1 Answers for Problems

1.D.2: Need to show that for any $B \subset \mathcal{X}$ there is at least one alternative $x \in B$ such that $x \succeq y$ for all $y \in B$. Choose any $x_1 \in B$. By completeness, $x_1$can be compared with every $y \in B$. If $x_1 \succeq y$ for all $y \in B$, we are finished. So suppose this is not the case. Then there is an $x_2 \neq x_1$ such that $x_2 \succeq x_1$ but not $x_1 \succeq x_2$. If $x_2 \succeq y$ for all $y \in B$ we are finished, if not there exists and $x_3 \succeq x_2$ but not $x_2 \succeq x_3$. Continue this procedure for k steps and suppose there is an $x_k \succeq x_{k-1}$ but not $x_{k-1} \succeq x_k$. Then writing $x \succeq y$ and not $y \succeq x$ as $x \succ y$, observe that this construction gives $x_k \succ x_{k-1} \succ \ldots x_1$ so that $x_k = x_j$ for any $j < k$ leads to an immediate contradiction if preferences are trasitive. Hence if there are n elements in $\mathcal{X}$, there can be at most n elements in B. So this construction has to stop after at most n iterations. When it stops, the terminal element x_k is revealed at least as good as any other element in B.

1.D.3 $C(\{x, y, z\})$ must contain at least one element (possibly more). Each of the options x, y, and z is the unique choice taken from some 2 element set. Since each of these two element sets is a subset of $\{x, y, z\}$ the weak axiom requires that $C(\{x, y, z\}) = \{x, y, z\}$. If that is true however, applying the weak axiom in the other direction, requires that both elements in each 2 element set lie in the choice correspondence for that set, which is false.

1.D.4. A rationalizing preference relation exists for C if $x \in C(B)$ if and only if $x \succeq y \; \forall y \in B$. Suppose $x \in C(B_1 \cup B_2)$. Then $x \succeq y$ for all y in $B_1 \cup B_2$ and $C(B_1) \cup C(B_2) \subset B_1 \cup B_2$. So $x \succeq y$ for all $y \in C(B_1) \cup C(B_2)$ or because $\succeq$ rationalizes the preference relation $x \in C(C(B_1) \cup C(B_2))$. On the other hand if $x \in C(C(B_1) \cup C(B_2))$, then $x \succeq y$ for all $y \in C(B_1) \cup C(B_2)$. Since each $y \in C(B_1)$ has the property that $y \succeq z$ for all $z \in B_1$ then $x \succeq z$ for all $z \in B_1$ by transitivity, and similarly $x \succeq z \forall z \in B_2$. Thus $x \in C(B_1 \cup B_2)$.

1.D.5. The strict preference relations are $x \succ y \succ z$; $x \succ z \succ y$; $z \succ x \succ y$; $z \succ y \succ x$; $y \succ x \succ z$; $y \succ z \succ x$. Assign these orderings probabilities q_1 through q_6. If $C(\{x, y\}) = \left\{\frac{1}{2}, \frac{1}{2}\right\}$, then the first three orderings pick x over y while the last three pick y over x. So we need

$$q_1 + q_2 + q_3 = q_4 + q_5 + q_6 = \frac{1}{2}$$

Similarly for $C(\{y, z\})$we need

$$q_1 + q_5 + q_6 = q_2 + q_3 + q_4 = \frac{1}{2}$$

(the orderings that pick y over z are as likely as the orderings that pick z over y, and finally from $C(\{z, x\}) = \left\{\frac{1}{2}, \frac{1}{2}\right\}$

$$q_3 + q_4 + q_6 = q_1 + q_2 + q_5 = \frac{1}{2}$$

So pretty obviously $q_1 = q_2 = \ldots q_6$ will work.

(b)It is more complicated for $C(\{x,y\}) = C(\{y,z\}) = C(\{x,z\}) = \left(\frac{1}{4}, \frac{3}{4}\right)$. Then

$$q_1 + q_2 + q_3 = \frac{1}{4} = q_1 + q_5 + q_6 = q_3 + q_4 + q_6$$

$$q_4 + q_5 + q_6 = q_2 + q_3 + q_4 = q_1 + q_2 + q_5 = \frac{3}{4}$$

From the first equation observe that $q_2 + q_3 = \frac{1}{4} - q_1$ while from the second equation $q_2 + q_3 = \frac{3}{4} - q_4$ which means that $q_4 = \frac{2}{4} + q_1$ or $q_4 \geq \frac{1}{2}$. Do a similar thing to show that q_2 and q_5 are also at least $\frac{1}{2}$ which is pretty clearly impossible if the q_i are to sum up to 1.

(c)Just brute force solution. From the argument above it isn't too hard to see that $1 - 2\alpha$ has to be smaller than $\frac{1}{3}$. Try completely solving the equation.