The Theory of Assortative Matching Based on Costly Signals

Heidrun C. Hoppe, Benny Moldovanu and Aner Sela*

27.3.2007

Abstract

We study two-sided markets with a finite numbers of agents on each side, and with two-sided incomplete information. Agents are matched assortatively on the basis of costly signals. We first analyze how signaling and welfare on each side of the market change when we vary the number of agents. Next we show that asymmetries in signaling activity between the two sides of the market can be explained either by asymmetries in size or in heterogeneity. Our main results identify general conditions under which the potential increase in expected output due to assortative matching (relative to random matching) is offset by the costs of signaling. Finally, we look at a version with a continuum of agents, and we establish the differences and similarities to the finite version. In particular, we characterize instances where the insights obtained for the continuum version apply also to small markets, and illustrate situations where they do not. Technically, the paper is based on the elegant theory about stochastic order relations among differences of order statistics, pioneered by R. Barlow and F. Proschan (1966, 1975) in the framework of reliability theory.

*We wish to thank Larry Ausubel, Ilan Eshel, John Moore, Georg Nöldeke, Avner Shaked, Moshe Shaked, Xianwen Shi, Lones Smith, Asher Wolinsky and participants at the conference "Matching and Two-Sided Markets", Bonn, 2006, for helpful remarks. Thomas Tröger gave an illuminating discussion during a seminar at the University of Bonn. This research has been partially financed by the Max Planck Research Prize (Moldovanu), and by the German Science Foundation through SFB 15-TR (Hoppe, Moldovanu, Sela). Hoppe: Department of Economics, University of Hannover, Königsworther Platz 1, 30167 Hannover, hoppe@mik.uni-hannover.de; Moldovanu: Department of Economics, University of Bonn, Lennestr. 37, 53113 Bonn, mold@uni-bonn.de; Sela: Department of Economics, Ben Gurion University, 84105 Beer Sheva, Israel, anersela@bgu.ac.il

1 Introduction

We study two-sided markets where a finite number of privately informed agents on each side of the market compete for potential matching partners on the other side. Examples include marriage, labor and education markets, markets for venture capital and new technologies. In these markets, agents typically differ in their attributes, and they gain from being matched with a better partner. If, as we assume here, the marginal product of any agent's attribute is increasing in his/her partner's attribute, total surplus is maximized by assortative matching.[1] But, assortative matching cannot work (at least not directly) if types are private information - here signaling can fulfill a crucial role. By revealing private information about types, signaling can determine who is to be matched with whom, thus increasing aggregate output. On the other hand, this increase in output may be offset by the costs of signaling. The precise characterization and various consequences of this trade-off (including a discussion of the similarities and distinctions between small and large markets) constitute the theme of the present paper. A main result pertains to the welfare comparison between assortative matching based on wasteful signaling and random matching for which no signaling is needed. We also offer two explanations for aggregate asymmetries in signaling behavior among the two market sides: these may be due either to differences in the sizes, or to differences in the degree of heterogeneity of the two market sides.

Two-sided signaling appears in virtually every matching market. For example, in labor markets employers signal their relative attractiveness through investments in working conditions, wage and non-wage benefits[2], while workers signal through investments in education; universities and scientists engage in grant acquisition activity to signal their relative academic standing as employers and competent faculty, respectively (see Arnow, 1983); in student-university relations, universities signal through hiring top faculty and building flashy facilities (library, sports, etc.), while students compete by attending prestigious high-schools, taking various tests, and engaging in extracurricular acclivities; marriage markets exhibit a broad range of signaling activities that serve courtship, from the production of art and music, to career decisions, to fashion and sports (for a more detailed description of some of these examples, see Miller, 2000 and the literature cited therein)

[1] The efficiency properties of assortative matching have been emphasized by Becker's (1973) classical study of populations vertically differentiated by an unique, linearly ordered attribute. Becker and many other contributors focused on centralized matching schemes. Shimer and Smith (2000) derive conditions under which a decentralized search equilibrium leads to assortative matching.

[2] As can be easily verified, the aggregate amount of signaling in our model does not change when we change the assumption of wasteful signals with the assumption that agents on one side signal through payments to the agents on the other side.

The present paper studies matching based on two-sided signaling by combining three main features:

1. We consider a finite number of agents on both sides of the market. More precisely, we "multiply" two tournament models with several heterogenous agents and several heterogenous prizes, as developed by Moldovanu and Sela (2001, 2006). In their one-sided model, bids are submitted and ranked, and then prizes are awarded such that the highest prize goes to the highest bidder, the second-highest prize goes to the second-highest bidder, etc.[3] Here we let "prizes come to life": agents on one side represent the prizes for which the agents on the other side compete. Thus, both sides are active, and the signaling behavior of each agent is affected by features (such as number of agents and distribution of characteristics) of both sides of the market[4]. As we shall demonstrate below, the size of the market is important, and the analysis of markets with a small, finite number of traders sometimes exhibits phenomena that are different from those in large markets with a continuum of agents.

2. We allow for incomplete information on both sides[5]. Since there is a finite number of agents, no agent knows here for sure his/her rank in its own population, nor the quality of a prospective equilibrium partner. This should be contrasted with the situation in models with a continuum of agents, or with complete information, where knowledge of the own attribute and of the distributions of attributes on both sides of the market completely determines own and equilibrium-partner rank, and thus the value of the equilibrium match. In our model these values are interdependent, and agents need to form expectations about the attributes of other agents on both sides of the market.[6]

3. We introduce a new mathematical methodology to the study of two-sided markets. This is based on the elegant work on stochastic orders among normalized spacings (i.e., differences)

[3] Their focus is on the revenue effects of changes in the number and size of the various prizes, in the bidding costs, and in the tournament's structure (e.g., one-stage or two-stage competition over prizes).

[4] Complete information matching models with one sided offers are analyzed, among others, by Bulow and Levin (2005), and by Felli and Roberts (2002). The latter paper focuses on costly investments, undertaken prior to matching in order to increase the match value. This important variety of complete information models has been pioneered by Cole, Mailath and Postlewaite (1992, 2001a,b).

[5] Many of our results have immediate implications for models with incomplete information on one side, or with complete information, as have been often used in the literature. We give several examples in the text.

[6] In contrast to the standard case in the double auction literature, our signals (that can be interpreted as bids) only determine who trades with whom, but not the terms of trade. On the other hand, in most of the double auction literature all traded units are identical (so that the optimal matching problem is fairly simple), while they are heterogenous here.

and other linear combinations of order statistics, pioneered by Richard Barlow and Frank Proschan (1965, 1966, 1975) in the framework of reliability theory.[7] Barlow and Proschan have shown how monotonic failure rates induce monotonicity properties of (normalized) differences of order statistics, and how such properties are further connected to the size of the respective coefficients of variation. This is important here since strategic behavior is determined here by a delicate interplay between probabilities of getting various partners (governed by order statistics on one's own side) and properties of the marginal gains from getting stochastically better partners (governed by differences in order statistics on the opposite side). If the distribution of abilities on one side of the market has an increasing (decreasing) failure rate, then the agents on the other side perceive decreasing (increasing) successive marginal gains. Thus, high ability agents on the other side perceive relatively low (high) expected marginal gains from making more effort. As we shall see, this and related properties play a crucial role in our study.

The general insight that agents may choose costly signals to reveal private information is of course well-known: Spence (1973) has shown how investment in education may serve as a signal to prospective employers even if the content of the education is itself negligible.[8] We adopt here, as a benchmark, Spence's assumption that signals are wasted (but see below an interpretation where the signals are payments to a third party). Spence and most of the following literature focused on a one-sided activity model (only workers are active), and assumed that firms are homogenous. Therefore matching concerns did not play a role. Several notable exceptions keep the one-sided activity structure, but introduce a role for matching. Chao and Wilson (1987) and Wilson (1989) consider a seller facing a continuum of customers who differ in their private valuations for service quality. They show how customers can be "matched" to different service qualities by offering them price menus that fully, or partially induce them to reveal their type. Fernandez and Gali (1999) compare markets to matching tournaments in a model with a continuum of uniformly distributed agents on each side. Only one side is active. Their main result is that, in spite of the wasteful signaling, tournaments may be welfare superior to markets if the active agents have budget constraints. Damiano and

[7] Basic texts on order statistics and stochastic orders are David and Nagaraja, (2003), and Shaked and Shanthikumar (1994), respectively. Boland et al. (2002) is a good survey of the material most relevant for the present study.

[8] Related ideas appear in biology: animals signal their fitness, i.e., their propensity to survive and reproduce, to potential mating partners. According to the *handicap principle (*Zahavi, 1975), signals must be disadvantageous in order to be honest. The handicap principle is widely used to relate the evolution of animal and human traits to *sexual selection*, but we are not aware of a full-fledged signaling-cum matching model in the biological literature (see the survey in Maynard-Smith and Harper, 2003).

Li (2004) allow for two-sided incomplete information in a model of price discrimination with a continuum of types on each side. Their focus is on the intermediary's revenue in such situations.

Despite the fact that in any given economic situation the number of agents is obviously finite, economists often work with a continuum of agents because it is much more convenient, and because this modeling assumption well captures the meaning of "perfect competition"[9]. While it is intuitive to expect that some sort of continuity will ensure that results obtained for large, finite markets will continue to hold in the perfectly competitive limit, it is less natural (but often tempting, given that the continuum model is usually more amenable to analysis) to "extrapolate backwards" by using insights obtained for the limit market in order to make predictions about small, finite markets. It is of course well-known (e.g., from oligopoly theory) that such attempts often fail, and the same happens here in general. But, the present careful analysis of small markets reveals that there are large and interesting classes of situations where the backward extrapolation from very large to small markets is indeed valid for certain insights.

The paper is organized as follows: In Section 2, we describe the matching model and introduce some useful definitions.

In Section 3, we derive a side-symmetric signaling equilibrium in strictly monotonic strategies. In this equilibrium, assortative matching based on the ranking of signals is equivalent (in terms of output) to assortative matching based on the ranking of true attributes. The amount of signaling is only a fraction of total output - there is no full rent dissipation even if the market gets very large. The reason is that signaling allowing for assortative matching (and thus increased output) creates externalities on both sides of the market. But only the externality on one's own side due to the incentive to win a better partner gets dissipated by competition.

Armed with the equilibrium characterization, we next focus on aggregate measures of signaling and welfare. It turns out that some the phenomena exhibited by these aggregate measures are particular to relatively small markets, and cannot occur in large populations: the difference stems from the fact that, in the model with a continuum of agents (see Section 5), total signaling is very tightly related to output, while with a finite number of agents there is a leeway among output and signaling (caused by the strictly positive differences among order statistics) that may get larger or smaller in various circumstances.

In Section 3.2, we analyze the effects of increasing the number of agents (i.e., entry) on one

[9] Gretsky, Ostroy and Zame (1999) formalize the meaning of perfect competition in the assignment game with either a finite number or a continuum of agents. Perfect competition (where agents appropriate their marginal products, and where the core is a singleton) is typical for the continuum version, but rare for the finite version.

side.[10] Entry affects the expected matching surplus, but also the agents' signaling activity. We show that the effects of entry (e.g., net effect on welfare on each side of the market) are determined by the failure rates of the underlying distributions of characteristics. These results are also methodologically useful since some of our subsequent proofs proceed by considering a market with equal numbers of agents on each side to which we add agents in order to create a long side.

In Section 3.3, we use heterogeneity differentials among the two sides of the market in order to identify circumstances where one side is signaling more than the other. Intuitively, the side whose high ability agents perceive relatively higher marginal gains from getting better partners will signal more. This result has immediate applications for intermediated markets where signals are replaced by payments to a matchmaker, and it provides a new explanation for asymmetries in payments schedules among the two sides of the market.

In Section 4, we compare random matching (without any signaling) with assortative matching (based on costly and wasteful signaling) in terms of total expected net welfare.[11] For distribution functions having a decreasing failure rate average (DFRA), assortative matching with signaling is welfare-superior, while for distribution functions having an increasing failure rate average (IFRA), random matching is superior. In the latter case, we also show that agents may be trapped: given that all others signal, signaling is individually optimal, even though each type of each agent may be better off under random matching.[12]

Let us note here that, besides the two focal equilibria analyzed here (strictly separating equilibrium and strictly pooling equilibrium yielding random matching) there exist many other intermediate, partially separating equilibria that yield a "coarse" matching of agents.[13]

In Section 5, we look at the continuum version of our matching model. We show that, instead of failure rates, the welfare comparison between random and assortative matching based on signaling

[10] Our comparative statics results in this and the next section focus on aggregate measures of signaling and welfare. We briefly point out the implications for individual measures - these are governed by the same properties of failure rates.

[11] While examples where pure pooling is welfare-superior to full separation in the Spence signaling model are known, there are no general results. Rege (2003) studies a model of status consumption with a continuum of agents and uniformly drawn attributes. For certain parameters of a Cobb-Douglas production function, she notes that the increased matching efficiency due to the consumption of status goods (which serves as a signal) may be offset by the needed expenditure

[12] Charles Darwin once remarked: "The sight of a peacock tail, whenever I gaze at it, makes me sick".

[13] These equilibria combine features of the two focal ones. That is, they involve random matching of agents within corresponding, assortatively matched subsets - see Damiano and Li (2004) for a model with a continuum of agents. Hoppe, Moldovanu and Ozdenoren (2006) estimate an intermediary's revenue loss from coarse matching. McAfee (2002) showed that a coarse matching involving only two distinct classes may achieve no less output than the average of assortative matching and random matching.

hinges now on the size of the coefficient of covariation among the two populations. For example, in symmetric settings, assortative matching with signaling is welfare-superior (welfare inferior) to random matching if and only if the coefficient of variation of the common distribution of attributes is larger (smaller) than unity.

We observe that IFRA (DFRA) distributions have coefficients of variations smaller (larger) than unity, but the converse is not true. Together with the main result for finite markets (see Section 4), this demonstrates that, for the IFRA and DFRA classes of distributions, the welfare comparison (which is easily obtained in the continuum limit) is in fact preserved in markets of **any** finite size - this is an example of the backward extrapolation mentioned above. In general though, for distributions that are not IFRA or DFRA, the welfare comparison crucially depends on market size, and the result for continuum limit may be completely reversed in small markets.

Section 6 concludes. Appendix A contains several useful results from the statistical literature, while Appendix B contains the proofs of our results.

2 The matching model

There is a finite set $N = \{1, 2, ..., n\}$ of men, and a finite set $K = \{1, 2, ..., k\}$ of women, where $n \geq k$. Each man is characterized by an attribute x, each woman by an attribute y. If a man and a woman are matched, the utility of each is the product of their attributes. Thus, total output from a match between agents with types x and y is $2xy$.[14]

Agent's i attribute is private information to i. Attributes are independently distributed over the interval $[0, \tau_F]$, $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, according to distributions F (men) and G (women), respectively. For all distributions used in the paper we assume, without mentioning it again, that $F(0) = G(0) = 0$, that F and G have continuous densities, $f > 0$ and $g > 0$, respectively, and finite first and second moments. The last requirement ensures that all integrals used below are well defined (e.g., all order statistics have finite expectations).

We study the following matching contest: Each agent sends a costly signal b, and signals are submitted simultaneously. Agents on each side are ranked according to their signals, and are then matched assortatively. That is, the man with the highest signal is matched with the woman with the highest signal, the man with the second-highest signal is matched with woman with the second-highest signal, and so forth. Agents with same signals are randomly matched to the corresponding

[14] All our results can be immediately extended to asymmetric production functions having the form $2\delta(x)\rho(y)$, where δ and ρ are strictly increasing and differentiable (see Section 5.1 for an example with a Cobb-Douglas production function).

partners. The utility of a man with attribute x that is matched to a woman with attribute y after sending a signal b is given by $xy - b$ (and similarly for women).[15] Thus, signals are costly. In contrast to standard models, costs differentials are not required here in order to sustain signaling. The reason is that different types of agents expect different marginal gains from signaling.

For the subsequent welfare comparisons we assume that, apart from their function enabling matching, signaling efforts are wasted from the point of view of our men and women. In other variations, not explicitly considered here, these may accrue as rents to a third party. The equilibrium analysis is invariant to such alternative specifications.

For the equilibrium characterization we will need several pieces of notation. Let $X_{(1,n)} \leq X_{(2,n)} \leq \cdots \leq X_{(n,n)}$ and $Y_{(1,k)} \leq Y_{(2,k)} \leq \cdots \leq Y_{(k,k)}$, denote the order statistics of men's and women's characteristics, respectively. We define $X_{(0,n)} \equiv 0$ ($Y_{(0,k)} \equiv 0$).

Let $F_{(i,n)}$ ($G_{(i,k)}$) denote the cumulative distribution of $X_{(i,n)}$ ($Y_{(i,k)}$). The density of $X_{(i,n)}$ is given by :

$$f_{(i,n)}(x) = \frac{n!}{(i-1)!\,(n-i)!} F(x)^{i-1} [1 - F(x)]^{n-i} f(x),$$

and similarly for $Y_{(i,k)}$.

Let EX (EY) be the expectation of $F(G)$. We denote by $EX_{(i,n)}$ ($EY_{(i,k)}$) the expected value of the order statistic $X_{(i,n)}$ ($Y_{(i,k)}$), and define $EX_{(0,n)} = EY_{(0,k)} = 0$. A useful identity, repeatedly used below, is :

$$\sum_{i=1}^{n} EX_{(i,n)} = nEX$$

3 Equilibrium analysis

In this section, we focus on a symmetric, strictly separating equilibrium where all agents on one side of the market use the same signaling strategy. Obviously, the model has other symmetric equilibria. For example, a strictly pooling equilibrium where no agent ever signals (which yields random matching) can also be sustained. In Section 4 below, we will compare the outcomes of these two focal equilibria.[16]

[15] This is a straightforward generalization of the standard one-sided independent private value model considered in the auction literature. From that literature it is well-known that results beyond the case of risk neutrality are seldom analytically tractable. An advantage of this formulation (which is also used in most of the related matching literature, e.g., McAfee (2002), Damiano and Li (2004)) is that all our results can be stated solely in terms of properties of the distribution functions.

[16] Other, partially separating equilibria yield a "coarse" matching of agents. These equilibria involve random matching of agents within corresponding, assortatively matched subsets - see Damiano and Li (2004) who study such equilibria in a model with a continuum of agents.

3.1 Existence of signaling equilibrium

Assume then that all men [women] use the same, strictly monotonic and differentiable equilibrium signaling function β $[\gamma]$. Because a man with attribute x has the option to behave as though his attribute is s, he effectively faces the following maximization problem:

$$\max_s \quad \left\{\sum_{i=n-k+1}^{n} F_i^n(s)\, xEY_{(k-(n-i),k)} - \beta(s)\right\}$$

where $F_i^n(s)$ denotes the probability that a man with type s meets $n-1$ competitors such that $i-1$ have a lower type and $n-i$ have a higher type[17]. Note that for $i = 2, \ldots n-1$, we have:

$$F_i^n(s) = F_{(i-1,n-1)}(s) - F_{(i,n-1)}(s) = \frac{(n-1)!}{(i-1)!\,(n-i)!} F(s)^{i-1} [1 - F(s)]^{n-i},$$

and we let $F_n^n(s) = F_{(n-1,n-1)}(s)$, and $F_1^n(s) = 1 - F_{(1,n-1)}(s)$.

Using the above observation, and the fact that, in equilibrium, the maximum should be indeed attained for $s = x$, we obtain the following differential equation:

$$\begin{aligned}\beta'(x) \quad &= \sum_{i=n-k+1}^{n-1} x\left[f_{(i-1,n-1)}(x) - f_{(i,n-1)}(x)\right] EY_{(k-(n-i),k)} \\ &\quad + f_{(n-1,n-1)}(x)\, xEY_{(k,k)}\end{aligned}$$

The man with the lowest type either never wins a woman (if $n > k$) or wins for sure the woman with the lowest type (if $n = k$). Hence, the optimal signal of this type is always zero, yielding the boundary condition $\beta(0) = 0$. The solution of the differential equation gives candidate equilibrium effort functions.

Proposition 1 *The profile of strategies where each man employs the strictly increasing signaling function*

$$\begin{aligned}\beta(x) \quad &= \int_0^x s\left\{\sum_{i=n-k+1}^{n-1}\left[f_{(i-1,n-1)}(s) - f_{(i,n-1)}(s)\right] EY_{(k-n+i,k)}\right\} ds \\ &\quad + \int_0^x s f_{(n-1,n-1)}(s)\, EY_{(k,k)} ds \qquad (1)\end{aligned}$$

and each woman employs the analogously derived signaling function $\gamma(y)$ constitutes an equilibrium of the matching contest.

[17] Similarly, we denote by $G_i^k(s)$ the probability that a woman with type s meets $k-1$ competitors such that $i-1$ have a lower type and $k-i$ have a higher type.

Note that the spacings of order statistics $(X_{(i,n)} - X_{(i-1,n)})$ represent the marginal gains from winning a stochastically better partner. The next proposition reveals that the aggregate signaling effort (say women's) is a weighted sum of expectations of *normalized spacings* of order statistics on the men's side, $(n-i+1)(EX_{(i,n)} - EX_{(i-1,n)})$, where the weights $EY_{(i-(n-k)-1,k)}$ are expectations of the women's order statistics (necessarily increasing in i). The same observation holds for the net welfare terms - here we introduce the assumption of wasteful signaling.

Proposition 2 *For any F, G, n, k, it holds that:*

1. *Men's total signaling effort and (net) welfare are given by :*

$$\begin{aligned} S_m(n,k) &= n\int_0^{\tau_F} \beta(x) f(x)\, dx \\ &= \sum_{i=n-k+1}^{n} (n-i+1)\left(EY_{(k-n+i,k)} - EY_{(i-(n-k)-1,k)}\right) EX_{(i-1,n)}, \qquad (2) \\ W_m(n,k) &= \sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} - S_m(n,k) \\ &= \sum_{i=n-k+1}^{n} (n-i+1)\left(EX_{(i,n)} - EX_{(i-1,n)}\right) EY_{(i-(n-k),k)} \qquad (3) \end{aligned}$$

2. *Women's total signaling effort and (net) welfare are given by :*

$$\begin{aligned} S_w(n,k) &= k\int_0^{\tau_G} \gamma(y) g(x)\, dx \\ &= \sum_{i=n-k+1}^{n} (n-i+1)\left(EX_{(i,n)} - EX_{(i-1,n)}\right) EY_{(i-(n-k)-1,k)} \qquad (4) \\ W_w(n,k) &= \sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} - S_w(n,k) \\ &= \sum_{i=n-k+1}^{n} (n-i+1)\left(EY_{(i-(n-k),k)} - EY_{(i-(n-k)-1,k)}\right) EX_{(i,n)} \qquad (5) \end{aligned}$$

3. *Total expected (net) welfare in assortative matching based on costly signaling is at least half the expected output (or, in other words, aggregate signaling efforts are less than half output).*

$$\begin{aligned} W(n,k) &= 2\sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} - S_m(n,k) - S_w(n,k) \qquad (6) \\ &\geq \sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} \qquad (7) \end{aligned}$$

If the normalized spacings are increasing in i, both total signaling effort and total welfare will be relatively high (we have "assortative matching" between weights and normalized spacings in the above expressions). The opposite holds if normalized spacings are decreasing in i. A main result by Barlow and Proschan identifies large classes of distributions where such monotonicity properties hold - it provides a basic tool for our analysis below. We first need the following definition:

Definition 1 *Let H be a distribution on $[0, \tau_H]$ with density h.*

1. *The failure (or hazard) rate of H is given by the function $\lambda(x) \equiv h(x) / [1 - H(x)]$, $x \in [0, \tau_H)$.*
2. *H is said to have an increasing (decreasing) failure rate (IFR) (DFR) if $\lambda(x)$ is increasing (decreasing) in x.*[18]
3. *H is said to have an increasing (decreasing) failure rate on average (IFRA) (DFRA) if $(\int_0^x \lambda(t)\, dt)/x$ is increasing (decreasing) in x.*[19]

Theorem 1 *(Barlow and Proschan, 1966) Assume that F is IFR (DFR). Then the normalized spacings $(n - i + 1)\left(X_{(i,n)} - X_{(i-1,n)}\right)$ are stochastically decreasing (increasing) in $i = 1, 2, ..., n$ for fixed n , and stochastically increasing (decreasing) in $n \geq i$, for fixed i.*

3.2 Effects of changes in the number of agents

We analyze here the effects on both sides of the market occurring when we change the number of agents on one side.[20] A main advantage of a model with a finite number of agents is that this analysis can be directly performed, thus shedding new light on effects that may be lost when passing to the continuum limit.

[18] The *IFR (DFR)* conditions are also equivalent to the *logconcavity (logconvexity)* of the survivor function $1 - F$. The exponential, uniform, normal, power (for $\alpha \geq 1$), Weibull (for $\alpha \geq 1$), gamma (for $\alpha \geq 1$) distributions are *IFR*. The exponential, Weibull (for $0 < \alpha \leq 1$), gamma (for $0 < \alpha \leq 1$) are *DFR*. See Barlow and Proschan (1975).

[19] The exponential distribution has a constant failure rate, and it is the only distribution that is both *IFRA* and *DFRA*. Clearly, the family of *IFRA (DFRA)* distributions includes all *IFR (DFR)* distributions.

[20] In complete information models, Kelso and Crawford (1982), Mo (1988), and Crawford (1991) studied changes in core payoffs following entry on one side in two-sided markets such as Shapley and Shubik's (1972) assignment game. Entry lowers the payoffs of same-side agents already in the market, while increasing the payoff of all agents on the other side.

Additional agents unambiguously increase the expected matching output. If there is entry on the long side (i.e. entry by men), the number of matches remains unchanged, but the expected value of the i'th man is increased. If there is entry on the short side (i.e. entry by women), both the number of matches and the expected value of the i'th woman gets higher. On the other hand, an increase in the number of agents also affects the agents' signaling activity. The next proposition shows how the effect on signaling and welfare may depend on certain properties of the distribution of agents' types.

Proposition 3 *Suppose there is entry on the men's side.*[21] *Then, for all G,*

1. *men's total signaling increases for all F,*
2. *women's total signaling increases (decreases) if F is DFR (IFR),*
3. *men's total welfare increases (decreases) if F is DFR (IFR),*
4. *women's total welfare increases for all F.*

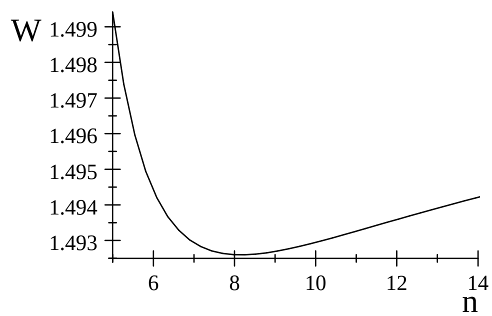
Entry by men leads to stiffer competition among men. While the effort of low types gets smaller (these types are "discouraged"), the effort of high types gets larger, leading to an increase in total signaling. In contrast, the effect on women's signaling is ambiguous: By Theorem 1, the marginal gains from winning a better, highly ranked man are relatively high (small) if F is DFR (IFR). Moreover, by Theorem 7 (Appendix A), if F is DFR (IFR), the difference of successive order statistics is stochastically increasing (decreasing) jointly in i and n. Thus, additional men further increase (reduce) the relatively high (small) marginal gains from winning a better, highly ranked man if F is DFR (IFR). As a consequence, total signaling by women goes up (down) if F is DFR (IFR).

To understand the welfare effects reported in Proposition 3, note that the increase in expected output due to entry is relatively large (small) if F is DFR (IFR). This output effect outweighs the increase in total signaling on the women's side if F is *DFR*. Thus, since women's total signaling goes down if F is *IFR*, women's total welfare is always increasing in the number of men. On the men's side, the output effect outweighs the increase in men's signaling if F is *DFR*, similarly to women. But, in contrast to women, men's total signaling goes up if F is *IFR*. In this case the signaling effect outweighs the output effect, leading to a reduction in men's total welfare.

[21] Entry by women (i.e., on the short side) has similar effects to entry by men, except that it leads to a higher number of matches, and hence a higher number of prizes for men. This increase has, ceteris paribus, a positive effect on the men's total signaling effort. Therefore, even if the distribution of women's types G is *IFR*, men's total signaling may increase due to the presence of additional women.

Combining the above observations, we can conclude that total welfare is always increased by entry if F is *DFR*, but may be reduced if F is *IFR*. Such a welfare loss due to entry is illustrated in the following example. The example will also highlight that the results can be quite different when the number of agents is modeled as a continuous variable. For such a setting, we find that entry on the men's side in this situation always increases total welfare (see Section 5).

Example 1 *Suppose* $F = x^{10}$, $G = y$, *and* $\tau_F = \tau_G = 1$. *Fix* $k = 3$. *Then* $W(3,3) \simeq 2.078$. *As depicted by the graph below,* $W(n,3)$ *is decreasing in* n *for* $n \leq 8$ *and increasing for* $n > 8$. *We also have* $\lim_{n\to\infty} W(n,3) = 1.5$.



3.3 Which side signals more ? The effects of heterogeneity differentials

In many applied studies (e.g., biological studies of sexual selection in various species, development studies on marriage markets in various rural societies) it has been noted that one side of the market engages in much more signaling activity than the other. Similarly, in intermediated markets where payments from agents accrue to a third party, prices are often uneven, with one side paying much more than the other.

In order to explain the asymmetry, the biological literature has noted that, although the sex-ratio is approximately 50-50 in most species, at any point in time, there are more males ready to mate than females. Thus, males perceive more competition, and need to signal more. Proposition 3 above shows that this heuristic argument neatly applies here if the distribution of attributes on the men's side is IFR: in that case, the addition of more men increases men's signaling while decreasing women's signaling. But, if that distribution is DFR, the precise effect is more subtle since an increase in the number of men also leads to higher women's signaling.

The recent IO literature on two-sided markets has identified the presence of network externalities as a source of asymmetric payment patterns: attracting customers on one side by lowering price may be profitable if this side creates externalities on the other side. As a consequence, intermediaries may find it profitable to charge even below-cost price to one side while charging high prices to the other.[22]

We identify here another cause for discrepancies in signaling activity among the two sides: relatively higher signaling activity on one side of the market may be due to a relatively smaller degree of heterogeneity on that side.[23] Two effects matter for this result: first, competition among agents on the same side becomes more intense as they become more homogeneous; and second, agents are willing to pay more as the other side, i.e., the perceived "prize structure", becomes more heterogeneous. To understand the second effect, note that an increase in heterogeneity implies that the marginal gains in terms of winning a better partner get larger at the high end of the type range, increasing agents' willingness to pay on the other side of the market.

We need the following result involving the function $G^{-1}F$. This function has an important meaning here: it describes the assortative matching function in the continuous version of our model (see Section 5).

Theorem 2 *(Barlow and Proschan, 1966) Let $X(Y)$ have distributions $F(G)$ such that $F(0) = G(0) = 0$. Assume that the function $G^{-1}F(x)$ is star-shaped, that is, $G^{-1}F(x)/x$ is increasing for $x \geq 0$.*[24] *Then, as x increases from 0 to ∞, the function $1 - F(x)$ crosses $1 - G(x)$ at most once, and then from above. In particular, if F and G have the same mean, then a crossing must occur, and F has a smaller variance than G.*

In Appendix A we detail the consequences of single-crossing on order statistics - these are the mathematical results used for the next result[25].

Proposition 4

1. *Let $n = k$ and let $G^{-1}F$ be star-shaped. Then men's signaling effort is higher than women's. Thus, if signals are wasteful, women are better-off than men.*
2. *Let either F or G be IFR, and let $G^{-1}F$ be convex. Then, for any $n \geq k$, men's signaling effort is higher than women's*

[22] See Tirole and Rochet (2003) and *The Economist*, "Matchmakers and trustbusters", p.84, Dec 10th, 2005..

[23] Consider, for example, an insightful excerpt taken from the empirical study of marriage in rural Ethiopia due to Fafchamps and Quisumbing (2005): "if the difference between grooms is large relative to the difference between brides, brides must bring more to fend off competition from lower ranked brides who wish to improve their ranking".

[24] Since convex functions ϕ on $[0, \tau)$ such that $\phi(0) \leq 0$ are star-shaped, the convexity of $G^{-1}F(x)$ on the support of F implies here that $G^{-1}F(x)$ is star-shaped. If G is the exponential distribution, then $G^{-1}F(x)$ being convex (concave) is equivalent to F being *IFR (DFR),* and $G^{-1}F(x)$ $(F^{-1}G(x))$ star-shaped is equivalent to F being *IFRA (DFRA).*

[25] Those insights can also be used to obtain comparative statics results about the two-sided effects of: 1) an increase in the heterogeneity of the attributes on one side of the market; 2) an increase in the overall quality of attributes on one side of the market.

A simple and intuitive corollary is as follows:

Corollary 1 *Let F be convex and G be concave. Then, for any $n \geq k$, men's total signaling effort is higher than women's.*

If F has an increasing density while G has a decreasing density, then men with relatively high types face a stiffer competition than those with relatively low types, while the opposite holds for women. In addition, if F is convex and G concave, the differences in successive order statistics is decreasing on the men's side, and increasing on the women's side (see Boland et al., 2001). This implies that the marginal gains in terms of winning a better partner are larger for highly-ranked men than highly-ranked women, which tends to further increase total men's effort relative to women's.

Remark 1 *We have mentioned in Section 2 that our results can be easily extended to models with a production function of the form $2\delta(x)\rho(y)$ where δ and ρ are strictly increasing, non-negative functions. Here is an example: consider the Cobb-Douglas production $2\delta(x)\rho(y) = 2x^c y^d$, $c, d > 0$. Let $n = k$, and assume that men's and women's attributes are uniformly distributed on $[0, 1]$. This model is equivalent to the one where the types are $\tilde{x}$, $\tilde{y}$, the production function is $2\tilde{x}\tilde{y}$, and the distributions of attributes are $\tilde{F}(\tilde{x}) = \tilde{x}^{\frac{1}{c}}$, $\tilde{G} = \tilde{y}^{\frac{1}{d}}$. Thus, men signal more and are worse-off if $c \leq d$.*

4 Assortative versus random matching

We now compare the equilibrium outcome of assortative matching with signaling to the outcome where agents are matched randomly. Random matching can also be seen as the outcome of a completely pooling equilibrium where nobody signals.

While the matching surplus generated through assortative matching is clearly larger than the one obtainable through random matching, assortative matching involves the cost of signaling efforts. The main questions we address are: 1) Under which conditions is the increase in total expected output achieved by assortative matching completely offset by the increased cost of signaling ? 2) Which types prefer random matching, and which types prefer assortative matching with signaling?

4.1 Total welfare

Total welfare in random matching is given by:

$$W^r(n, k) = 2\min(n, k) EX \cdot EY \tag{8}$$

Comparing assortative and random matching in terms of total welfare, we obtain the following result:

Proposition 5 *1. Suppose that $n = k$. Then random matching is welfare superior (inferior) to assortative matching based on signaling if F and G are IFRA (DFRA).*

2. For any $n \geq k$, assortative matching based on signaling is welfare superior to random matching if F and G are DFR.

The normalized spacings of order statistics, representing the marginal gains from winning a better partner, are i.i.d. for the exponential distribution. If, in this case, the market sides are of equal size, the increase in total expected output achieved by assortative matching coincides with the increase in signaling cost. When the normalized spacings are increasing in i, as in the DFR case, the increases in both total signaling cost and total output due to assortative matching is relatively high. Part 1 establishes that, in this case, the output increase outweighs the increase in signaling costs. The opposite holds in the IFR case.[26] Part 2 uses the entry result, Proposition 3, to extend the characterization to the case where $n \geq k$. This is straightforward since an increase in n does not increase the number of pairs, and hence does not affect output in random matching.

4.2 Individual welfare

We now compare between assortative matching and random matching from each agent's point of view. Obviously, agents with low types prefer random matching since in that case they expect relatively good (i.e., average) partners without incurring any cost, while in assortative matching with signaling they get partners with low types after having wasted some resources on signaling.

Lemma 1 *For any distributions F and G, and for any $n \geq k$, there exists at most one cutoff type $\hat{x} \in [0, \tau_F]$ such that all men $x < \hat{x}$ are better-off under random matching, while all men $x \geq \hat{x}$ are better-off under assortative matching based on signaling (and analogously for women).*

From Proposition 5 we know that assortative matching based on signaling yields a higher total welfare than random matching if F and G are DFR. Together with the above Lemma, this implies that, if F and G are DFR, there must exist some types of agents that actually prefer assortative matching with signaling to random matching, i.e., the cutoff point is interior.

[26] The results in Section 5 below will provide some more intuition for these results in terms of a measure of the heterogeneity of the respective populations.

Suppose now that F and G are not DFR, such that the cutoff defined in Lemma 1 does not necessarily exist. An interesting question is whether it is possible that all agents, including those with high types, are better-off under random matching ? The answer is affirmative:

Proposition 6 *Let $n = k$, and assume that $\tau_F < \infty$. For any G, if F stochastically dominates the uniform distribution on $[0, \tau_F]$, then all types of men prefer random matching to assortative matching based on signaling.*[27] *Analogous results hold for women.*

Note that in the Spence model with two types of workers and homogenous firms, the workers' payoffs in the separating equilibrium are independent of the distribution of worker types.[28] Changes in the distribution only affect the pooling equilibrium wage, which is equal to the workers' expected marginal product. It is known for that model that workers prefer the pooling equilibrium to the separating one if the fraction of high-ability workers is sufficiently high. In contrast payoffs in both pooling and separating equilibria depend here on the distributions of types, and the comparison between the two is somewhat subtler.

5 Large populations

We now consider the version of our matching model where, instead of a finite number of agents, there are measures of men and women, distributed according to F and G, respectively. Our analysis will focus on the connections between this model, and the finite model analyzed so far. The leeway between perfect competition in the continuum version versus less than perfect competition in the finite version plays an important role.

We first normalize both measures of agents to one. Random matching yields now an expected total output (and welfare) of $2EXEY$. Under assortative matching, a man with attribute x is matched with a woman with attribute $y = \psi(x)$, where $\psi(x) = G^{-1}F(x)$. Let $\varphi = \psi^{-1}$. Expected total output under assortative matching is given by $2\int_0^{\tau_F} x\psi(x) f(x)dx$. The signaling activity that enables assortative matching is characterized next:

Proposition 7 *In the continuum model, the equilibrium signaling function for men and women are given by $\beta(x) = \int_0^x z\psi'(z)\, dz$ and $\gamma(y) = \int_0^y z\varphi'(z)\, dz$, respectively. In each matched pair, exactly half the output from assortative matching is wasted through signaling in this equilibrium.*

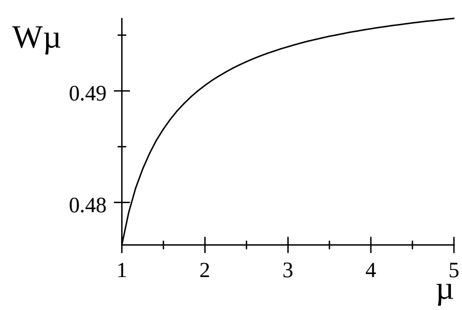
[27] If F is stochastically dominated by the uniform distribution, then some types of men prefer random matching to assortative matching with signaling.

[28] For a generalization to a continuum of workers' types, see Mailath (1987).

Note that half the total output is dissipated through signaling for any level of heterogeneity.[29] The reason is that each agent shares half of the incremental rents from winning a better partner with his/her matching partner, while dissipating the other half in the contest with agents on the same side. Thus, the dissipated rents on both sides add up to half the output.[30]

It is straightforward to incorporate asymmetric market sizes in the above model. Start with F and G, two distributions with normalized mass of one, as before. Consider then men's distributions having the form $F_\mu(x) = \mu F(x)$ where $\mu \geq 1$, and let $a_\mu = F_\mu^{-1}(\mu - 1)$. Then, in assortative matching among the populations represented by F_μ and G, men having attributes in the interval $[a_\mu, \tau_F]$ are matched to women having attributes in the interval $[0, \tau_G]$ according to the function $\psi_\mu(x) = G^{-1}(F_\mu(x) - \mu + 1)$. The equilibrium derivations follow then analogously to those in Proposition 7.

Remark 2 *We now use the above observation in order to reconsider the situation described in Example 1 (Section 3): let $F(x) = x^{10}$, $G(y) = y$, $\tau_F = \tau_G = 1$, and let W_μ denote total welfare as a function of $\mu \geq 1$. Recall that in Example 1 we fixed the number of women (and thus of pairs) to be three, and note that $\lim_{\mu\to\infty} W_\mu = \lim_{n\to\infty} \frac{1}{3} W(n,3) = 0.5$. But, whereas in the discrete setting entry on the men's side initially reduced welfare, we obtain here that W_μ is monotonically increasing in μ.*



To understand the difference, note that in a small population agent i faces a considerable uncertainty about the type of the same-side agent who is ranked just below i: there is a positive probability that this type is much lower than i's own type, while in the continuum model almost perfect substitutes

[29] For example, consider a sequence of distributions converging to the Dirac distribution on $\tau < \infty$. As the distributions become more concentrated, the limiting value of aggregate signaling is half the limiting value of the expected output, and thus bounded away from zero. That is, even when the probability that a potential matching partner is worse than τ gets arbitrarily small, agents still engage in signaling in order to prevent being matched with a low type.

[30] The proof of Proposition 7 establishes close relations between signaling, net welfare and stable (i.e., core) payoffs for each matched pair. These type of relations are more general, and hold for any fixed sharing rule among partners.

to i are always present. As a consequence, the equilibrium signaling efforts are lower in the discrete model. Faced with entry, the amount of signaling may initially rise faster than total output in a small market, while the opposite holds in the large, "perfectly competitive" market.

With respect to heterogeneity differentials we obtain the following result, which turns out to be analogous to Proposition 4 for the discrete model:

Proposition 8 *In the continuum model, men's total signaling is larger (smaller) than women's total signaling if the matching function $\psi = G^{-1}F$ is convex (concave).*

As we demonstrate below, in the continuum model the welfare comparison between assortative matching based on signaling and random matching exhibits some analogies but also some subtle differences versus the finite, discrete model. By applying Proposition 7 we obtain:

Proposition 9 *In the continuum model, assortative matching based on signaling is welfare superior (inferior) to random matching if and only if*

$$\frac{Cov(X, \psi(X))}{EX \cdot E\psi(X)} \geq (\leq)\ 1$$

The above result can be more easily explained in the symmetric setting where $F = G$. Let $CV \equiv \sqrt{Var(X)}/EX$ be the coefficient of variation of the common distribution of abilities. A smaller coefficient means that types are less heterogeneous. Proposition 9 immediately yields:

Corollary 2 *Let $F = G$ in the continuum model. Then, assortative matching based on signaling is welfare superior (inferior) to random matching if and only if $CV \geq (\leq)\ 1$[31]. In particular, assortative matching based on signaling is welfare superior (inferior) to random matching if F is DFRA (IFRA).*

When the coefficient of variation is greater than 1, the increase in total output through assortative matching (relative to the random output) outweighs the increase in the costs of signaling, and vice versa for a coefficient of variation smaller than 1. Note that the difference in total output between assortative matching and random matching gets smaller as the degree of heterogeneity in the population decreases, while signaling remains proportional to output for any degree of heterogeneity. Hence, as the coefficient of variation gets smaller, total welfare achieved by assortative matching eventually falls below the level achieved by random matching.

[31] Note that $CV = 1$ for the exponential distribution: as in the finite case with equal numbers of men and women, total welfare in random matching equals then total welfare in assortative matching.

Remark 3 *The last part of Corollary 2 follows from Barlow and Proschan's (1975) result whereby F being DFRA (IFRA) implies that its coefficient of variation is larger (smaller) than unity. For distributions with monotone average hazard rates, this observation, together with Proposition 5-1, show that the insight obtained in the continuum model holds in fact for finite markets of* ***any*** *size (with equal numbers of men and women). But, it is important to note that the converse relation among coefficients of variation and monotone average hazard rates is* ***not true:*** *a coefficient of variation larger (smaller) that unity does not necessarily imply that the respective distribution is DFRA (IFRA). As a consequence, the "if and only if" result for the continuum model applies to a much larger class of distributions than those covered by the weaker "if" result of Proposition 5-1 covering the discrete model. Moreover, this discrepancy can be used to illustrate that the "backward extrapolation" from very large to very small markets may yield wrong insights and predictions if the distributions of attributes are not IFRA or DFRA, thus justifying the need for the precise (if somewhat more tedious) analysis of the discrete model.*

Example 2 *Let* $F = G = x^{\frac{9}{20}}$ *and let* $\tau_F = \tau_G = 1$. *We obtain that* $CV = \sqrt{\frac{400}{441}} < 1$. *For the continuum model, Corollary 2 immediately yields that random matching is welfare-superior to assortative matching. Consider now the finite model with an equal number of agents on each side. Since* F *is not* $IFRA$, *Proposition 5 is not applicable. In fact, it turns out that the welfare comparison for this example crucially depends on market size. For* $n \geq 10$, *we indeed have* $W(n,n) \leq W^r(n,n)$, *i.e., random matching is welfare superior to assortative matching, but, for* $n < 10$, *the exact opposite holds:* $W(n,n) > W^r(n,n)$, *i.e., assortative matching is welfare superior to random matching*

The above results have been obtained by direct arguments applied to the continuum model. Of course, it is also possible to explicitly take limits in the discrete model[32]. It can be then shown that per-capita output and signaling effort in the discrete model converge to their continuous counterparts when the number of agents goes to infinity. Such an exercise immediately yields, for example, that, in the limit, assortative matching based on signaling is welfare superior (inferior) to random matching if F *is DFRA (IFRA),* but it cannot yield (at least not by the methods described in this paper) the stronger if and only if result involving the coefficient of (co)variation.

Finally, we have mentioned in the introduction that our insights deliver immediate results for other, less general models. Here is an example: Consider a continuous setting where the attributes of

[32] Peters (2004) studies the limit, as the number of agents goes to infinity, of mixed strategy equilibria arising in a complete-information model where a finite number of agents on each side of the market make costly investments prior to the match. In his model, the limit need not correspond to the hedonic equilibrium in the market with a continuum of agents.

one side of the market (say men) are known. Then, signaling is only performed by one side, and the waste from signaling is halved. Thus, assortative matching via signaling becomes more attractive relative to random matching, and, by an argument similar to the one in Corollary 2, we obtain that assortative matching is welfare superior (inferior) to random matching if $CV \geq (\leq) \sqrt{1/3}$. In particular, the two alternatives are now equivalent for the uniform distribution on a bounded interval.

6 Conclusion

We have studied two-sided matching models where privately informed agents on each side are matched on the basis of costly signals. For the welfare analysis we have assumed that signals are wasted. Our study revealed how welfare on both sides of the market is affected by changes in primitives of the model such as the number of the agents and the distributions of their attributes, and it yielded a new explanation for the often observed asymmetry in signaling activity. We have also identified conditions under which assortative matching based on wasteful signaling is welfare superior (inferior) to random matching. Thus, the effects of policies that attempt to curb "wasteful" signaling need to be carefully examined in each particular situation.[33]

The analysis of markets with finite numbers of agents has revealed several phenomena that are particular to such markets, and do not occur in very large ones. This analysis been made possible by the application of results and methods from mathematical statistics. We believe that the applications of these methods will be fruitful also in other areas, such as double auctions. Finally, we hope that our model (or some of its many possible variations) will be useful as a sound, theoretical basis around which to organize observations in a variety of empirical studies, e.g., of marriage, labor and education markets.

[33] Alternatively, this holds for policies that attempt to manipulate the rent accruing to a third party, such as an intermediary (see Hoppe, Moldovanu, Ozdenoren, 2006).

7 Appendix A: Order statistics and stochastic orders

Definition 2 *For any two non-negative random variables, X and Z, with distributions F and H and hazard rates λ_x and λ_z, respectively, X is said to be smaller than Z in the hazard rate order (denoted as $X \leq_{hr} Z$) if $\lambda_x(s) \geq \lambda_z(s)$, for all $s \geq 0$. X is said to be smaller than Z in the usual stochastic order (denoted as $X \leq_{st} Z$) if $F(s) \geq H(s)$ for all $s \geq 0$.*

Theorem 3 *(see Shaked and Shanthikumar ,1994):*

1. *If X and Z are two random variables such that $X \leq_{hr} Z$, then $X \leq_{st} Z$.*
2. *Let $X_1, X_2, ..., X_n$ be independent random variables. Then:*
   - $X_{(i,n)} \leq_{hr} X_{(i+1,n)}$ *for* $i = 1, 2, ..., n-1$,
   - $X_{(i-1,n-1)} \leq_{hr} X_{(i,n)}$ *for* $i = 2, 3, ..., n$
   - $X_{(i,n-1)} \geq_{hr} X_{(i,n)}$ *for* $i = 2, 3, ..., n-1$.

A basic consequence of single crossing for random variables ordered by the star-shaped order is:

Theorem 4 *(see Barlow and Proschan, 1966) Let X, Z two random variables with distributions F, H respectively, such that $F(0) = H(0) = 0$, and such that $H^{-1}F$ is star-shaped. Then:*

1. *The function $\omega(i,n) = EX_{(i,n)} - EZ_{(i,n)}$ changes sign at most once when i (n) increases and then from positive to negative (negative to positive), if at all. If $EX = EZ$ then a change of sign when i increases must occur.*
2. *The ratio $EX_{(i,n)}/EZ_{(i,n)}$ is decreasing (increasing) in i (n).*
3. *The ratio $EX_{(n-i,n)}/EZ_{(n-i,n)}$ is decreasing in n.*

Many of our proofs rely on a conjunction of the above result with the following Theorem:

Theorem 5 *(see Barlow and Proschan, 1966): Consider $\alpha_i > 0$, $\beta_i \geq 0$, $i = 1, 2, , n$, such that β_i/α_i is increasing in i. Then $(\sum_1^n a_i\beta_i)/(\sum_1^n \beta_i) \leq (\sum_1^n a_i\alpha_i)/(\sum_1^n \alpha_i)$ for any $a_1 \geq a_2 ... \geq a_n$.*

Two simple, but important consequences are:

Theorem 6 *(see Barlow and Proschan, 1966)*

1. *If $H^{-1}F$ is star-shaped, and if $EX = EZ$ then*

$$\sum_{i=1}^{n} a_i[(n-i+1)(EX_{(i,n)} - EX_{(i-1,n)})] \geq \sum_{i=1}^{n} a_i[(n-i+1)(EZ_{(i,n)} - EZ_{(i-1,n)})]$$

2. *If F is IFRA (DFRA) then:*

$$\sum_{i=1}^{n} a_i[(n-i+1)(EX_{(i,n)} - EX_{(i-1,n)})] \geq (\leq) EX \sum_{i=1}^{n} a_i$$

We also use the following generalizations of Barlow and Proschan's results:

Theorem 7 *(see Hu and Wei, 2001) Define $U_{(j,i,n)} \equiv X_{(j,n)} - X_{(i,n)}$ for $0 \leq i < j \leq n$. Let F be DFR (IFR). Then $U_{(j-1,i-1,n-1)} \leq_{hr} (\geq_{hr}) U_{(j,i,n)}$.*

Theorem 8 *(see Khaledi and Kochar, 1999):*

1. *If $X \leq_{hr} Z$ and either X or Z is DFR, then $(n-i+1)(X_{(i,n)} - X_{(i-1,n)}) \leq_{st} (n-i+1)(Z_{(i,n)} - Z_{(i-1,n)})$, $i = 1, 2, 3, ..., n$.*

8 Appendix B: Proofs

Proof of Proposition 1. We first show that the function β in (1) is strictly monotonically increasing. Note that (1) can be written as

$$\begin{aligned}\beta(x) &= \int_0^x \left\{ \sum_{i=n-k+1}^{n-1} f_{(i,n-1)}(s) \left[EY_{(k-n+i+1,k)} - EY_{(k-n+i,k)} \right] \right\} sds \\ &\quad + \int_0^x f_{(n-k,n-1)}(s) \, EY_{(1,k)} sds\end{aligned}$$

Taking the derivative with respect to x yields

$$\begin{aligned}\beta'(x) &= \sum_{i=n-k+1}^{n-1} f_{(i,n-1)}(x) \left[EY_{(k-n+i+1,k)} - EY_{(k-n+i,k)} \right] x \\ &\quad + f_{(n-k,n-1)}(x) \, EY_{(1,k)} x\end{aligned}$$

which is strictly positive because $Y_{(k-n+i+1,k)} \geq_{st} Y_{(k-n+i,k)}$.

Next, we check whether the second-order condition is satisfied. Integrating the RHS of (1) by parts, yields

$$\begin{aligned}
\beta(x) &= x\sum_{i=n-k+1}^{n-1} EY_{(k-(n-i),k)}\left[F_{(i-1,n-1)}(x)-F_{(i,n-1)}(x)\right] \\
&\quad +xF_{(n-1,n-1)}(y)\,EY_{(k,k)} \\
&\quad -\int_0^x \left\{\sum_{i=n-k+1}^{n-1} EY_{(k-(n-i),k)}\left[F_{(i-1,n-1)}(s)-F_{(i,n-1)}(s)\right]\right\} ds \\
&\quad -\int_0^y F_{(n-1,n-1)}(s)\,EY_{(k,k)}ds \\
&= x\sum_{i=n-k+1}^{n} F_i^n(x)\,EY_{(k-(n-i),k)} - \int_0^x \sum_{i=n-k+1}^{n} EY_{(k-(n-i),k)}F_i^n(s)\,ds
\end{aligned}$$

Let $z=\beta^{-1}(b)$ be the type for which the equilibrium effort is b. The expected payoff of a man with type x from exerting effort $\beta(z)$ is thus given by:

$$\begin{aligned}
U(b,x) &= \sum_{i=n-k+1}^{n}\left[F_{(i-1,n-1)}(z)-F_{(i,n-1)}(z)\right]xEY_{(k-(n-i),k)}-\beta(z) \\
&= \sum_{i=n-k+1}^{n} F_i^n(z)\,EY_{(k-(n-i),k)}(x-z) \\
&\quad +\int_0^z \sum_{i=n-k+1}^{n} EY_{(k-(n-i),k)}F_i^n(s)\,ds
\end{aligned}$$

Hence, the difference between the expected payoffs of type x when he exerts efforts of $\beta(x)$ and $\beta(z)$ is:

$$\begin{aligned}
U(\beta(x),x)-U(\beta(z),x) = {} & \textstyle\sum_{i=n-k+1}^{n} F_i^n(z)\,EY_{(k-(n-i),k)}(z-x) \\
& -\textstyle\int_x^z \sum_{i=n-k+1}^{n} EY_{(k-(n-i),k)}F_i^n(s)\,ds
\end{aligned} \tag{9}$$

Since β is strictly increasing, the function $H(s)=\sum_{i=n-k+1}^{n} F_i^n(s)\,EY_{(k-(n-i),k)}$ increases in s and therefore the difference in (9) is always positive. ■

Proof of Proposition 2. **1)** Substituting (1) into (2) yields:

$$\begin{aligned}
S_m(n,k) &= n\int_0^{\tau_F}\int_0^x \sum_{i=n-k+1}^{n-1} f_{(i-1,n-1)}(s)\,EY_{(k-(n-i),k)}sdsf(x)\,dx \\
&\quad -n\int_0^{\tau_F}\int_0^x \sum_{i=n-k+1}^{n-1} f_{(i,n-1)}(s)\,EY_{(k-(n-i),k)}sdsf(x)\,dx \\
&\quad +n\int_0^{\tau_F}\int_0^x f_{(n-1,n-1)}(s)\,EY_{(k,k)}sdsf(x)\,dx
\end{aligned} \tag{10}$$

Integrating the first plus the third terms of (10) by parts and rearranging terms, we obtain

$$\begin{aligned}
& n\int_0^{\tau_F}\int_0^x \sum_{i=n-k+1}^{n} f_{(i-1,n-1)}(s)\, EY_{(k-(n-i),k)} s ds f(x)\, dx \\
= \; & n\int_0^{\tau_F} [1-F(x)] \sum_{i=n-k+1}^{n} x f_{(i-1,n-1)}(x)\, EY_{(k-(n-i),k)} dx \\
= \; & n\int_0^{\tau_F} \sum_{i=n-k+1}^{n} \frac{n-i+1}{n} x f_{(i-1,n)}(x)\, EY_{(k-(n-i),k)} dx \\
= \; & \sum_{i=n-k+1}^{n} (n-i+1)\, EX_{(i-1,n)} EY_{(k-(n-i),k)}
\end{aligned}$$

Similarly, integrating the second term of (10) by parts, we obtain

$$\begin{aligned}
& -n\int_0^{\tau_F}\int_0^x \sum_{i=n-k+1}^{n-1} f_{(i,n-1)}(s)\, EY_{(k-(n-i),k)} s ds f(x)\, dx \\
= \; & -n\int_0^{\tau_F} [1-F(x)] \sum_{i=n-k+1}^{n-1} x f_{(i,n-1)}(x)\, EY_{(k-(n-i),k)} dx \\
= \; & -n\int_0^{\tau_F} \sum_{i=n-k+1}^{n-1} \frac{n-i}{n} x f_{(i,n)}(x)\, EY_{(k-(n-i),k)} dx \\
= \; & \sum_{i=n-k+1}^{n} (n-i)\, EX_{(i,n)} EY_{(k-(n-i),k)}
\end{aligned}$$

Collecting terms, yields:

$$S_m(n,k) = \sum_{i=n-k+1}^{n} \left[(n-i+1)\, EX_{(i-1,n)} - (n-i)\, EX_{(i,n)}\right] EY_{(k-(n-i),k)}$$

$$= \sum_{i=n-k+1}^{n} (n-i+1)\, EX_{(i-1,n)} \left(EY_{(k-n+i+1,k)} - EY_{(k-n+i,k)}\right) \tag{11}$$

Men's total welfare follows from the definition of gross surplus and (11).

2) Analogous to the above.

3) Note that the only difference in the expressions for $W(n,k)$ on the one hand, and $S_m(n,k) + S_w(n,k)$ on the other, is that the normalized spacings appearing in $W(n,k)$ are multiplied by a higher weight, corresponding to the expectation of a higher order statistic. Thus

$$\begin{aligned} S_m(n,k) + S_w(n,k) &\leq W(n,k) \Leftrightarrow \\ W(n,k) + S_m(n,k) + S_w(n,k) &\leq 2W(n,k) \Leftrightarrow \\ 2\sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} &\leq 2W(n,k) \Leftrightarrow \\ \sum_{i=n-k+1}^{n} EX_{(i,n)} EY_{(k-(n-i),k)} &\leq W(n,k) \end{aligned} \tag{12}$$

as desired. ■

Proof of Proposition 3. **1)** For $j = i - (n-k)$,we rewrite total men's signaling as:

$$S_m(n,k) = \sum_{j=1}^{k} (k-j+1) \left(EY_{(j,k)} - EY_{(j-1,k)} \right) EX_{(j+n-k-1,n)} \tag{13}$$

The result follows since $EY_{(j,k)} \geq_{st} EY_{(j-1,k)}$ and since $EX_{(j+n-k-1,n)}$ is stochastically increasing in n (see Theorem 3, Appendix A).

2) For $j = i - (n-k)$,we rewrite total women's signaling as:

$$S_w(n,k) = \sum_{j=1}^{k} (k-j+1) \left(EX_{(j+n-k,n)} - EX_{(j+n-k-1,n)} \right) EY_{(j-1,k)} \tag{14}$$

The result follows by Theorem 7 (Appendix A).

3) Men's total welfare is given by:

$$W_m(n,k) = \sum_{j=1}^{k} (k-j+1) \left(EX_{(j+n-k,n)} - EX_{(j+n-k-1,n)} \right) EY_{(j,k)} \tag{15}$$

which is is similar to women's signaling (expression (14)). The proof is analogous to that at point 2 above, and we omit it here.

4) Women's total welfare is given by :

$$W_w(n,k) = \sum_{j=1}^{k} (k-j+1) \left(EY_{(j,k)} - EY_{(j-1,k)} \right) EX_{(j+n-k,n)} \tag{16}$$

which is is similar to men's signaling (expression (13)). The proof is analogous to that at point 1 above, and we omit it here. ■.

Proof of Proposition 4. **1)** Using (2) and (4), we get:

$$S_m(n,n) - S_w(n,n) = \sum_{i=1}^{n}(n-i+1)\left(EY_{(i,n)}EX_{(i-1,n)} - EY_{(i-1,n)}EX_{(i,n)}\right) \geq 0$$

The last inequality follows from Theorem 4-(2) (Appendix A) which says that the ratio $EX_{(i,n)}/EY_{(i,n)}$ is decreasing in i .

2) From Proposition 3 we know that:

(i) for any $n \geq k$, and any F, G, $S_m(n,k) \geq S_m(k,k)$,

(ii) for any $n \geq k$, for any G, and for F IFR, $S_w(n,k) \leq S_w(k,k)$

Since $G^{-1}F$ convex implies $G^{-1}F$ star-shaped, the result follows directly from **1)** and (i), (ii) if F is IFR.

Assume now that G is IFR. This means that $H^{-1}G$ convex, where H is the exponential distribution. Thus, $H^1GG^{-1}F = H^{-1}F$ is convex (since it is a composition of increasing convex functions), which means that F is IFR. The result follows as above. ■

Proof of Proposition 5 **1)** Welfare in random matching can be written as:

$$\begin{aligned} W^r(n,n) &= 2nEX \cdot EY = nEX\frac{\sum_{i=1}^{n}EY_{(i,n)}}{n} + nEY\frac{\sum_{i=1}^{n}EX_{(i,n)}}{n} \\ &= EX\sum_{i=1}^{n}EY_{(i,n)} + EY\sum_{i=1}^{n}EX_{(i,n)} \end{aligned} \tag{17}$$

Welfare in assortative matching is given by (6). Let $a_i = -EY_{(i,n)}$, and note that a_i is decreasing in i. Applying Theorem 6-(2) (Appendix A) yields: if F is IFRA (DFRA), then

$$-\sum_{i=1}^{n}EY_{(i,n)}[(n-i+1)\left(EX_{(i,n)} - EX_{(i-1,n)}\right)] \geq (\leq) - EX\sum_{i=1}^{n}EY_{(i,n)} \tag{18}$$

Multiplying by (-1), we obtain: if F is IFRA (DFRA), then

$$\sum_{i=1}^{n}EY_{(i,n)}[(n-i+1)\left(EX_{(i,n)} - EX_{(i-1,n)}\right)] \leq (\geq) EX\sum_{i=1}^{n}EY_{(i,n)} \tag{19}$$

Similarly, we obtain: if G is IFRA (DFRA), then

$$\sum_{i=1}^{n}EX_{(i,n)}[(n-i+1)\left(EY_{(i,n)} - EY_{(i-1,n)}\right)] \leq (\geq) EX\sum_{i=1}^{n}EX_{(i,n)} \tag{20}$$

The combination of (19) and (20) completes the proof.

2) The result for the general case $n \geq k$ follows by applying the entry results of Proposition 3: Recall that, by Proposition 3, entry by men increases welfare in assortative matching based on signaling if F is DFR (and hence DFRA). The result follows by noting that entry on the long side does not affect welfare from random matching since the number of matched pairs remains constant.
■

Proof of Lemma 1 Let $U^a(x), U^r(x)$ denote the expected utility of type x under assortative matching with signaling, and under random matching, respectively.

Note that $U^a(x) = \max_s \left\{\sum_{i=n-k+1}^{n} F_i^n(s)\, xEY_{(k-n+i,k)} - \beta(s)\right\}$ is an increasing convex function (since it is the maximum of linear increasing functions), while U^r is an increasing linear function with slope EY. Thus, these functions can cross at most once. Note further that the derivative of $U^a(x)$ at $x = 0$ is

$$\left.\frac{dU^a(x)}{dx}\right|_{x=0} = \sum_{i=n-k+1}^{n} F_i^n(0)\, EY_{(k-n+i,k)} \leq EY_{(1,k)} < EY$$

where the first inequality follows either by $\sum_{i=n-k+1}^{n} F_i^n(0)\, EY_{(k-n+i,k)} = 0$ if $n > k$, (since $F_i^n(0) = 0$ if $i > 1$) or by $\sum_{i=n-k+1}^{n} F_i^n(0)\, EY_{(k-n+i,k)} \leq EY_{(1,n)}$ for $n = k$, (since $F_1^n(0) = \lim_{\varepsilon \to 0} F(\varepsilon)^{\varepsilon} \leq 1$). Thus, $U^a(x) \leq U^r(x)$ in a neighborhood of zero, and the wished result follows.
■

Proof of Proposition 6 By Lemma 1, it is clear that if the man with the highest type prefers random matching, then all other types of men prefer random matching as well (and analogously for women). Under assortative matching based on signaling, the expected utility of the type τ man is

$$U^a(\tau) = \tau EY_{(n,n)} - \sum_{i=1}^{n-1} EX_{(i,n-1)}\left(EY_{(i+1,n)} - EY_{(i,n)}\right)$$

The expected utility of this type under random matching is

$$U^r(\tau) = \tau \cdot EY = \frac{\tau}{n}\left(\sum_{i=1}^{n} EY_{(i,n)}\right)$$

If F stochastically dominates (is stochastically dominated by) the uniform distribution, we obtain that $EX_{(i,n-1)} \geq (\leq) \tau \frac{i}{n}$. Then

$$\begin{aligned} U^a(\tau) &\leq (\geq)\, \tau EY_{(n,n)} - \frac{\tau}{n}\sum_{i=1}^{n-1} i\left(EY_{(i+1,n)} - EY_{(i,n)}\right) \\ &= \frac{\tau}{n}\left(\sum_{i=1}^{n} EY_{(i,n)}\right) = U^r(\tau) \quad \blacksquare \end{aligned}$$

Proof of Proposition 7 Consider men's types $x, \hat{x}$, $x > \hat{x}$, with equilibrium bids $\beta(x)$, $\beta(\hat{x})$. In equilibrium, type x is assortatively matched with type $\psi(x)$, and $\hat{x}$ is matched with $\psi(\hat{x})$. Type x should not pretend that he is $\hat{x}$ (thus being matched with $\psi(\hat{x})$ and paying $\beta(\hat{x})$), and vice-versa for type $\hat{x}$. This yields:

$$\begin{aligned} x\psi(x) - \beta(x) &\geq x\psi(\hat{x}) - \beta(\hat{x}) \\ \hat{x}\psi(\hat{x}) - \beta(\hat{x}) &\geq \hat{x}\psi(x) - \beta(x) \end{aligned}$$

Combining the above and dividing by $x - \hat{x}$, gives:

$$\frac{\hat{x}\psi(x) - \hat{x}\psi(\hat{x})}{x - \hat{x}} \leq \frac{\beta(x) - \beta(\hat{x})}{x - \hat{x}} \leq \frac{x\psi(x) - x\psi(\hat{x})}{x - \hat{x}}$$

Taking the limit $\hat{x} \to x$ gives $\beta'(x) = x\psi'(x)$. Together with $\beta(0) = 0$, this yields $\beta(x) = \int_0^x z\psi'(z)\,dz$. Letting $\varphi = \psi^{-1}$, we analogously obtain $\gamma(y) = \int_0^y z\varphi'(z)\,dz$.[34]

We now show in each matched pair exactly half the output from assortative matching is wasted through signaling. It is well known that the unique stable (i.e., core) payoff configuration for our two-sided market with a continuum of agents (and with complete information) is given by:

$$w(x) = 2\int_0^x \psi(t)dt;\ v(y) = 2\int_0^y \varphi(t)dt$$

Since in the core there are no transfers outside matched pairs, it must hold that

$$\forall x,\ w(x) + v(\psi(x)) = 2x\psi(x)$$

By the above calculations, we know that

$$\beta(x) = \int_0^x t\psi'(t)dt = x\psi(x) - \int_0^x \psi(t)dt$$

and similarly for women. This yields:

$$x\psi(x) = \beta(x) + \frac{1}{2}w(x) = \gamma(\psi(x)) + \frac{1}{2}v(\psi(x))$$

This yields:

$$\beta(x) + \gamma(\psi(x)) = 2x\psi(x) - \frac{1}{2}[w(x) + v(\psi(x)] = x\psi(x)$$

as claimed. ■

[34] The results of Mailath (1987) can be applied to show that the second order conditions are satisfied, and that these are indeed equilibrium signaling strategies.

Proof of Proposition 8 Comparing total signaling on the two sides of the market (after integrating by parts) yields:

$$\begin{aligned} & S_m(\infty) \geq (\leq)\ S_w(\infty) \\ \Leftrightarrow \quad & \textstyle\int_0^{\tau_F} x\psi'(x) \frac{1-F(x)}{f(x)} dF(x) \leq (\geq) \int_0^{\tau_F} \psi(x) \frac{1-F(x)}{f(x)} dF(x) \\ \Leftrightarrow \quad & \textstyle\int_0^{\tau_F} \left[\psi(x) - x\psi'(x)\right] \frac{1-F(x)}{f(x)} dF(x) \leq (\geq)\ 0 \end{aligned}$$

The result follows by noting that $\psi(x) - x\psi'(x) < (>)\, 0$ if $\psi(x)$ is convex (concave). ■

Proof of Proposition 9 By Proposition 7, total net welfare in assortative matching based on signaling is given by $\int_0^\tau x\psi(x) f(x)dx$. Thus, assortative matching with signaling is welfare superior (inferior) to random matching if:

$$\begin{aligned} \int_0^\tau x\psi(x) f(x)dx &\geq [\leq]\ 2\int_0^\tau xf(x)\,dx \int_0^\tau yg(y)dy \Leftrightarrow \\ E(X\psi(X)) &\geq [\leq]\ 2EX \cdot EY \Leftrightarrow \\ E(X\psi(X)) &\geq [\leq]\ 2EX \cdot E\psi(X) \Leftrightarrow \\ \frac{Cov(X\psi(X))}{EX \cdot E\psi(X)} &\geq [\leq]\ 1 \end{aligned}$$

(Note that $EY = E\psi(X)$; the proof uses the well-known fact that for any random variable Z with cumulative distribution H, $EZ = \int_0^1 H^{-1}(z)\,dz$) ■.

References

[1] Arnow, K. S. (1983) “The university’s entry fee to federal research programs”, *Science* 219, 27-32.

[2] Barlow, R. E. and Proschan, F. (1965) *Mathematical Theory of Reliability,* Wiley, New York.

[3] Barlow, R. E. and Proschan, F. (1966) ”Inequalities for linear combinations of order statistics from restricted families” *Ann. Math. Statistic.* 37, 1593-1601.

[4] Barlow, R. E. and Proschan, F. (1975) *Statistical Theory of Reliability and Life Testing*, McArdle Press, Silver Spring.

[5] Becker, G. S. (1973) ”A theory of marriage: part 1”, *Journal of Political Economy* 81, 813-846.

[6] Boland, Philip J., Taizhong Hu, Moshe Shaked and J. George Shanthikumar (2002), “Stochastic ordering of order statistics II,” in *Modeling Uncertainty: An Examination of Stochastic Theory, Methods, and Applications,* M. Dror, P. L’Ecuyer, and F. Szidarovszky (eds), Boston: Kluwer.

[7] Bulow, J. and Levin, J. (2005): ”Matching and price competition”, *American Economic Review* forthcoming.

[8] Chao, H. and Wilson, R. (1987): ”Priority service: pricing, investment, and market organization” *American Economic Review* 77, 899-916.

[9] Cole, H.J, Mailath, G. and Postlewaite, A. (1992): ”Social norms, savings behavior, and growth”, *Journal of Political Economy* 100, 1092-1125.

[10] Cole, H.J, Mailath, G. and Postlewaite, A. (2001a): ”Efficient non-contractible investments in large economies”, *Journal of Economic Theory* 101, 333-373.

[11] Cole, H.J., Mailath, G. and Postlewaite, A. (2001b): ”Efficient non-contractible investments in finite economies”, *Advances in Theoretical Economics* 1, Article 2.

[12] Crawford, V.P. (1991): "Comparative statics in matching markets", *Journal of Economic Theory* 54, 389-400.

[13] Damiano, E. and Li, H. (2004): ”Price discrimination and efficient matching”, *Economic Theory*, forthcoming.

[14] David, H. A. and Nagaraja, H. N. (2003) *Order Statistics*, Wiley & Sons, New Jersey.

[15] Fafchamps, M. and Quisumbing, A. (2005) "Assets at marriage in rural Ethiopia" *Journal of Development Economics* 77, 1-25

[16] Felli, L. and Roberts, H. (2002): " Does competition solve the hold-up problem ?", CEPR discussion paper No. 3535

[17] Fernandez, R. and Gali, J. (1999): "To each according to...? Markets, tournaments and the matching problem with borrowing constraints" *Review of Economic Studies* 66, 799-824.

[18] Gretzky, N.E., Ostroy, J.M. and Zame, W. (1999): "Perfect competition in the continuous assignment model" *Journal of Economic Theory* 88, 60-118.

[19] Hoppe, H.C., Moldovanu, B., and Ozdenoren, E. (2006) "Intermediation and Matching", Discussion paper, University of Bonn.

[20] Hu, T. and Wei, Y. (2001) "Stochastic comparisons of spacings from restricted families of distributions", *Statistics and Probability Letters* 52, 91-99.

[21] Kelso, A.S. and Crawford, V.P. (1982) "Job matching, coalition formation, and gross substitutes", *Econometrica* 50, 1483-1504.

[22] Khaledi, B.E. and Kochar, S.C. (1999) "Stochastic orderings between distributions and their sample spacings -II" *Statistics and Probability Letters* 44, 161-166.

[23] Mailath, G. F. (1987), "Incentive compatibility in signaling games with a continuum of types", *Econometrica* 55, 1349-1365.

[24] Maynard Smith, J. and D. Harper (2003) *Animal Signals*, Oxford University Press, Oxford.

[25] McAfee, R. P. (2002) "Coarse matching" *Econometrica* 70, 2025-2034.

[26] Miller, G. (2001), *The Mating Mind*, Anchor Books, New York.

[27] Mo, J.P. (1988): "Entry and structures of interest groups in assignment games", *Journal of Economic Theory* 46, 66-96

[28] Moldovanu, B. and Sela, A. (2001) "The optimal allocation of prizes in contests" *American Economic Review* 91, 542-558.

[29] Moldovanu, B. and Sela, A. (2006) "Contest architecture" *Journal of Economic Theory*, forthcoming.

[30] Peters, M. (2004) "The pre-marital investment game", Discussion paper, University of British Columbia.

[31] Rege, M. (2003) "Why do people care about social status", discussion paper, Case Western Reserve University.

[32] Rochet, J. C. and Tirole, J. (2003), "Platform competition in two-sided markets", *Journal of the European Economic Association* 1, 990-1029.

[33] Shaked, M. and Shanthikumar, J. G. (1994) *Stochastic Orders and their Applications*, Academic Press, Boston.

[34] Shapley, L.S. and Shubik, M. (1972) "The assignment game I: the core", *International Journal of Game Theory* 1, 111-130.

[35] Shimer, R. and Smith, L. (2000) "Assortative matching and search" *Econometrica* 68, 343-369.

[36] Spence, M. (1973) "Job market signaling" *Quarterly Journal of Economics* 87, 296-332.

[37] Wilson, R. (1989) "Efficient and competitive rationing", *Econometrica* 57, 1-40.

[38] Zahavi, A. (1975) "Mate selection - a selection for a handicap" *Journal of Theoretical Biology* 53, 205-214.