Games

The theory of games provides a description of games that fits common games like poker or the board game “Monopoly” but will cover many other situations as well. In any game, there is a list of players. Games generally unfold over time; at each moment in time, players have information, possibly incomplete, about the current state of play, and a set of actions they can take. Both information and actions may depend on the history of the game prior to that moment. Finally, players have payoffs, and are assumed to play in such a way as to maximize their expected payoff, taking into account their expectations for the play of others. When the players, their information and available actions, and payoffs have been specified, we have a game.

Matrix Games

The simplest game is called a matrix payoff game with two players. In a matrix payoff game, all actions are chosen simultaneously. It is conventional to describe a matrix payoff game as played by a row player and a column player. The row player chooses a row in a matrix; the column player simultaneously chooses a column. The outcome of the game is a pair of payoffs where the first entry is the payoff of the row player and the second is the payoff of the column player. Table 7 -1 provides an example of a “2 $\times$ 2” matrix payoff game, the most famous game of all, which is known as the *prisoner’s dilemma*.

Table 7-1: The Prisoner’s Dilemma

Row \ Column	Confess	Don’t
Confess	(-10,-10)	(0,-20)
Don’t	(-20,0)	(-1,-1)

In the prisoner’s dilemma, two criminals named Row and Column have been apprehended by the police and are being questioned separately. They are jointly guilty of the crime. Each player can choose either to confess or not. If Row confesses, we are in the top row of the matrix (corresponding to the row labeled Confess). Similarly, if Column confesses, the payoff will be in the relevant column. In this case, if only one player confesses, that player goes free and the other serves twenty years in jail. (The entries correspond to the number of years lost to prison. The first entry is always Row’s payoff, the second Column’s payoff.) Thus, for example, if Column confesses and Row does not, the relevant payoff is the first column and the second row, in reverse color in Table 7 -2.

Table 7-2: Solving the Prisoner's Dilemma

Row \ Column	Confess	Don’t
Confess	(-10,-10)	(0,-20)
Don’t	(-20,0)	(-1,-1)

If Column confesses and Row does not, Row loses twenty years, and Column loses no years, that is, goes free. This is the payoff (-20,0) in reverse color in Table 7-2. If both confess, they are both convicted and neither goes free, but they only serve ten years each. Finally, if neither confesses, there is a ten percent chance they are convicted anyway (using evidence other than the confession), in which case they average a year lost each.

The prisoner’s dilemma is famous partly because it is readily solvable. First, Row has a strict advantage to confessing, no matter what Column is going to do. If Column confesses, Row gets -10 from confessing, -20 from not, and thus is better off from confessing. Similarly, if Column doesn’t confess, Row gets 0 from confessing, -1 from not confessing, and is better off confessing. Either way, no matter what Column does, Row should choose to confess.[1] This is called a *dominant strategy*, a strategy that is optimal no matter what the other players do.

The logic is exactly similar for Column: no matter what Row does, Column should choose to confess. That is, Column also has a dominant strategy, to confess. To establish this, first consider what Column’s best action is, when Column thinks Row will confess. Then consider Column’s best action when Column thinks Row won’t confess. Either way, Column gets a higher payoff (lower number of years lost to prison) by confessing.

The presence of a dominant strategy makes the prisoner’s dilemma particularly easy to solve. Both players should confess. Note that this gets them ten years each in prison, and thus isn’t a very good outcome from their perspective, but there is nothing they can do about it in the context of the game, because for each, the alternative to serving ten years is to serve twenty years. This outcome is referred to as (Confess, Confess), where the first entry is the row player’s choice, and the second entry is the column player’s choice.

Consider an entry game, played by Microsoft (the row player) and Piuny (the column player), a small start-up company. Both Microsoft and Piuny are considering entering a new market for an online service.

[1] If Row and Column are friends are care about each other, that should be included as part of the payoffs. Here, there is no honor or friendship among thieves, and Row and Column only care about what they themselves will get.

The payoff structure is

Table 7-3: An Entry Game

MS \ Piuny	Enter	Don't
Enter	(2,-2)	(5,0)
Don't	(0,5)	(0,0)

In this case, if both companies enter, Microsoft ultimately wins the market, and earns 2, and Piuny loses 2. If either firm has the market to itself, they get 5 and the other firm gets zero. If neither enters, both get zero. Microsoft has a dominant strategy to enter: it gets 2 when Piuny enters, 5 when Piuny doesn't, and in both cases does better than when Microsoft doesn't enter. In contrast, Piuny does not have a dominant strategy: Piuny wants to enter when Microsoft doesn't, and vice-versa. That is, Piuny's optimal strategy depends on Microsoft's action, or, more accurately, Piuny's optimal strategy depends on what Piuny believes Microsoft will do.

Piuny can understand Microsoft's dominant strategy, if it knows the payoffs of Microsoft.[2] Thus, Piuny can conclude that Microsoft is going to enter, and this means that Piuny should not enter. Thus, the *equilibrium* of the game is for MS to enter and Piuny not to enter. This equilibrium is arrived at by the *iterated elimination of dominated strategies*, which sounds like jargon but is actually plain speaking. First, we eliminated Microsoft's *dominated strategy* in favor of its dominant strategy. Microsoft had a dominant strategy to enter, which means the strategy of not entering is dominated by the strategy of entering, so we eliminated the dominated strategy. That leaves a simplified game in which Microsoft enters:

Table 7-4; Eliminating a Dominated Strategy

MS \ Piuny	Enter	Don't
Enter	(2,-2)	(5,0)

In this simplified game, after the elimination of Microsoft's dominated strategy, Piuny also has a dominant strategy: not to enter. Thus, we *iterate* and eliminate dominated strategies again, this time eliminating Piuny's dominated strategies,

[2] It isn't so obvious that one player will know the payoffs of another player, and that often causes players to try to signal that they are going to play a certain way, that is, to demonstrate commitment to a particular advantageous strategy. Such topics are taken up in business strategy and managerial economics.

and wind up with a single outcome: Microsoft enters, and Piuny doesn't. The *iterated elimination of dominated strategies* solves the game.[3]

Here is another game, with three strategies for each player.

Table 7-5: A 3 X 3 Game

Row \ Column	Left	Center	Right
Top	(-5,-1)	(2,2)	(3,3)
Middle	(1,-3)	(1,2)	(1,1)
Bottom	(0,10)	(0,0)	(0,-10)

The process of iterated elimination of dominated strategies is illustrated by actually eliminating the rows and columns, as follows. A reverse color (white writing on black background) indicates a dominated strategy.

Middle dominates bottom for Row, yielding:

Table 7-6: Eliminating a Dominated Strategy

Row \ Column	Left	Center	Right
Top	(-5,-1)	(2,2)	(3,3)
Middle	(1,-3)	(1,2)	(1,1)
Bottom	(0,10)	(0,0)	(0,-10)

With bottom eliminated, Left is now dominated for Column by either Center or Right, which eliminates the left column.

Table 7-7: Eliminating Another Dominated Strategy

Row \ Column	Left	Center	Right
Top	(-5,-1)	(2,2)	(3,3)
Middle	(1,-3)	(1,2)	(1,1)
Bottom	(0,10)	(0,0)	(0,-10)

With Left and Bottom eliminated, Top now dominates Middle for Row.

[3] A strategy may be dominated not by any particular alternate strategy but by a randomization over other strategies, which is an advanced topic not considered here.

Table 7-8: Eliminating a Third Dominated Strategy

Column

Row		Left	Center	Right
	Top	(-5,-1)	(2,2)	(3,3)
	Middle	(1,-3)	(1,2)	(1,1)
	Bottom	(0,10)	(0,0)	(0,-10)

Finally, Column chooses Right over Center, yielding a unique outcome after the iterated elimination of dominated strategies, which is (Top, Right).

Table 7-9: Game Solved

Column

Row		Left	Center	Right
	Top	(-5,-1)	(2,2)	(3,3)
	Middle	(1,-3)	(1,2)	(1,1)
	Bottom	(0,10)	(0,0)	(0,-10)

The iterated elimination of dominated strategies is a useful concept, and when it applies, the predicted outcome is usually quite reasonable. Certainly it has the property that no player has an incentive to change their behavior given the behavior of others. However, there are games where it doesn't apply, and these games require the machinery of a *Nash equilibrium*, named for Nobel laureate John Nash (1928 –).

Nash Equilibrium

In a *Nash equilibrium*, each player chooses the strategy that maximizes their expected payoff, given the strategies employed by others. For matrix payoff games with two players, a Nash equilibrium requires that the row chosen maximizes the row player's payoff, given the column chosen by the column player, and the column, in turn, maximizes the column player's payoff given the row selected by the row player. Let us consider first the prisoner's dilemma, which we have already seen.

Table 7-10: Prisoner's Dilemma Again

Column

Row		Confess	Don't
	Confess	(-10,-10)	(0,-20)
	Don't	(-20,0)	(-1,-1)

Given that the row player has chosen to confess, the column player also chooses confession because -10 is better than -20. Similarly, given that the column player chooses confession, the row player chooses confession, because -10 is better than -20. Thus, for both players to confess is a Nash equilibrium. Now let us consider whether any other outcome is a Nash equilibrium. In any outcome, at least one player is not confessing. But that player could get a higher payoff by confessing, so no other outcome could be a Nash equilibrium.

The logic of dominated strategies extends to Nash equilibrium, except possibly for ties. That is, if a strategy is strictly dominated, it can't be part of a Nash equilibrium. On the other hand, if it involves a tied value, a strategy may be dominated but still part of a Nash equilibrium.

The Nash equilibrium is justified as a solution concept for games as follows. First, if the players are playing a Nash equilibrium, no one has an incentive to change their play or re-think their strategy. Thus, the Nash equilibrium has a "steady state" aspect in that no one wants to change their own strategy given the play of others. Second, other potential outcomes don't have that property: if an outcome is not a Nash equilibrium, then at least one player does have an incentive to change what they are doing. Outcomes that aren't Nash equilibria involve mistakes for at least one player. Thus, sophisticated, intelligent players may be able to deduce each other's play, and play a Nash equilibrium

Do people actually play Nash equilibria? This is a controversial topic and mostly beyond the scope of this book, but we'll consider two well-known games: Tic-Tac-Toe (see, e.g. http://www.mcafee.cc/Bin/tictactoe/index.html) and Chess. Tic-Tac-Toe is a relatively simple game, and the equilibrium is a tie. This equilibrium arises because each player has a strategy that prevents the other player from winning, so the outcome is a tie. Young children play Tic-Tac-Toe and eventually learn how to play equilibrium strategies, at which point the game ceases to be very interesting since it just repeats the same outcome. In contrast, it is known that Chess has an equilibrium, but no one knows what it is. Thus, at this point we don't know if the first mover (White) always wins, or the second mover (Black) always wins, or if the outcome is a draw (neither is able to win). Chess is complicated because a strategy must specify what actions to take given the history of actions, and there are a very large number of potential histories of the game thirty or forty moves after the start. So we can be quite confident that people are not (yet) playing Nash equilibria to the game of Chess.

The second most famous game in game theory is *the battle of the sexes*. The battle of the sexes involves a married couple who are going to meet each other after work, but haven't decided where they are meeting. Their options are a baseball game or the ballet. Both prefer to be with each other, but the man prefers the baseball game and the woman prefers the ballet. This gives payoffs something like this:

Table 7-11: The Battle of the Sexes

	Woman	
Man	Baseball	Ballet
Baseball	(3,2)	(1,1)
Ballet	(0,0)	(2,3)

The man would rather that they both go to the baseball game, and the woman that they both go to the ballet. They each get 2 payoff points for being with each other, and an additional point for being at their preferred entertainment. In this game, iterated elimination of dominated strategies eliminates nothing. You can readily verify that there are two Nash equilibria: one in which they both go to the baseball game, and one in which they both go to ballet. The logic is: if the man is going to the baseball game, the woman prefers the 2 points she gets at the baseball game to the single point she would get at the ballet. Similarly, if the woman is going to the baseball game, the man gets three points going there, versus zero at the ballet. Thus, for both to go to the baseball game is a Nash equilibrium. It is straightforward to show that for both to go to the ballet is also a Nash equilibrium, and finally that neither of the other two possibilities, involving not going to the same place, is an equilibrium.

Now consider the game of *matching pennies.* In this game, both the row player and the column player choose heads or tails, and if they match, the row player gets the coins, while if they don't match, the column player gets the coins. The payoffs are provided in the next table.

Table 7-12: Matching Pennies

Row \ Column	Heads	Tails
Heads	(1,-1)	(-1,1)
Tails	(-1,1)	(1,-1)

You can readily verify that none of the four possibilities represents a Nash equilibrium. Any of the four involves one player getting -1; that player can convert -1 to 1 by changing his or her strategy. Thus, whatever the hypothesized equilibrium, one player can do strictly better, contradicting the hypothesis of a Nash equilibrium. In this game, as every child who plays it knows, it pays to be unpredictable, and consequently players need to *randomize.* Random strategies are known as mixed strategies, because the players mix across various actions.

Mixed Strategies

Let us consider the matching pennies game again.

Table 7-13: Matching Pennies Again

Row \ Column	Heads	Tails
Heads	(1,-1)	(-1,1)
Tails	(-1,1)	(1,-1)

Suppose that Row believes Column plays Heads with probability p. Then if Row plays Heads, Row gets 1 with probability p and -1 with probability (1-p), for an expected value of $2p – 1$. Similarly, if Row plays Tails, Row gets -1 with probability

p (when Column plays Heads), and 1 with probability $(1-p)$, for an expected value of $1 – 2p$. This is summarized in the next table.

Table 7-14: Mixed Strategy in Matching Pennies

	Column		
Row	Heads	Tails	
Heads	(1,-1)	(-1,1)	$1p + -1(1-p)=2p-1$
Tails	(-1,1)	(1,-1)	$-1p + 1(1-p)=1-2p$

If $2p – 1 > 1 – 2p$, then Row is better off on average playing Heads than Tails. Similarly, if $2p – 1 < 1 – 2p$, Row is better off playing Tails than Heads. If, on the other hand, $2p – 1 = 1 – 2p$, then Row gets the same payoff no matter what Row does. In this case Row could play Heads, could play Tails, or could flip a coin and randomize Row's play.

A *mixed strategy Nash equilibrium* involves at least one player playing a randomized strategy, and no player being able to increase their expected payoff by playing an alternate strategy. A Nash equilibrium without randomization is called a *pure strategy Nash equilibrium.*

Note that that randomization requires equality of expected payoffs. If a player is supposed to randomize over strategy *A* or strategy *B*, then both of these strategies must produce the same expected payoff. Otherwise, the player would prefer one of them, and wouldn't play the other.

Computing a mixed strategy has one element that often appears confusing. Suppose Row is going to randomize. Then Row's payoffs must be equal, for all strategies Row plays with positive probability. But that equality in Row's payoffs doesn't determine the probabilities with which Row plays the various rows. Instead, that equality in Row's payoffs will determine the probabilities with which Column plays the various columns. The reason is that it is Column's probabilities that determine the expected payoff for Row; if Row is going to randomize, then Column's probabilities must be such that Row is willing to randomize.

Thus, for example, we computed the payoff to Row of playing Heads, which was $2p – 1$, where p was the probability Column played Heads. Similarly, the payoff to Row of playing Tails was $1 – 2p$. Row is willing to randomize if these are equal, which solves for $p = ½$.

Let q be the probability that Row plays Heads. Show that Column is willing to randomize if, and only if, $q = ½$. (Hint: First compute Column's expected payoff when Column plays Heads, and then Column's expected payoff when Column plays Tails. These must be equal for Column to randomize.)

Now let's try a somewhat more challenging example, and revisit the battle of the sexes.

Table 7-15: Mixed Strategy in Battle of the Sexes

Man \ Woman	Baseball	Ballet
Baseball	(3,2)	(1,1)
Ballet	(0,0)	(2,3)

This game has two pure strategy Nash equilibria: (Baseball,Baseball) and (Ballet,Ballet). Is there a mixed strategy? To compute a mixed strategy, let the Woman go to the baseball game with probability p, and the Man go to the baseball game with probability q. Table 7 -16 contains the computation of the mixed strategy payoffs for each player.

Table 7-16: Full Computation of the Mixed Strategy

Man \ Woman	Baseball (p)	Ballet (1-p)	Man's E Payoff
Baseball (prob q)	(3,2)	(1,1)	$3p + 1(1\text{-}p)=1+2p$
Ballet (prob 1-q)	(0,0)	(2,3)	$0p + 2(1\text{-}p)=2\text{-}2p$
Woman's E Payoff	$2q + 0(1\text{-}q)=2q$	$1q + 3(1\text{-}q)=3\text{-}2q$	

For example, if the Man (row player) goes to the baseball game, he gets 3 when the Woman goes to the baseball game (probability p) and otherwise gets 1, for an expected payoff of $3p + 1(1\text{-}p) = 1 + 2p$. The other calculations are similar but you should definitely run through the logic and verify each calculation.

A mixed strategy in the Battle of the Sexes game requires both parties to randomize (since a pure strategy by either party prevents randomization by the other). The Man's indifference between going to the baseball game and the ballet requires $1+2p = 2 - 2p$, which yields $p = \frac{1}{4}$. That is, the Man will be willing to randomize which event he attends if the Woman is going to the ballet ¾ of the time, and otherwise to the baseball game. This makes the Man indifferent between the two events, because he prefers to be with the Woman, but he also likes to be at the baseball game; to make up for the advantage that the game holds for him, the woman has to be at the ballet more often.

Similarly, in order for the Woman to randomize, the Woman must get equal payoffs from going to the game and going to the ballet, which requires $2q = 3 - 2q$, or $q = \frac{3}{4}$. Thus, the probability that the Man goes to the game is ¾, and he goes to the ballet ¼ of the time. These are independent probabilities, so to get the probability that both go to the game, we multiply the probabilities, which yields $\frac{3}{16}$. The next table fills in the probabilities for all four possible outcomes.

Table 7-17: Mixed Strategy Probabilities

Man \ Woman	Baseball	Ballet
Baseball	$\frac{3}{16}$	$\frac{9}{16}$
Ballet	$\frac{1}{16}$	$\frac{3}{16}$

Note that more than half the time, (Baseball, Ballet) is the outcome of the mixed strategy, and the two people are not together. This lack of coordination is a feature of mixed strategy equilibria generally. The expected payoffs for both players are readily computed as well. The Man's payoff was $1+2p = 2 – 2p$, and since $p = ¼$, the Man obtained 1 ½. A similar calculation shows the Woman's payoff is the same. Thus, both do worse than coordinating on their less preferred outcome. But this mixed strategy Nash equilibrium, undesirable as it may seem, is a Nash equilibrium in the sense that neither party can improve their payoff, given the behavior of the other party.

In the Battle of the sexes, the mixed strategy Nash equilibrium may seem unlikely, and we might expect the couple to coordinate more effectively. Indeed, a simple call on the telephone should rule out the mixed strategy. So let's consider another game related to the Battle of the Sexes, where a failure of coordination makes more sense. This is the game of "Chicken." Chicken is played by two drivers driving toward each other, trying to convince the other to yield, which involves swerving into a ditch. If both swerve into the ditch, we'll call the outcome a draw and both get zero. If one swerves and the other doesn't, the swerver loses and the other wins, and we'll give the winner one point.[4] The only remaining question is what happens when both don't yield, in which case a crash results. In this version, that has been set at four times the loss of swerving, but you can change the game and see what happens.

Table 7-18: Chicken

Row \ Column	Swerve	Don't
Swerve	(0,0)	(-1,1)
Don't	(1,-1)	(-4,-4)

This game has two pure strategy equilibria: (Swerve, Don't) and (Don't, Swerve). In addition, it has a mixed strategy. Suppose Column swerves with probability p. Then Row gets $0p + -1(1-p)$ from swerving, $1p + (-4)(1-p)$ from not swerving, and Row will randomize if these are equal, which requires $p = ¾$. That is, the probability that Column swerves, in a mixed strategy equilibrium is ¾. You can

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[4] Note that adding a constant to a player's payoffs, or multiplying that player's payoffs by a positive constant, doesn't affect the Nash equilibria, pure or mixed. Therefore, we can always let one outcome for each player be zero, and another outcome be one.

verify that the Row player has the same probability by setting the probability that Row swerves equal to *q* and computing Column's expected payoffs. Thus, the probability of a collision is $\frac{1}{16}$ in the mixed strategy equilibrium.

The mixed strategy equilibrium is more likely in some sense in this game; if the players already knew which player would yield, they wouldn't actually need to play the game. The whole point of the game is to find out who will yield, which means it isn't known in advance, which means the mixed strategy equilibrium is in some sense the more reasonable equilibrium.

Paper, Scissors, Rock is a child's game in which two children simultaneously choose paper (hand held flat), scissors (hand with two fingers protruding to look like scissors) or rock (hand in a fist). The nature of the payoffs is that paper beats rock, rock beats scissors, and scissors beat paper. This game has the structure

Table 7-19: Paper, Scissors, Rock

Row \ Column	Paper	Scissors	Rock
Paper	(0,0)	(-1,1)	(1,-1)
Scissors	(1,-1)	(0,0)	(-1,1)
Rock	(-1,1)	(1,-1)	(0,0)

Show that, in the Paper, Scissors, Rock game, there are no pure strategy equilibria. Show that playing all three actions with equal likelihood is a mixed strategy equilibrium.

Find all equilibria of the following games:

1

Row \ Column	Left	Right
Up	(3,2)	(11,1)
Down	(4,5)	(8,0)

2

Row \ Column	Left	Right
Up	(3,3)	(0,0)
Down	(4,5)	(8,0)

3

Row	Column	
	Left	Right
Up	(0,3)	(3,0)
Down	(4,0)	(0,4)

4

Row	Column	
	Left	Right
Up	(7,2)	(0,9)
Down	(8,7)	(8,8)

5

Row	Column	
	Left	Right
Up	(1,1)	(2,4)
Down	(4,1)	(3,2)

6

Row	Column	
	Left	Right
Up	(4,2)	(2,3)
Down	(3,8)	(1,5)

Examples

Our first example concerns public goods. In this game, each player can either contribute, or not. For example, two roommates can either clean their apartment, or not. If they both clean, the apartment is nice. If one cleans, that roommate does all the work and the other gets half of the benefits. Finally, if neither clean, neither is very happy. This suggests payoffs like:

Table 7-20: Cleaning the Apartment

Row	Column	
	Clean	Don't
Clean	(10,10)	(0,15)
Don't	(15,0)	(2,2)

You can verify that this game is similar to the prisoner's dilemma, in that the only Nash equilibrium is the pure strategy in which neither player cleans. This is a game theoretic version of the tragedy of the commons – even though the roommates would both be better off if both cleaned, neither do. As a practical matter, roommates do solve this problem, using strategies that we will investigate when we consider dynamic games.

Table 7-21: Driving on the Right

Row \ Column	Left	Right
Left	(1,1)	(0,0)
Right	(0,0)	(1,1)

The important thing about the side of the road the cars drive on is not that it is the right side but that it is the *same* side. This is captured in the Driving on the Right game above. If both players drive on the same side, then they both get one point, otherwise they get zero. You can readily verify that there are two pure strategy equilibria, (Left,Left) and (Right,Right), and a mixed strategy equilibrium with equal probabilities. Is the mixed strategy reasonable? With automobiles, there is little randomization. On the other hand, people walking down hallways often seem to randomize whether they pass on the left or the right, and sometimes do that little dance where they try to get past each other, one going left and the other going right, then both simultaneously reversing, unable to get out of each other's way. That dance suggests that the mixed strategy equilibrium is not as unreasonable as it seems in the automobile application.[5]

Table 7-22: Bank Location Game

LA \ NYC	No Concession	Tax Rebate
No Concession	(30,10)	(10,20)
Tax Rebate	(20,10)	(20,0)

Consider a foreign bank that is looking to open a main office and a smaller office in the United States. The bank narrows its choice for main office to either New York (NYC) or Los Angeles (LA), and is leaning toward Los Angeles. If neither city does anything, LA will get $30 million in tax revenue and New York ten million. New York, however, could offer a $10 million rebate, which would swing the main office to New York, but now New York would only get a net of $20 M. The discussions are carried on privately with the bank. LA could also offer the

[5] Continental Europe drove on the left until about the time of the French revolution. At that time, some individuals began driving on the right as a challenge to royalty who were on the left, essentially playing the game of chicken with the ruling class. Driving on the right became a symbol of disrespect for royalty. The challengers won out, forcing a shift to driving on the right. Besides which side one drives on, another coordination game involves whether one stops or goes on red. In some locales, the tendency for a few extra cars to go as a light changes from green to yellow to red forces those whose light changes to green to wait, and such a progression can lead to the opposite equilibrium, where one goes on red and stops on green. Under Mao Tse-tung, the Chinese considered changing the equilibrium to going on red and stopping on green (because 'red is the color of progress') but wiser heads prevailed and the plan was scrapped.

concession, which would bring the bank back to LA.

Verify that the bank location game has no pure strategy equilibria, and that there is a mixed strategy equilibrium where each city offers a rebate with probability ½.

Table 7-23: Political Mudslinging

Dem \ Republican	Clean	Mud
Clean	(3,1)	(1,2)
Mud	(2,1)	(2,0)

On the night before the election, a Democrat is leading the Wisconsin senatorial race. Absent any new developments, the Democrat will win, and the Republican will lose. This is worth 3 to the Democrat, and the Republican, who loses honorably, values this outcome at one. The Republican could decide to run a series of negative advertisements ("throwing mud") against the Democrat, and if so, the Republican wins although loses his honor, which he values at 1, and so only gets 2. If the Democrat runs negative ads, again the Democrat wins, but loses his honor, so only gets 2. These outcomes are represented in the Mudslinging game above.

Show that the only Nash equilibrium is a mixed strategy with equal probabilities of throwing mud and not throwing mud.

Suppose that voters partially forgive a candidate for throwing mud when the rival throws mud, so that the (Mud, Mud) outcome has payoff (2.5,.5). How does the equilibrium change?

You have probably had the experience of trying to avoid encountering someone, who we will call Rocky. In this instance, Rocky is actually trying to find you. The situation is that it is Saturday night and you are choosing which party, of two possible parties, to attend. You like party 1 better, and if Rocky goes to the other party, you get 20. If Rocky attends party 1, you are going to be uncomfortable and get 5. Similarly, Party 2 is worth 15, unless Rocky attends, in which case it is worth 0. Rocky likes Party 2 better (these different preferences may be part of the reason you are avoiding him) but he is trying to see you. So he values Party 2 at 10, party 1 at 5 and your presence at the party he attends is worth 10. These values are reflected in the following table.

Table 7-24: Avoiding Rocky

You \ Rocky	Party 1	Party 2
Party 1	(5,15)	(20,10)
Party 2	(15,5)	(0,20)

(i) Show there are no pure strategy Nash equilibria in this game. (ii) Find the mixed strategy Nash equilibria. (iii) Show that the probability you encounter Rocky is $7/12$.

Our final example involves two firms competing for customers. These firms can either price high or low. The most money is made if they both price high, but if one prices low, it can take most of the business away from the rival. If they both price low, they make modest profits. This description is reflected in the following table:

Table 7-25: Price Cutting Game

Firm 1 \ Firm 2	High	Low
High	(15,15)	(0,25)
Low	(25,0)	(5,5)

Show that the firms have a dominant strategy to price low, so that the only Nash equilibrium is (Low, Low).