1 General Equilibrium

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- n individuals
- unknown types Θ_i for individual i characterize things about i that are unknown to traders other than i
- Θ_0 is a set of things unknown to all traders - call them unknown states of nature
- G is the joint distribution of types and states of nature
- the set of outcomes is given by X
- $Y : \Theta_0 \to \mathcal{P}(X)$ represents the set of outcomes that are feasible in each state of nature

- $u_i : \Theta \times X \rightarrow \mathbb{R}$ is agent i's utility function (observe that each agent cares directly about every other agent's type and about the whole allocation).

- an outcome x is feasible in state θ_0 if $x \in Y(\theta_0)$.

- an *allocation rule* is a mapping $\alpha : \Theta \rightarrow X$

- an allocation rule is feasible if $\alpha(\theta)$ is feasible for each $\theta \in \Theta$.

- an allocation rule is *ex post efficient* if there does not exist an alternative feasible allocation rule α' such that

  $$u_i(\alpha'(\theta), \theta) \geq u_i(\alpha(\theta), \theta)$$

  for each i and for each θ, with strict inequality holding for some i and θ.

- write $\theta = \{\theta_i, \theta_{-i}\}$. An allocation rule is *interim efficient if there does not exist an alternative rule α' such that*

  $$\int u_i(\alpha'(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i) \geq$$

$$\int u_i(\alpha(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i)$$

for every θ_i and i with strict inequality for some θ_i and i.

- an allocation rule is *(Bayesian) incentive compatible* if it is feasible and

$$\int u_i(\alpha(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i) \geq$$

$$\int u_i(\alpha(\theta'_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i)$$

for each i, every θ_i and every θ'_i

- an allocation rule is interim incentive efficient if it is feasible, incentive compatible and if there does not exist an alternative feasible incentive compatible decision rule α'such that

$$\int u_i(\alpha'(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i) \geq$$

$$\int u_i(\alpha(\theta_i, \theta_{-i}), (\theta_i, \theta_{-i}))dF(\theta_{-i}|\theta_i)$$

for every θ_i and i with strict inequality for some θ_i and i.

- note that interim incentive efficiency and interim efficiency differ in two ways - first only rules that are incentive compatible can be interim incentive efficient. There are interim efficient rules that aren't incentive compatible. Second, interim incentive efficient rules don't have to be interim efficient, since they are only compared against other incentive compatible allocation rules. In other words, there might an alternative allocation rule that interim dominates α - but if it isn't incentive compatible, that doesn't matter.

- Example 1: Private value exchange environment with complete information

    - $X = \mathbb{R}^{Jn}$ - an allocation is a list of consumption vectors (with J goods) one for each of the traders

  - $Y[\theta_0] = \{x \in X : \sum_{i=1}^{n} x_{ij} \leq \omega_j(\theta_0) \forall j = 1, \ldots, n\}$ where $\omega_j(\theta_0)$ is the aggregate endowment of good j in state θ_0.
  - $\Theta = \{\underline{\theta}\}$ - there is one state that everyone knows
  - $u_i(x, \theta) = \overline{u}_i(x_{i1}, \ldots, x_{iJ})$

- variants we will study involve allowing consumption externalities and allowing the aggregate endowment to vary with the state

- in such a problem ex post efficiency is sometimes referred to as Pareto optimality.

- since no one has any private information interim efficiency and interim incentive efficiency are the same thing

- Example 2: Public Goods with Complete Information

  - $X = \mathbb{R}^{n+1}$ - some level of production of the public good along with a list of payments by each agent

  - $Y[\theta_0] = \left\{x \in \mathbb{R}^{n+1} : C(x_1, \theta_0) \leq \sum_{i=2}^{n} x_i\right\}$ where $C(x_1, \theta_0)$ is the cost of producing the public good in state θ_0.
  - $\Theta = \left\{\overline{\theta}_0, \overline{\theta}_1, \ldots, \overline{\theta}_n\right\}$ complete information
  - $u_i(x) = \overline{u}_i(x_1) - x_i$ value of the public good less the tax paid

- Example 3: Private Values insurance with symmetric information

  - $X = \mathbb{R}^n$ - an allocation is a list showing how much money each trader has
  - $Y[\theta_0] = \{x \in X : \sum_{i=1}^{n} x_i \leq \omega(\theta_0)\}$ where $\omega(\theta_0)$ is the total amount of wealth available in state θ_0.
  - $\Theta = \{\theta_{01}, \ldots, \theta_{0S}\}$ (a finite set of states of the world)
  - $u_i(x) = \overline{u}_i(x_i)$ and $\overline{u}$ is a state independent utility for wealth function

- all feasible allocation rules are ex post efficient, a rule is interim incentive efficient allocation if and only if it is also interim efficient because of the fact that no one has private information

- Example 4: Insurance with Asymmetric Information - 2 traders - 2 states

  - $X = \mathbb{R}$ - a list of monetary transfers from agent 1 (the insurance buyer) to agent 2 (the insurance seller)

  - $$Y[\theta_0] = \begin{cases} 0 \leq x \leq W & \text{if } \theta_0 = N \\ 0 \leq x \leq W - D & \text{if } \theta_0 = A \end{cases}$$

    the insurance buyer may or may not have an accident with a monetary cost D

  - $\Theta = \Theta_0 \times \Theta_1 = \{\{N, L\}, \{N, H\}, \{A, L\}, \{A, H\}\}$ - an accident occurs, and the insurance buyer may be one of two types, L means low accident probability, H means high accident probability.

- an allocation rule $\alpha(\theta) = \alpha(\theta_0, \theta_1)$ specifies a transfer in each state for each type of insurance buyer
- the joint distribution is given by
-

	A	N	Conditional
L	q_{la}	q_{ln}	$q_l = \frac{q_{la}}{q_{la}+q_{ln}}$
H	q_{ha}	q_{hn}	$q_h = \frac{q_{ha}}{q_{ha}+q_{hn}}$
Pr	$\lambda = q_{la} + q_{ha}$	$1 - \lambda$	

- the allocation rule $\alpha(\theta_0, \theta_1)$ gives the premium that an insurance buyer of type θ_1 pays when $\theta_0 = N$, and gives the net benefit the buyer receives (again depending on type) when $\theta_0 = A$.
- payoffs are for the buyer

$$u_1\left(\alpha(\theta_0, \theta_1), (\theta_0, \theta_1)\right) = \begin{cases} \overline{u}\left(w - \alpha(\theta_0, \theta_1)\right) & \text{if } \theta_0 = N \\ \overline{u}\left(w - \alpha(\theta_0, \theta_1) - d\right) & \text{otherwise} \end{cases}$$

and

$$u_2\left(\alpha(\theta_0, \theta_1), (\theta_0, \theta_1)\right) = \alpha(\theta_0, \theta_1)$$

- again, every allocation is ex post efficient.
- the interim payoffs of the buyer of type L are

  $$q_l \overline{u}\left(w - \alpha(A, L) - d\right) + (1 - q_l)\overline{u}\left(w - \alpha\left(N, L\right)\right)$$

  and type H

  $$q_h \overline{u}\left(w - \alpha(A, H) - d\right) + (1 - q_h)\overline{u}\left(w - \alpha\left(N, H\right)\right)$$

- the interim payoff for the seller is

  $$q_{la}\alpha(A, L) + q_{ln}\alpha(N, L) + q_{ha}\alpha(A, H) + q_{hn}\alpha(N, H)$$

- the usual insurance result is that interim efficient allocations are such that each type of buyer enjoys the same income in each state because the seller is risk neutral. This will only be interim incentive efficient if both types of buyers have the same constant income (if one type has a higher constant income, both types will want to pretend to be that type).

- there are interim incentive efficient allocations in which the lower accident probability buyer gets a constant income, while the lower probability buyer has a random income.

- in modern literature the fact that allocation rules have to be incentive compatible is taken for granted, so the term 'interim efficiency' is usually intended to mean interim incentive efficiency - so read carefully.

- Example 5: Private Value Auctions
  - $X = \mathbb{R}^{2(n-1)}$ - a list giving the probability with which each of $(n-1)$bidder is awarded an indivisible unit of some good, and the payment they are required to make to the auctioneer (agent n).
  - $Y[\theta_0] = \left\{ x = \{q_1, \ldots q_{n-1}, p_1, \ldots, p_{n-1}\} \in \mathbb{R}^{2(n-1)} : \sum_{i=1}^{n-1} q_i \leq 1 \right\}$
  - $\Theta = \left\{ \overline{\theta}_0, \Theta_1, \ldots, \Theta_{n-1} \right\}$ each bidder has an unknown type

- for each bidder

  $$u_i(x,\theta) = q_i\theta_i - t_i$$

  for the seller

  $$u_n(x,\theta) = \sum_{i=2}^{n} t_i$$

- these examples cover all the different variants you will see in the course

- it is important to see the connection between them - each model involves a number of specialized assumptions

- strong theorems come from these very specialized assumptions