Public Goods

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December 1, 2008

1 Introduction

A *public good* is a good having the property that the total output of the good is enjoyed by everyone. In contrast, a *private good* has the property that if I consume it, you cannot. Some of the more important goods in the modern economics are actually public. For example, an idea or a bit of information is a public good. Whether or not you know the idea, or have the information does not impact on my ability to know the same thing or have the same information (though whether or not you have the same information as I do may determine whether or not I can make money off you). A music file shared on my computer is a public good – if you take it from me, my ability to enjoy it is not affected at all. A theorem, like the first welfare theorem that we studied last week, is a public good: I do not forget it when you learn it. Contrast this with my car or my lunch which are both private goods: if you take them from me I cannot enjoy them at all.

This note describes an equilibrium for the *voluntary contribution game*, which is a common way to think about how public goods are provided. It explains why the amount of the public good in the voluntary contribution game is underproduced (because the outcome is not Pareto optimal: there is another outcome that will make everyone better off). This note then explains how public goods can be priced to ensure Pareto optimal supply.

1.1 The voluntary contribution game

Suppose there are two goods, one public (y) and one private (x). Let $f(x)$ denote the amount of the public good that can be produced from x units of the private good. Suppose there are two consumers with utility functions $u_1(x, y)$ and $u_2(x, y)$ respectively. Their endowments of the private good are ω_1 and ω_2. The set of points $\{(x, y) : y = f(\omega_1 + \omega_2 - x)\}$ is the *production possibilities frontier*.

If the first consumer decides to consume x_1 (and devote the rest of his endowment ω_1 to production of the public good) while consumer 2 decides to consume x_2 the utilities of each of the consumers are given by

$$u_1(x_1, f(\omega_1 + \omega_2 - x_1 - x_2))$$

for consumer 1 and

$$u_2\left(x_2, f\left(\omega_1+\omega_2-x_1-x_2\right)\right)$$

for consumer 2. The important point is that if consumer 1 say decides to consume a bit less of the private good and produce a bit more of the public good, then consumer 2 will enjoy the additional public good too without any cost at all.

The voluntary contribution game equilibrium is one way to predict how much of the private good each consumer will choose. Each of the consumers simply picks the amount of the private good they want on their own. This is a bit hard to do because the amount that each consumer will choose to contribute depends on how much they expect the other consumer to contribute. A good way to do this is to use a *Nash equilibrium* in place of a Walrasian equilibrium. Instead of taking prices to be fixed, each consumer takes the contribution of the other consumer to be fixed and chooses the contribution that maximizes his utility given this expectation. In a Walrasian equilibrium, when a consumer acts as if he believes that prices are fixed, he has to be physically able to purchase the bundle that maximizes his utility at these prices. That is why the price expectations have to be such that markets clear. Analogously, when the consumer chooses his optimal contribution given a fixed expectation about the contribution of the other consumer, he has to end up with exactly the amount of the public good that he expected to get.

Formally, a Nash equilibrium for the voluntary contribution game is a pair of private consumptions x_1^* and x_2^* such that

$$u_1\left(x_1^*, f\left(\omega_1+\omega_2-x_1^*-x_2^*\right)\right) \geq u_1\left(x', f\left(\omega_1+\omega_2-x'-x_2^*\right)\right)$$

for any alternative contribution $x' \in [0, \omega_1]$ and

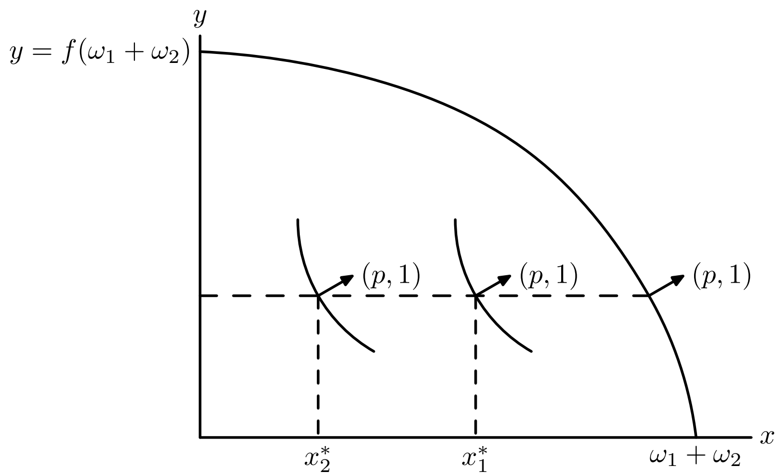
$$u_2\left(x_2^*, f\left(\omega_1+\omega_2--x_1^* x_2^*\right)\right) \geq u_2\left(x', f\left(\omega_1+\omega_2-x_1^*-x'\right)\right)$$

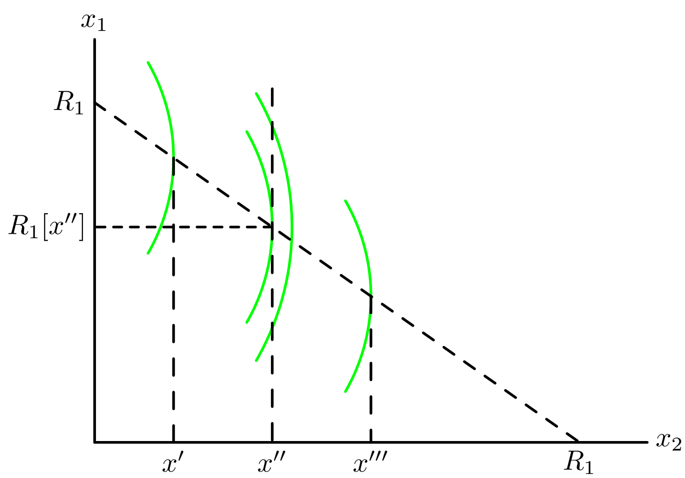
for any alternative contribution $x' \in [0, \omega_2]$.

One way to view the outcome of this game is given in Figure 1 where the two consumers choose x_1^* and x_2^*. The 'budget line' that consumer 1 faces, for example, when consumer 2 chooses consumption x_2^* is the set of all pairs $\{(x_1, y) : y = f\left(\omega_1+\omega_2-x_1-x_2^*\right)\}$. The slope of this is exactly the same as the slope of the production possibilities frontier at the point $(x_1^*+x_2^*, y^*)$. The same is true for consumer 2. So, in the equilibrium of the voluntary contribution game, each consumer has the same marginal rate of substitution and the same marginal rate of transformation in production.

With private goods, this is exactly what you want. Recall that, in the Edgeworth box, both consumers' indifference curves were tangent and the common slope of their indifference curves was equal to the slope of the production possibilities frontier. With public goods, this is not the outcome that you want.

An alternative approach – that helps to explain how contributions are determined and why these contributions are not Pareto optimal – is to try find the best choice for consumer 1 to make for all the different possible contribution





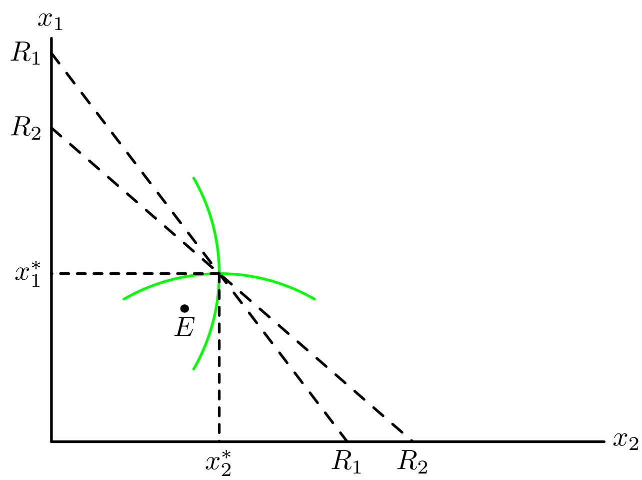
levels that consumer 2 might choose. This approach is more common in game theory and involves the construction of something called the *best reply function*. The best reply functions are put together to understand the final outcome.

In Figure 2, the various *consumption choices* that player 2 can make are given along the bottom axis. Each such choice implies a contribution to the production of the public good - just take the difference between the consumption level and the endowment to find it (this figure would look a bit different if player 2's *contributions* to production of the public good were listed on the bottom axis – that approach is more common).

The green lines represent iso-utility curves for consumer 1. They are solutions to equations of the form

$$u_1\left(x_1, f\left(\omega_1 + \omega_2 - x_1 - x_2\right)\right) = K$$

where K is some constant. Consumer 1 achieves higher utility (holding his own



consumption level constant) the *lower* is the consumption level of consumer 2. Lower consumption by consumer 2 means that consumer 2 is contributing more of her endowment to produce the public good.

In the Nash equilibrium, consumer 1 forms some belief about the consumption level that consumer 2 will pick. Suppose for the moment, that he believes that consumer 2 will choose consumption x''. Then he can attain any (x_1, x_2) combination that lies on the vertical line through x''. The best such point is the one that lies on the highest iso-utility curve. That is the one through the point $(R_1[x''], x'')$ where an iso-utility curve is just tangent to this vertical line. (If he were to lower his planned consumption by moving down this vertical line, he would end up on a lower iso-utility curve like the one that lies just to the right of the point $(R_1[x''], x'')$.

There would be a different best choice like this for every different choice that consumer 2 makes. The picture shows the corresponding tangencies at point x' and x'''. If you joined all these best choices together they would form a line (not necessarily straight as it is in the picture) called consumer 1's reaction function. This is the line R_1R_1 in the picture. It explains what consumer 1 would choose to do for every possible different belief that he might have about the consumption choice of consumer 2.

Doing the same exercise for consumer 2 yields a similar curve, which is drawn in Figure 3 as R_2R_2. The point where these two curves intersect is the Nash equilibrium. Each consumer chooses his best consumption given what he expects the other consumer to choose; and, as it turns out, the other consumer always does exactly what he expects him to do.

When you try to construct the iso-utility curves for person 2, he will choose one that is tangent to the flat line through x_1^* since he expects consumer 1's consumption choice to x_1^* no matter what he does. This means that the iso-utility curves for the two consumers must cut through each other as shown in the diagram. There must be a point like E where both of the consumers would be better off if they could jointly agree to move there. That would involve each of them reducing their own consumption of the private good and using that to increase production of the public good.

2 Resolving the Public Good Problem

There is a rather surprising way to resolve this problem. The entire difficulty with public goods arises because a consumer who raises his or her contribution to the public good increases the utility of the other trader. To get consumers to choose the right amounts something has to be done to 'internalize' this externality. Here is one way to do it.

First declare that there are actually 3 goods, y_1, y_2, and x. The first is public good for person 1, the second public good for person 2 and the third the private good. All production of these three goods will be undertaken by a single profit-maximizing firm whose production possibilities frontier is just

$$\{(y_1, y_2, x) : y_1 = y_2 = f(\omega_1 + \omega_2 - x)\}$$

All endowments are owned by the firm, but consumer 1 owns the share $\omega_1/(\omega_1 + \omega_2)$ of the firm and will receive that share of its profits. Consumer 2 will own the complementary share $\omega_2/(\omega_1 + \omega_2)$. We will make one big assumption, which is that this single firm is a price-taker.

Consumer 1 only cares about his consumption of the private good and his consumption of good 1; consumer 2 only cares about her consumption of the private good and good 2. Consumer 1 doesn't care how much y_2 consumer 2 consumes, so there are no externalities in consumption. The firm 'internalizes' all the externalities associated with the public good. The physical connection between y_1 and y_2 is simply a part of its production process, so there are no production externalities. So, all we need to do is to find the Walrasian equilibrium of this economy with production and that will give us a Pareto optimal allocation by the first welfare theorem that we studied last week. The solution is given in Figure 4.

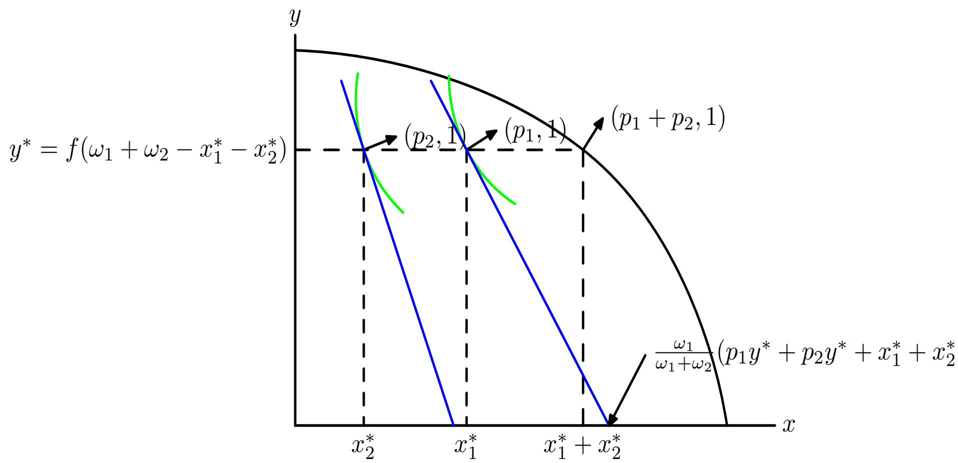
Notice that, when the firm increases production of the public good, it receives revenue twice on each unit it produces. Consumer 1 pays p_1 for that unit, but consumer 2 also pays p_2 for it. So, the iso-profit curve for the profit maximizing firm that must be tangent to the production possibilities frontier has slope $\frac{1}{p_1+p_2}$. The firm earns its profits for its production decision then distributes these profits to its owners. Consumer 1 receives income

$$\frac{\omega_1}{\omega_1 + \omega_2}(p_1 y^* + p_2 y^* + x_1^* + x_2^*)$$

which is labelled on the horizontal axis in Figure 4 . Consumer 2 receives

$$\frac{\omega_2}{\omega_1 + \omega_2}(p_1 y^* + p_2 y^* + x_1^* + x_2^*)$$

which is the point where 2's budget line intersects the horizontal axis (that has not been labeled in the figure to keep things simpler).



Consumer 1 now faces a budget line (the blue line in the picture) along which he chooses his best consumption bundle. Notice that since consumer 1 only buys good y_1 at price p_1 – the slope of this budget line is $\frac{1}{p_1}$ *not* $\frac{1}{p_1+p_2}$. So in this equilibrium, consumers marginal rates of substitution will be different from the firm's marginal rate of substitution and generally different from each other.

The market clearing conditions are twofold. First, consumers must both choose to purchase the common level of output of the public good that has been offered by the firm. Second, the sum of the private good demand of each consumer must be equal to the total amount of the private good that the firm has chosen to produce.

The prices that support this outcome are often know as Lindahl prices. The reciprocal of the slope of the production possibilities frontier is the marginal cost of producing one extra unit of the public good (expressed in terms of units of the private good). Since the iso-profit line must be tangent to the production possibilities curve, this marginal cost is equal to $p_1 + p_2$. The reciprocal of the slopes of the consumers indifference curves are equal to their marginal willingness to pay for the public good (again expressed in terms of the private good). Since the indifference curves are tangent to the individual budget lines, these willingnesses to pay are p_1 and p_2 respectively. So, the Lindahl prices ensure that the marginal cost of producing the public good is exactly equal to the sum of the two consumers' willingness to pay.