## Assignment 2

Spring 2020
(1) Find all the equilibria for the following games.

| $(3,2)$ | $(11,1)$ | $(7,2)$ | $(0,9)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(4,5)$ | $(8,0)$ | $(3,3)$ | $(0,0)$ |
| $(1,1)$ | $(2,4)$ | $(8,7)$ | $(8,8)$ |
| $(10,10)$ | $(0,15)$ |  |  |
| $(4,1)$ | $(3,2)$ | $(4,5)$ | $(8,0)$ |
| $(15,0)$ | $(2,2)$ | $(2,3)$ |  |

(2) Starting with our original network game

| $\left(\frac{1}{2}, \frac{1}{2}\right)$ | $(1, \beta)$ |
| :--- | :--- |
| $(\beta, 1)$ | $\left(\frac{\beta}{2}, \frac{\beta}{2}\right)$ |

suppose that the network administrator has to decide whether to replace the existing router (the one that gets packets through with probability $\beta$ ) with a new router that gets packets through for sure (as long as two packets don't arrive at the same time), or whether to add a third router that gets packets through with probability $\beta$. Suppose the cost of those two options is the same. Which of the two options will will achieve a higher expected number of packets through the network under anarchy (websites choose their own strategies). How does your answer depend on $\beta$.
(3) Find the payoffs in the following game for players 1 and 2.


Figure 1: Problem set 3
Give the normal form payoffs in this game.
(4) Find all the equilibria in this game

| $(1,3)$ | $(3,2)$ |
| :--- | :--- |
| $(4,1)$ | $(2,4)$ | and contrast them with the payoffs when row moves first, column moves first.

(5) (Imperfect information) In this game, we'll consider the router choice problem again. As before, two computers will have the choice between to routers. Each router randomly selects a packet to transmit from the packets they receive. The first router (Router 1) directs its packet on and sends it successfully with probability 1. The second router (Router 2) directs its packet onward, but the packet is lost with probability $(1-\beta)$ just as before. The change here is that routers 1 and 2 will also have to decide where to direct their packets. They will have two choices, Router 3 and Router 4. Router 3 will randomly select a packet and retransmit it with probablity 1 (dropping any other packets it has received). Router 4 will randomly select a packet and transmit it. This transmission will be successful with probability $\beta$.
Let $\pi$ be the probablity with which each computer sends their packet to Router 1. Let $\rho$ be the probablity with which Router 1 tries to transmit its packet to Router 3, and let $\gamma$ be the probability with which Router 2 tries to transmit its packet to Router 3. You can assume that both Router 1 and 2 want to maximize the probability with which their packets are transmitted on to the internet.
The problem is to find the values of $\pi, \rho$ and $\gamma$. You should assume that when Router 1 and Router 2 decide where to send their packet they know whether or not they have dropped a packet. In other words, if Router 1 has a packet to transmit from some computer, it knows whether it has dropped the other packet, or whether it hasn't (in which case the packet was sent to the other router).

