Insurance

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1 Introduction

In this chapter, we study a very simple model of insurance using the ideas and concepts developed in the chapter on risk aversion. You may recall from the previous chapter, the concept of a *lottery*: a collection of outcomes that are each assigned a different probability of occurring. We are going to assume that the independence axiom holds, and that our consumers and decision makers have preferences that are linear in the probabilities with which these different outcomes occur.

Expected utility says that we should start with a list of all possible *outcomes*. Let's just think of the set of outcomes as a finite set S (later on we will refer to them as *states*). Sometimes, we let S refer to the number of outcomes in this set. We let s refer to a particular outcome in this set. A lottery is a vector of probabilities p consisting of S non-negative components, the sum of which must equal 1. The expected utility theorem says that there is a vector of real numbers $\{u_1, \ldots, u_S\}$ such that two lotteries p and q can be ranked in the following way: $p \succeq q$ if and only if

$$\sum_{s=1}^{S} p_s u_s \geqslant \sum_{s=1}^{S} q_s u_s$$

In most of what we do in economics, including the insurance problem, we fix the probabilities p then vary the numbers u_s in some fashion. This is exactly what we are going to do here with the insurance problem.

On the surface, the expected utility theorem doesn't seem to support this approach. In fact, it is a very restrictive special case which you can understand if you think of the set of outcomes as a continuum, and the set of lotteries as a set of probability measures on this continuum. At the moment, the details of this argument aren't so important. All you need to remember is that we will use the expected utility theorem to support the arguments here.

Let's apply these ideas to insurance. We start with a consumer who has an income y but who expects to have an accident with some probability p. The accident has a known monetary cost d. The consumer deals with a competitive insurance company which sells policies to many similar consumers and knows the probability with which the consumer has an accident.

A good example might be a farmer whose fields produce a yearly income y unless there is a late frost which kills off some of his crop. The probability of a late frost is p. A bad example would be car insurance. There are a couple of reasons why. The first is that the probability of an accident is something that a driver has some control over. If you are insured against the costs of an accident, then there is little incentive to take care. This is referred to as *moral hazard*. A farmer, on the other hand, has little control over the weather. Drivers also have a lot of information about their own accident probability that the insurance company doesn't. The weather, on the other hand, is unpredictable, but in a way that everyone agrees.

The insurance company will sell a *policy* to the consumer. A policy is a premium q and a *net* benefit b. The net benefit is the difference between the gross benefit B and the premium q. The way the insurance policy works is that the consumer pays the insurance company q up front. Then, if (and only if) the consumer has an accident, the insurance company pays the consumer B. Equivalently, if the consumer has an accident, the insurance company pays back the premium q and gives the consumer an additional b to make up for her loss.

The insurance company is doing this with many different consumers and is willing to offer the consumer any policy that breaks even in the sense that the company's expected profit from the policy is zero. The expected profit to the company from a policy with premium q and net benefit b is

$$(1-p)q - pb$$

Our consumer has expected utility preferences, so if she buys a policy with premium q and net benefit b, her expected utility is

$$(1-p)u(y-q) + qu(y-d+b)$$

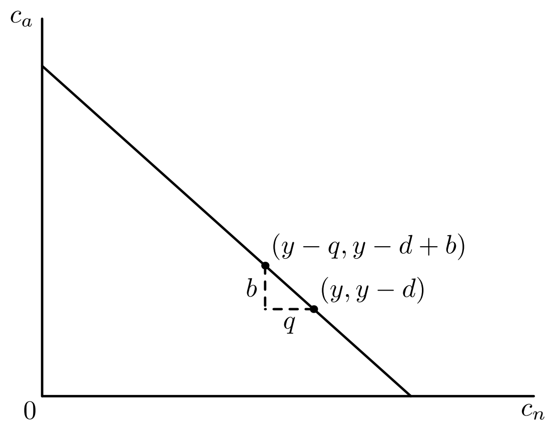

Figure 1: Budget Set

The consumer's problem is to buy a policy that maximizes her expected utility.

Let's solve the problem two different ways. First, we use the graphical approach. To see this approach, we can convert the problem into something that looks exactly like the consumer problems we have already studied.

The horizontal axis in the diagram measures the amount of income the consumer enjoys in the event that she does not have an accident. If she doesn't buy insurance, this will just be y. If she does buy insurance, it will be y less the premium that she pays. The vertical axis measures the level of consumption in the event that our consumer does have an accident. In this diagram, the consumer's *endowment* is the point $(y, y-d)$. The consumer can switch from her endowment to the consumption pair $(y-q, y-d+b)$ by buying a policy with premium q and net benefit b.

In the diagram, I have drawn this as if it created an entire budget line for the consumer. The reasoning is as follows: the insurance company will presumably be willing to sell the consumer *any* policy that generates zero expected profit, i.e., any policy that satisfies $(1-p)q - pb = 0$, or $b = q\frac{1-p}{p}$. The set of all such policies generates a feasible set that passes through the endowment point (since the policy $(0, 0)$ obviously gives the insurance company zero expected profit) and has slope $-\frac{1-p}{p}$.

The consumer's indifference curves are made up of all the pairs (c_n, c_a) that yield the same level of expected utility. So for example, the indifference

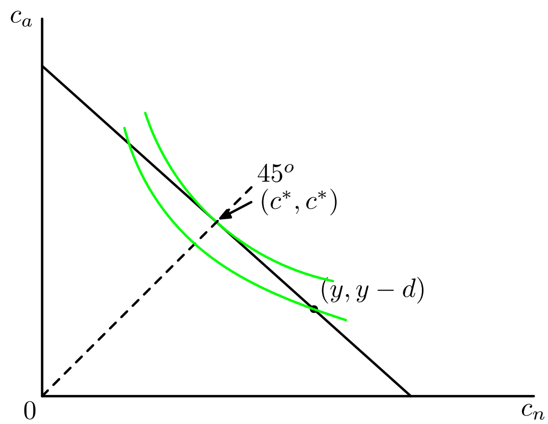

Figure 2: Constant Consumption

curve through the endowment is given by the set of solutions to

$$(1-p)u(c_n) + pu(c_a) = (1-p)u(y) + pu(y-d) \tag{1}$$

Using the method of total differentials, the slope of the indifference curve can be derived by solving

$$(1-p)u'(c_n)dc_n + pu'(c_a)dc_a = 0 \tag{2}$$

or

$$\frac{dc_a}{dc_n} = -\frac{(1-p)u'(c_n)}{pu'(c_a)} \tag{3}$$

A very nice property of expected utility preferences is that if $c_a = c_n$, then $u'(c_a) = u'(c_n)$. If, in addition, the function u is concave (has a decreasing first derivative), then the indifference curves will appear as convex curves as in the following picture:

Since the slope of every indifference curve is $-\frac{1-p}{p}$ on the 45^o line, the highest indifference curve's tangency point to the budget set will occur on the 45^o line. The nice implication is that a risk averse consumer who can buy insurance at 'actuarially fair' prices (i.e., prices that give the insurance company zero expected profit) will buy insurance up to the point where his consumption is the same whether or not she has an accident.

To be complete, let's solve this as well using Lagrangian methods because it proves insightful. First, the consumers problem (at least when dealing with a fair insurance company) is to maximize

$$(1-p)u(y-q) + pu(y-d+b) \tag{4}$$

by choosing a premium and benefit package (q, b) subject to the constraint that

$$(1-p)q - pb = 0 \tag{5}$$

Notice that there is nothing here about the premium and benefit being positive. It is conceivable that the consumer might want to bet with another consumer that she would *not* have an accident. In that case, she would receive money from the other consumer when she didn't have an accident, which coincides with a negative value for q. Then, to satisfy the constraint, she would have to pay out in the event that she did have an accident.

Since there are no inequality constraints, our Lagrangian theorem says that at the optimal solution, there will be a multiplier λ such that

$$-(1-p)u'(y-q) + \lambda(1-p) = 0 \tag{6}$$

and

$$pu'(y-d+b) - \lambda p = 0 \tag{7}$$

The probabilities cancel in this expression, so we get $u'(y-q) = u'(y-d+b) = \lambda$. If we assume that the marginal utility of income is monotonically declining, then this can only occur when $y - q = y - d + b$.