient condition for the investor to invest a positive amount of his wealth in the risky asset.

(b): Assuming that $\pi r_1 + (1-\pi)\, r_2 > R$, give a sufficient condition such that demand for the risky asset is an increasing function of r_1

(c): What happens to the demand for the risky asset when π increases, assuming that $r_1 > r_2$. Interpret this in terms of stochastic dominance.

(15) 3. Suppose there are two goods and two states. There are no securities, only spot markets in each of the two states. Trader 1 has preferences

$$\begin{aligned} u(x,y) &= \alpha \ln(x) + (1-\alpha)\ln(y) \quad \text{in state 1} \\ u(x,y) &= \beta \ln(x) + (1-\beta)\ln(y) \quad \text{in state 2} \end{aligned}$$

Trader 1 has a single unit of each good in each state. Trader 2 has preferences $u_2(x,y) = kx^{\frac{1}{2}}y^{\frac{1}{2}}$ in each state, where k is a positive constant.

Trader 1 does not observe the state, but trader 2 does since he has a state contingent endowment of $(1, 1)$ in state 1 and $(1, \omega)$ in state 2, where $\omega > 1$.

(a): Write down a restriction on α, β, and ω which will ensure that the rational expectations equilibrium will exist and will be fully revealing.

(b): Write down a restriction on α, β, and ω which will ensure that the rational expectations equilibrium exists, but reveals no information about price.

(c): Write down a restriction on α, β, and ω for which the rational expectations equilibrium will not exist.

(16) Use an Edgeworth box to show

(a) pareto optimal allocations may not be supportable as competitive equilibrium allocations even with redistribution of endowments if preferences are non-convex

(b) Walrasian equilibrium may not exist if demand functions are discontinuous.

(c) there may be more than one Walrasian equilibrium

(d) monopoly will not be pareto optimal (let consumer 1 choose the relative price)

(e) there is a risk neutral consumer and a risk averse consumer. The aggregate endowment is sure. The risk neutral consumer believes that the risk averse consumer will have an accident with probability π. The risk neutral consumer believes that the accident occurs with probability $\tilde{\pi} > \pi$. Then in a competitive equilibrium, the risk averse consumer will consume more when an accident does not occur than he will when the accident occurs.

(17) Consider a Radner equilibrium when there are S states, J securities and only a single consumption good ex post (i.e no spot markets). Suppose that asset demand functions are continuous. Prove that a Radner equilibrium exists.

(18) A pubic good y is produced using inputs of a private good x. The production possibilities frontier for the whole economy is given by $x + y^2 = W$ where W is the aggregate endowment of the private good. There are n consumers for whom public and private goods are perfect substitutes in the sense that every consumer is willing to give up exactly 1 unit of the private good to get an additional unit of the public good no matter how much of the private and public good the are currently consuming. Each consumer owns an endowment of the private good equal to $1/n$. What is the equilibrium of the 'voluntary

contribution game' for this problem in which each consumer voluntarily decides how much to contribute to production of the public good. How does total output of the public good vary with n? Describe this equilibrium as a Walrasian equilibrium with production externalities in which each consumer interacts with one and only one firm. Find the Lindahl prices for this problem by assuming that all consumers interact with a single price taking firm. In each case give a complete description of commodities traded and consumer budget constraints.

(19) There are two consumers in an exchange economy. There is only a single physical good in this economy, but consumers are uncertain about endowments. Consumer 2 has an endowment of the physical good that is always equal to 1. Consumer 1 has a variable endowment, and gets some information about what this endowment is likely to be. Consumer 1's endowment varies with the weather (which is either fair (F) or rainy (R)). Consumer 1 is not sure exactly how his endowment varies with the weather, though he does have some information which comes in the form of a signal which is either G, B, or M. The joint distribution of signals, endowments and weather is given in the following table

	$1+\omega, F$	$1+\omega, R$	$1, F$	$1, R$
G	$2/12$	0	0	$3/12$
B	0	0	$1/12$	$1/12$
M	0	$3/12$	$2/12$	0

The rows of the matrix correspond to the different signals that consumer 1 might receive. The columns describe consumers 1's endowment and the weather. For example, $(1+\omega, F)$ means that the weather is fine *and* consumer 1's endowment is $1+\omega$ and ω is some quantity of the physical commodity. The elements of the table are probabilities, so the probability that the weather is fine and consumer 1 has endowment 1 is $1/12$. There are two Radner securities one pays 1 unit of the physical commodity if and only if the weather is fine, the other pays 1 unit of the physical commodity if and only if the weather is rainy. Each consumer maximizes expected utility given by

$$b^i \ln\left(c_F^i\right) + \left(1-b^i\right) \ln\left(c_R^i\right)$$

where b^i is consumer i's belief about the probability with which it will rain (conditional on all his or her information). Describe the problem each consumer solves in the Radner equilibrium associated with this economy. Show that this problem is equivalent to a standard Cobb Douglas utility maximization problem and that the market clearing conditions in the Radner equilibrium can be rewritten as standard market clearing conditions for demand for contingent consumption when the weather is fine. Compute the full information price function. For what values of ω is it revealing? Give the rational expectations belief function assuming the full information price function is revealing. Is there a rational expectations equilibrium when $\omega = 1$?

(20) Write out a proof of *one* of the following theorems: show *either* that*:*
- an investor whose preferences satisfy increasing *relative* risk aversion and who chooses to invest a positive proportion of his total wealth in a risky asset will raise the *proportion* of his total wealth that he invests in a risky asset if his wealth increases; *or*
- a lottery F over income is preferred (at least weakly) to a lottery G over income by *every* investor whose utility for income function is non-decreasing if and only if $F(x) \leq G(x)$ for every x in the union of the supports of the two distributions; *or*
- the security prices that prevail in every Radner equilibrium satisfy the no-arbitrage condition.

3. Midterm

(1) Do each question:
- Suppose that the set of lotteries $\mathcal{L}$ is such that there is a best element b in $\mathcal{L}$ and a worst element w in $\mathcal{L}$ (meaning b is at least as good as every lottery in $\mathcal{L}$ and that every lottery in $\mathcal{L}$ is at least as good as w). Suppose that $\lambda > \lambda'$ implies that $\lambda b + (1-\lambda) w \succ \lambda' b + (1-\lambda') w$. Define $u(p) = \{\lambda : \lambda b + (1-\lambda) w\}$. Prove that $u(p) \geq u(p')$ if and only if $p \succeq p'$.
- Suppose there are two commodities x and y and that some consumer's indifference curves all given by equations of the form

$$y = \left(K - x^2\right)^{\frac{1}{2}}$$

for different values of K. Use the method we discussed in class to provide a utility function that represents these preferences. Can you show that these preferences can also be represented by a Cobb-Douglas utility function?

- Demand for two commodities has been estimated as

$$x = b_1 + b_2 p_1 + b_3 p_2 + b_4 w$$

and

$$y = c_1 + c_2 p_1 + c_3 p_2 + c_4 w$$

where p_1 and p_2 are prices and w is consumer wealth. All other parameters are estimates. Write down the symmetry conditions that are the consequences of utility maximization (ignore the negative semi-definiteness conditions).

- A vacation company offers a 'surprise' vacation package - you turn up at the airport then the company rolls a dice to determine whether you will fly to the Bahamas for a week on the beach, or to Aspen, for a week of skiing. If you pack your bag correctly (i.e. swimsuit for Bahamas or ski jacket for Aspen) the utility you get is 1 while if you pack your bag incorrectly (e.g, swimsuit for Aspen) your utility is zero. Of course, the catch is that you have to pack your bag before you learn the destination and you can only pack one type of cloths (either swimsuit, or ski jacket). Show that your preferences for lotteries over destinations won't satisfy the independence axiom.

(2) Do each question.

- There are two consumers, and two goods. Both consumers have preferences

$$u_i(x, y) = \ln x + y$$

and both have a single unit of each good as an endowment. Calculate the set of Pareto Efficient allocations and illustrate them in an Edgeworth box.

- Suppose there are n consumers, 2 goods and m firms. Each consumer owns $\frac{1}{n}$ of each firm and each of the firms produces good y from good x according to the function $y = \alpha \ln x$. Consumer 3 has a Cobb-Douglas utility function given by

$$\frac{1}{3} \ln x + \frac{2}{3} \ln y$$

At a certain pareto efficient allocation (x^*, y^*), consumer 3 gets the allocation $(x_3^*, y_3^*) = (10, 5)$. What are the competitive prices that support(x^*, y^*)

(3) Do each Question
- Suppose that preferences over lotteries satisfy $p, p' \in \mathcal{L}$, $\lambda \in [0,1]$ $p \sim p' \Rightarrow \lambda p + (1-\lambda) p' \sim p$. Prove that if preferences satisfy the independence axiom then they also satisfy this condition. Show that the utility function $u(p_1, p_2, p_3) = \frac{p_2}{1-p_1}$ satisfies this condition, but does not satisfy the independence axiom.
- Suppose there are two assets. One asset return 1$ for each dollar invested, one returns $1+s$ dollars for each dollar invested, where s is a random variable with positive expected value distributed F on the real line. An investor has wealth W. If the proportion α of this wealth is invested in the risky asset, then the investor's expected utility is $\int u\left(\left(1-\alpha\right)w + \alpha w\left(1+s\right)\right) dF\left(s\right)$. Show that if an investor has constant relative risk aversion, then the proportion of his or her wealth invested in the risky asset will be independent of wealth.

(4) Do Each Question.
- There are two consumers, and two goods. Both consumers have preferences
$$u_i\left(x, y\right) = \min\left[x, y\right]$$
Suppose that the aggregate endowment of good x is strictly larger than the aggregate endowment of good y. Show the set of Pareto Efficient allocations in an Edgeworth box.
- Suppose there are n consumers, k physical goods, that each consumer has the same endowment, and each consumer has the same preferences. Is the endowment pareto optimal? Provide a set of sufficient conditions for the endowment to be optimal, and give a proof of the optimality of the endowment under your conditions.
- In an exchange economy with 2 consumers and 2 goods, suppose that both consumers have preferences $u\left(x, y\right) = y + \ln\left(x\right)$. The endowment of consumer 1 is $\left(\frac{3}{2}, \frac{1}{2}\right)$ while the endowment of consumer 2 is $\left(\frac{1}{2}, \frac{3}{2}\right)$. Find the competitive equilibrium price and allocations.

(5) A consumer has an accident with probability q. If no accident occurs, she has income y, while if an accident occurs, this income is reduced to $y - d$. An insurance company is willing to

sell the consumer any insurance policy whose expected profit is zero, i.e., any policy such that $(1-q)p = qb$ where p is the premium associated with the policy and b is the net benefit the consumer receives when she has an accident. The consumer chooses a policy to maximize

$$g(q)u(y-d+b) + [1-g(q)]u(y-p)$$

where g is a continuous and montonically increasing function satisfying $g(0) = 0$ and $g(1) = 1$. Prove that the insurance buyer will choose to over insure (buy a policy in which her net income when she has an accident is lower than when she doesn't have an accident) if the function g is concave.

(6) The Allais paradox involves two experiments. The prizes in both experiments are the same $\{x_1, x_2, x_3\} = \{1000, 500, 0\}$. In the first experiment, you are offered two lotteries

$$p = \{0, 1, 0\}$$

and

$$p' = \{.1, .89, .01\}$$

In these lotteries, the first element of the vector represents the probability with which you win 1000, the second is the probability you win 500, the third is the probability you win nothing. In the second experiment, the lotteries are

$$q = \{0, .11, .89\}$$

and

$$q' = \{.1, 0, .9\}$$

The Allais paradox says that people will normally choose p over p' but then pick q'over q. This violates the independence axiom. Suppose that a consumer has preferences over lotteries that can be represented by the utility function

$$u(p) = g(p_1)U(1000) + g(p_2)U(500) + g(1-p_1-p_2)U(0)$$

where the utility numbers are three constants and g is a continuous differentiable function satisfying $g(0) = 0$ and $g(1) = 1$. Assuming that indifference curves are always convex and that $U(1000) > U(500) > U(0)$, show that these preferences will be consistent with the behavior described above ($p \succ p'$and $q' \succ q$) only if g is a strictly convex function.

(7) Let $x[p, w]$ be a demand *correspondence* associated with a rational preference ordering. Suppose that there is some pair (p', w') such that $x[p, w] \cap x[p', w'] \neq \emptyset$. Show that for any point

$x \in x[p, w]$ that lies outside the intersection $x[p, w] \cap x[p', w']$, $p'x > w'$.